GREEN VIEW PUBLIC SCHOOL VACATION 2024-25 HOLIDAY

HOME WORK-MATHS-12th

CH-1 Relation and functions

Q.1 Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2 \textit{ divides } a - b\}$. Show that the relation R transitive? Write the equivalence class [0]

Q.2 Show that the relation S in the set $A = \{x \epsilon Z: 0 \leq x \leq 12\}$ given by $S = \{(a, b): a, b \in Z, |a - b| \textit{ is divisible by } 4\}$ is an equivalence relation. Also find the equivalence class [1]. Show that the relation R defined by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation. Also, find the equivalence classes [(2,3)] and [(1,3)].

Q.3Show that the relation R in the set R of real numbers, defined as $R = \{(a, b): a \leq b\ 2\ \}$ neither

reflexive nor symmetric nor transitive.

Q.4Let f : {2, 3, 4, 5} → {3, 4, 5, 9} and g : {3, 4, 5, 9} → {7, 11, 15} be functions defined as $f(2) = 3$, $f(3) = 4$, $f(4) = f(5) = 5$ and 2 VINOD AV | JNV THRISSUR $g(3) = g(4) = 7$ and $g(5) = g(9) = 11$. Find gof.

Q.5 Determine whether each of the following relations are reflective, symmetric and transitive.

(i) Relation R on the set A = {1, 2, 3,………, 13, 14} defined as
R = {(x, y) : 3x – y = 0}

(ii) Relation R on the set N of all natural numbers defined as
R = {(x, y) : y = x + 5 and x < 4}

(iii) Relation R on the set A = {1, 2, 3, 4, 5, 6} defined as
R = {(x, y) : y is divisible by x}

(iv) Relation R on the set Z of all integer by x}
R = {(x, y) : x – y is an integer}

Q.6 Show that the function f : R → R given by f(x) = ax + b, where a, b □ R, a □ 0 is a bijection.

Q.7 Show that the function f : R → R given by f(x) = cos x for all x □ R, is neither one-one nor onto.

Q 8.Let A = R – {2} and B = R – {1}. If f : A → B is a mapping defined by f(x)= $\frac{x-1}{x-2}$

show that f is bijective.

Q.9 Classify the following functions as injection, surjection or bijection:

(i) f : N → N given by f(x) = x2
(ii) f : Z → Z given by f(x) = x
(iii) f : N → N given by f(x) = x3
(iv) f : Z → Z given by f(x) = x3
(v) f : R → R, defined by f(x) = | x |
(vi) f : Z → Z, defined by f(x) = x2 + x

(vii) $f : Z \to Z$, defined by $f(x) = x - 5$

(viii) $f : R \to R$, defined by $f(x) = \sin x$

(ix) $f : R \to R$, defined by $f(x) = x3 + 1$

(x) $f : R \to R$, defined by $f(x) = x3-x$

(xi) $f : R \to R$, defined by $f(x) = \sin x + \cos x$

(xii) $f : Q - \{3\} \to Q$, defined by $f(x) = \frac{2x+3}{x-3}$

(xiii) $f : Q \to Q$, defined by $f(x) = x3 + 1$

(xiv) $f : R \to R$, defined by $f(x) = 5x3 + 4$

(xv) $f : R \to R$, defined by $f(x) = 3 - 4x$

(xvi) $f : R \to R$, defined by $f(x) = 1 + x2$

Q.10 Let A = {1,2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not.

CH-2 Inverse Trigonometry

Q.1 For the principal values, evaluate the following :

(i) $\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}}$

(ii) $\sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(iii) $\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

(iv) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Q.2 Evaluate each of the following :

(i) $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

(ii) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$

(iii) $\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

(iv) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

(v) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(vi) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Q.3 Express each of the following in the simplest form :

(i) $\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), -\pi < x < \pi$

(ii) $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$

(iii) $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$

(iv) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$

Q.4 Prove that :

$$\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x2}\right) = \tan^{-1}\left(\frac{3x-x3}{1-3x2}\right), |x| < \frac{1}{\sqrt{3}}$$

Q.5 Prove that :

(i) $\tan^{-1}\left[\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right] = \frac{\pi}{4} - \frac{x}{2}$, if $\pi < x < \frac{3\pi}{3}$

(ii) $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{\pi}{2} - \frac{x}{2}$, if $\frac{\pi}{2} < x < \pi$

Q.6 Prove that :

(i) $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{\dfrac{x^2+1}{x^2+2}}$ (ii) $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\dfrac{x^2+1}{x^2+2}}$

Q.7 Prove the following :

(i) $4\tan^{-1}\dfrac{1}{5} - \tan^{-1}\dfrac{1}{70} + \tan^{-1}\dfrac{1}{99} = \dfrac{\pi}{4}$ (ii) $2\tan^{-1}\dfrac{1}{5} + \sec^{-1}\dfrac{5\sqrt{2}}{7} + 2\tan^{-1}\dfrac{1}{8} = \dfrac{\pi}{4}$

Q.8 Solve the following equations :

(i) $\tan^{-1}\dfrac{x-1}{x-2} + \tan^{-1}\dfrac{x+1}{x+2} = \dfrac{\pi}{4}$ (ii) $\tan^{-1}2x + \tan^{-1}3x = \dfrac{\pi}{4}$

(iii) $\tan^{-1}\dfrac{x-1}{x+1} + \tan^{-1}\dfrac{2x-1}{2x+1} = \tan^{-1}\dfrac{23}{36}$ (iv) $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\,x)$

Q.9 If $\sin^{-1}\dfrac{2a}{1+a^2} - \cos^{-1}\dfrac{1-b^2}{1+b^2} = \tan^{-1}\dfrac{2x}{1-x^2}$, then prove that $x = \dfrac{a-b}{1+ab}$

Q.10 Write each of the following in the simplest form :

(i) $\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$ (ii) $\tan^{-1}\left(\sqrt{1+x^2} + x\right), x \in R$

(iii) $\tan^{-1}\left(\sqrt{1+x^2} - x\right), x \in R$ (iv) $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$

(v) $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+1}{x}\right), x \neq 0$ (vi) $\tan^{-1}\sqrt{\dfrac{a-x}{a+x}}, -a < x < a$

(vii) $\tan^{-1}\left(\dfrac{x}{a+\sqrt{a^2-x^2}}\right), -a < x < a$ (viii) $\sin^{-1}\left(\dfrac{x+\sqrt{1-x^2}}{\sqrt{2}}\right), -1 < x < 1$

(ix) $\sin^{-1}\left(\dfrac{\sqrt{1+x}+\sqrt{1-x}}{2}\right), 0 < x < 1$ (x) $\sin\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)$

CH-3 Matrices

Q.1 Find x, y, a and b if $\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$

Q.2 Two farmers Ram Kishan and Gurcharan singh cultivate only three varities of rice namely Basmati, Permal and Naura. The sale (in Rs) of these varities of rice by both the farmers in the month of September and October are given by the following matrices A and B

September sales (in Rs)

	Basmati	Permal	Naura	
A =	10, 000	20, 000	30,000	RamKishan
	50, 000	30, 000	10, 000	Gurcharan Singh

	Basmati	Permal	Naura	
B =	5,000	10, 000	6,000	RamKishan
	20,000	10, 000	10, 000	Gurcharan Singh

Find :

(i) What were the combined sales in September and October for each farmer in each variety.

(ii) What was the change in sales from September to October ?

(iii) If both farmers receive 2% profit on gross rupees sales, compute the profit for each farmer and for each variety sold in October.

Q.3 If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, find

(i) A + B and B + C (ii) 2B + 3A and 3C – 4B.

Q.4 Let $A = \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix}$ and I be the identity matrix of order 2.

Show that $I + A = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Q.5 If. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I_2 = O$

Q.6 If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, then prove by principle of mathematical induction that

$A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$ for all $n \in N$.

Q.7 If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

Q.8 Find the inverse of each of the following matrices and verify that $A^{-1} A = I_3$.

(i) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

Q.9 Solve the following system of equations by matrix method :

(i) x + y - z = 3
2x + 3y + z = 10
3x - y - 7z = 1

(ii) x + y + z = 3
2x - y + z = - 1
2x + y - 3z = - 9

(iii) 6x - 12y + 25z = 4

(iv) 3x + 4y + 7z = 14

4x + 15y - 20z = 3

2x + 18y + 15z = 10

2x - y + 3z = 4

x + 2y-3z = 0

(v) $\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10$

$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$

(vi) 5x + 3y + z = 16

2x + y + 3z = 19

x + 2y + 4z = 25

(vii) 3x + 4y + 2z = 8

2y-3z = 3

x - 2y + 6z = - 2

(viii) 2x + y + z = 2

x + 3y-z = 5

3x + y- 2z = 6

(ix) 2x + 6y = 2

3x - z = - 8

2x - y + z = - 3

(x) x - y + z = 2

2x - y = 0

2y - z = 1

Q.10(i) If A= $\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$, find A-1. Using A-1 , solve the system of linear equations

x - 2y = 10, 2x + y + 3z = 8, - 2y + z = 7

(ii) A = $\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{bmatrix}$, find A-1 and hence solve the following system of equations:

3x- 4y + 2z = -1, 2x + 3y+5z = 7, x + z = 2

(iii) A = $\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$ and B $\begin{bmatrix}7 & 2 & -3\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$, find AB. Hence, solve the system of

equations: x - 2y = 10,2x + y + 3z = 8 and - 2y + z = 7

NOTE:- Solve all these questions in A-4 size papers and submit in a file on 21 June 2024.
