Brain International School

Vikas Puri, New Delhi

REVISION SHEET (sample papers)

SUBJECT:MATHEMATICS CLASS-XII

General Instructions :

- **Reading Time : 15 minutes**
- This Question paper contains - **Five Sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- **Section A** has **18 MCQ's** and **02** Assertion-Reason based questions of **1 mark** each.
- **Section B** has **5 Very Short Answer (VSA)**-type questions of **2 marks** each.
- **Section C** has **6 Short Answer (SA)**-type questions of **3 marks** each.
- **Section D** has **4 Long Answer (LA)**-type questions of **5 marks** each.
- **Section E** has **3 source based/case based/passage based/integrated units of assessment** (**4 marks** each) with sub parts.

SECTION – A

(Questions 1 to 20 carry 1 mark each.)

01. Given that A is a square matrix of order 3 and $|\text{adj.A}| = 49$, then $|A^{-1}|$ is equal to

(a) ± 7 (b) $\pm\frac{1}{49}$ (c) $\pm\frac{1}{7}$ (d) –7 only

02. For $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $A + A^T$ equals

(a) $2\begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos 2\theta & 0 \\ 0 & \cos 2\theta \end{bmatrix}$ (c) $\begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$ (d) $2\begin{bmatrix} 0 & \cos\theta \\ \cos\theta & 0 \end{bmatrix}$

03. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then which of the following is true?

(a) $\vec{a}.\vec{b} = 0$ (b) $\vec{a} = \lambda\vec{b}$ (c) $\vec{a}.\vec{b} = \vec{0}$ (d) $\vec{a}\times\vec{b} = 0$

04. Let $\tan^{-1} : R \to \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Then $\tan^{-1}(-1) =$

(a) $-\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

05. Let $A = \{i, s, h, a\}$. If $R : A \to A$ is given by $R = \{(i,i),(s,s),(a,a),(a,h)\}$, then which of the following ordered pair must be added to make R a reflexive relation?

(a) (h,a) (b) (s,h) (c) (h,h) (d) (h,i)

06. If m and n respectively, are the order and degree of the differential equation $x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} = 0$, then $(mn) =$

(a) 1 (b) 2 (c) 3 (d) 4

07. The feasible region, for the inequalities $x \geq 0,\ x + y \leq 1$ and, $y \geq 0$, lies in

(a) IV Quadrant (b) III Quadrant (c) II Quadrant (d) I Quadrant

08. What is the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$?

(a) 0 (b) 1
(c) 2 (d) infinitely many unit vectors are possible

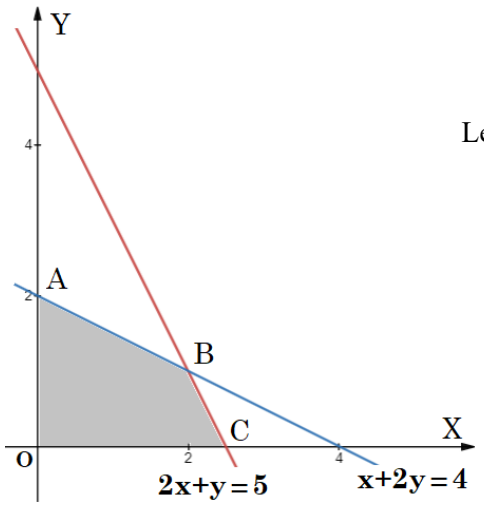
09. If $f(x)$ is an odd function, then the value of $\int_{-a}^{a} f(x)dx =$

(a) $2\int_{0}^{a} f(x)dx$ (b) $\int_{0}^{a} f(x)dx$ (c) $2\int_{0}^{\frac{a}{2}} f(x)dx$ (d) 0

10. Minor of the element 9 in $\Delta = \begin{vmatrix} 2 & 4 & 9 \\ 3 & 6 & -9 \\ -2 & -3 & 1 \end{vmatrix}$ is

(a) 0 (b) –3 (c) 3 (d) 1

11. The feasible region in a LPP is as shown in the graph below.
The corner points are denoted by A, B, C and O.



Let $Z = 2x + 5y$, then the value of $Z_{max} - Z_{min}$ equals

(a) 0 (b) 4
(c) 10 (d) 1

12. If $0 < x < \frac{\pi}{2}$, and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$, then the values of x is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}, \frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

13. Given that the matrices A and B of order $3\times m$ and $3\times n$ respectively, are such that AB and BA both exist, then order of A is

(a) 3×4 (b) 4×3 (c) 3×3 (d) cannot be determined

14. Two independent events A and B are such that $P(A) = 0.6$ and $P(B) = 0.5$.
Based on this information, which of the following options is **incorrect**?

(a) $P(A \cap B) = 0.3$ (b) $P(A \cup B) = 0.8$ (c) $P(A \mid B) = P(A)$ (d) $P(B \mid A) = P(A)$

15. Integration factor of the differential equation $\left(\frac{dy}{dx}\right) - \frac{y}{x} = x^2$ is denoted by $f(x)$. Then $f'(x) =$

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$ (c) $\frac{1}{x^2}$ (d) $-\frac{1}{x^2}$

16. If $y = x^e$, then $\frac{dy}{dx} =$

(a) x^e (b) $e.x^{e-1}$ (c) $e.x^e$ (d) $x^e \times \log x$

17. The direction angle made by the line $\frac{x-1}{1} = \frac{y+1}{\sqrt{2}} = \frac{z-2}{-1}$ with positive direction of x-axis, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}, \frac{5\pi}{6}$

18. If $|\vec{a}| = 2, |\vec{b}| = 2\sqrt{3}$ and $\vec{a} \perp \vec{b}$, then the value of $|\vec{a} + \vec{b}|$ is

(a) 16 (b) ± 4 (c) 4 (d) ± 16

*Followings are **Assertion-Reason based questions**.*

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A) :** There is no value of 'b' for which the function $f(x) = x + \cos x + b$ is strictly decreasing over R (set of real numbers).

Reason (R) : If $f'(x) \geq 0$ in $x \in [a, b]$ then, f (x) is an increasing function in $x \in [a, b]$.

20. **Assertion (A) :** $\hat{i} + \hat{j} + 2k$ is a vector parallel to the line $\vec{r} = \hat{i} - \hat{j} + k + \lambda(\hat{i} + \hat{j} + 2k)$.

Reason (R) : In the vector form of line $\vec{r} = x_1\hat{i} + y_1\hat{j} + z_1k + \lambda(a_1\hat{i} + b_1\hat{j} + c_1k)$, a vector parallel to the line is $a_1\hat{i} + b_1\hat{j} + c_1k$.

SECTION B

*(Question numbers 21 to 25 carry **2 marks** each.)*

21. Using principal values, evaluate $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{2\pi}{3}\right)$.

OR

Let $A = \{1, 2, 3\}$. Write all the possible equivalence relations defined on set A.

22. For the function $f(x) = 4x - \frac{1}{2}x^2, -2 \leq x \leq \frac{9}{2}$, find the absolute maximum value and absolute minimum value.

23. Find the vector equation of the line joining the points (1, 2, 3) and (–3, 4, 3). Also write its Cartesian equation.

OR

If a line makes the angles α, β and γ with the coordinate axes, then evaluate :
$\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

24. If $x = a\sec\theta, y = b\tan\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

OR

If $f(x) = \begin{cases} \frac{\log(1+4x) - \log(1-x)}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then find the value of k.

25. If $\vec{p} + \vec{q} + \vec{r} = \vec{0}$ and $|\vec{p}| = 3, |\vec{q}| = 5, |\vec{r}| = 7$, then find the angle between $\vec{p}$ and $\vec{q}$.

SECTION C

*(Question numbers 26 to 31 carry **3 marks** each.)*

26. Find : $\int \frac{1}{\sin(x-a)\cos(x-b)}dx$.

27. For events E and F, $P(E) = 0.4$, $P(F) = 0.5$ and $P(E \cup F) = 0.7$.
Using the concept of conditional probability, find $P(E|F) + P(F|E)$.

OR

A girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

28. Evaluate : $\int_0^{2\pi} |\cos x|\, dx$.

OR

Evaluate : $\int_0^3 \{|x| + |x-1|\}\, dx$.

29. Solve the differential equation : $x^2 y\,dx - (x^3 + y^3)dy = 0$.

OR

Find the particular solution of the following differential equation :

$\cos y\,dx + (1 + 2e^{-x})\sin y\,dy = 0;\ y(0) = \frac{\pi}{4}$.

30. A linear programming problem is as follows.

To maximize: $Z = (x + y)$

Subject to constraints: $2x + y \le 50,\ x + 2y \le 40,\ x \ge 0,\ y \ge 0$.

In the feasible region, find the point at which maximum value of Z occurs. Solve graphically.

31. Find : $\int \frac{x\,dx}{x^2 + 3x + 2}$.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. Using integration, find the area of the smaller region bounded between $y = \sqrt{36 - x^2}$ and $x = 4$.

33. Using differentiation, find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

OR

Two equal sides of an isosceles triangle with fixed base b (in centimeter) are decreasing at the rate of 3 cm/s. How fast is the area decreasing when two equal sides are equal to the base?

34. Determine the equations of a line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and; $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-z}{5}$.

OR

If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

35. If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the following system of equations :

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4;\ x, y, z \ne 0$.

SECTION E

*(Question numbers 36 to 38 carry **4 marks** each.)*

*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** of **2 marks** each.*

36. **CASE STUDY I :** Read the following passage and the answer the questions given below.

An organization conducted bike race under two different categories – Boys and Girls.

There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



(i) How many relations are possible from B to G?

(ii) Among all the possible relations which are defined from B to G, how many functions can be formed from B to G?

(iii) Let $R : B \to B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}$.

Check if R is an equivalence relation.

OR

(iii) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective (i.e., one-one and onto both). Justify your answer.

37. **CASE STUDY II :** Read the following passage and answer the questions given below.



Mr Neeraj Jha is a business analyst. He offers his expert-views to the companies.

A ball manufacturing company hires Mr Jha for his services.

Mr Jha observed that $P(x) = -5x^2 + 1250x + 30$ (in ₹) is the total profit function of this ball manufacturing company, where x is the production of the company.

(i) Differentiate $P(x)$ with respect to x.

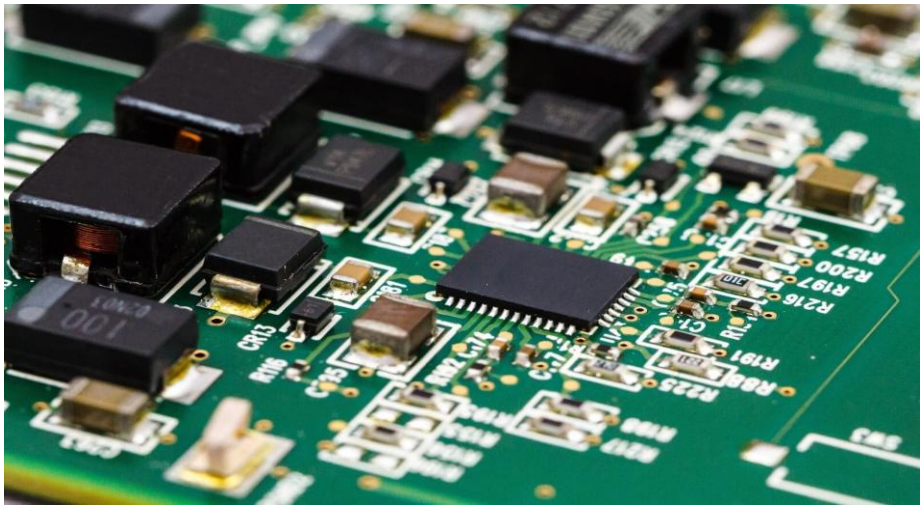
(ii) What will be the production when the profit is maximum?

(iii) What will be the maximum profit?

OR

(iii) Check if the profit function P(x) is strictly increasing in the interval $x \in (0, 125)$?

38. **CASE STUDY III :** Read the following passage and answer the questions given below.



An electronic assembly consists of two kinds of sub-systems *say*, A and B.
From previous testing procedures, the following probabilities are assumed to be known:

$$P(\text{A fails}) = 0.2,\ P(\text{B fails alone}) = 0.15,\ P(\text{A and B fail}) = 0.15.$$

(i) Find $P(\text{B fails})$ and, $P(\text{A fails alone})$.

(ii) Find $P(\text{A fails} \mid \text{B has failed})$.

SAMPLE PAPER 2

SECTION – A

(Questions 1 to 20 carry 1 mark each.)

01. For a square matrix P, if we have $P.(\text{adj}.P) = \begin{bmatrix} -2025 & 0 & 0 \\ 0 & -2025 & 0 \\ 0 & 0 & -2025 \end{bmatrix}$, then $|P| + |\text{adj}.P| =$

(a) $2025^2 \times 2024$ (b) -2024 (c) 2025×2024 (d) $(-2025)^2 + 2025$

02. X and Y are two matrices such that the transpose of $(X+Y)$ is $\begin{bmatrix} 2 & -1 \\ 7 & 6 \end{bmatrix}$. If $Y = \begin{bmatrix} 3 & -2 \\ 6 & 0 \end{bmatrix}$, then which of the following is correct?

(a) $X = \begin{bmatrix} -1 & 1 \\ 1 & 6 \end{bmatrix}$ (b) $X = \begin{bmatrix} -1 & 9 \\ -7 & 6 \end{bmatrix}$ (c) $X = \begin{bmatrix} 1 & -9 \\ 7 & -6 \end{bmatrix}$ (d) $X = \begin{bmatrix} -4 & 4 \\ 0 & 7 \end{bmatrix}$

03. Function f defined by $f(x) = e^{-x}$ is strictly increasing when

(a) $x \in \phi$ (b) $x \in (-\infty, 0)$ (c) $x \in [0, \infty)$ (d) $x \in (-\infty, \infty)$

04. If A, B and C are square matrices of order 3 and $\det(BC) = 2\det(A)$, then what is the value of $\det(2A^{-1}BC)$?

(a) 1 (b) 2^2 (c) 2^3 (d) 2^4

05. The value of 'n', such that the differential equation given by $\frac{dy}{dx} = \frac{x^2 + y^2}{x^n}$; where $x, y \in \square^+$ is homogeneous, is

(a) 0 (b) 1 (c) 2 (d) 3

06. If $\begin{vmatrix} 2 & -3 & 1 \\ k & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$, then

(a) $7k = 10$ (b) $7k + 10 = 0$ (c) $10k + 7 = 0$ (d) $10k = 7$

07. If A is a square matrix of order n, then the number of minors in the determinant of A are

(a) n (b) $n-1$ (c) n^2 (d) n^n

08. Delhi Metro is highly popular mode of transport among the commuters. The Metro connects numerous stations across Delhi NCR. Among them are Dwarka and Hauz Khas. Wishi takes the Metro scheduled at 8:25 AM from Dwarka station to Hauz Khas station every morning. The probability that the Metro is late is $\frac{3}{4}$ and, the probability that Wishi gets a seat in the Metro is $\frac{1}{15}$. The probability that the Metro is on time and she gets a seat in it is

(a) $\frac{1}{20}$ (b) $\frac{1}{4}$ (c) $\frac{1}{60}$ (d) $\frac{7}{10}$

09. There are two non-zero vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}.\vec{b} = 0$. Then the projection of $\vec{a}$ on $\vec{b}$ is

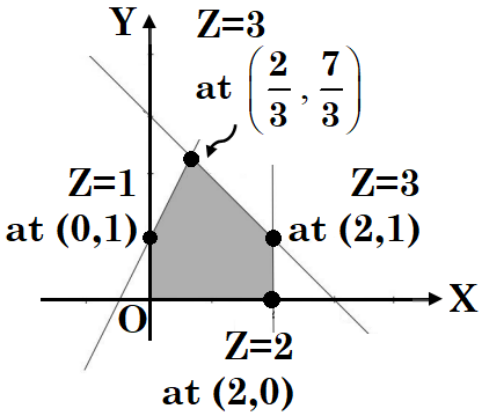
(a) 1 (b) 0 (c) 2 (d) can not be determined

10. Let $y = f(x)$ be a real function such that its first-order derivative is same as its second-order derivative. Then $f(x) =$

(a) x (b) 2^x (c) e^x (d) no such function exists

11. Shown below is the feasible region of a linear programming problem (L.P.P.) whose objective function is : Maximize $Z = x + y$.

A student Drishti claimed that there exists **no optimal solution** for the L.P.P. as there is no unique maximum value at the corner points of its feasible region. Based on her statement, choose most appropriate option.



(a) Her claim is correct, as there are two corner points of its feasible region at which maximum value of Z occurs.

(b) Her claim is false, as there are exactly two corner points i.e., $\left(\frac{2}{3}, \frac{7}{3}\right)$ and $(2, 1)$ at which the maximum value of Z occurs, which is 3.

(c) Her claim is false, as every point on the line joining $\left(\frac{2}{3}, \frac{7}{3}\right)$ and $(2, 1)$ gives the maximum value of Z, which is 3.

(d) Her claim is false, as the maximum value of Z occurs at (2, 0), which is 2.

12. If $f(x)$ is continuous for all real values of x, then $\int_{\frac{a}{4}}^{\frac{b}{4}} f(4x)dx$ equals

(a) $4\int_a^b f(x)dx$ (b) $\frac{1}{4}\int_{4a}^{4b} f(x)dx$ (c) $\frac{1}{4}\int_a^b f(x)dx$ (d) $4\int_{4a}^{4b} f(x)dx$

13. For the function $y = \sin^{-1}\{|x| - 1\}$

(a) Domain $= x \in [-1, 1]$, Range $= y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) Domain $= x \in (-2, 2)$, Range $= y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) Domain $= x \in (-1, 1)$, Range $= y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

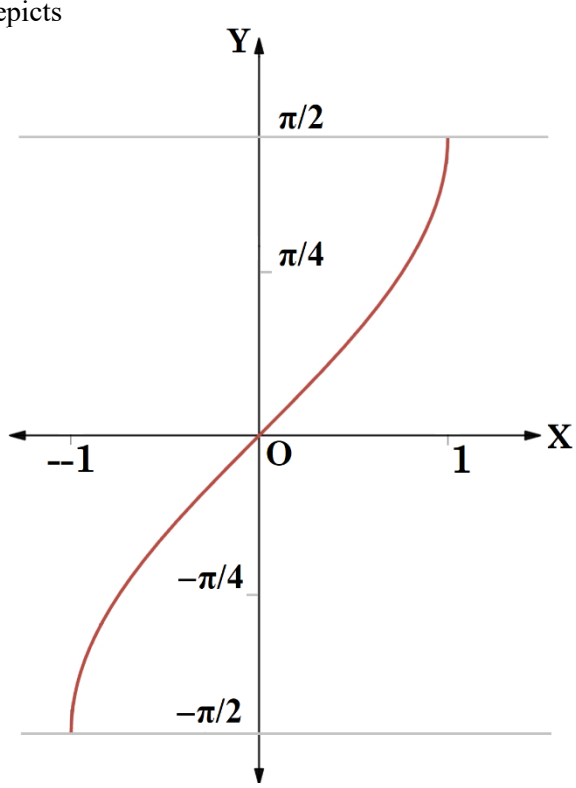
(d) Domain $= x \in [-2, 2]$, Range $= y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

14. A differential equation has an order of 3 and a degree of 2. Which of the following could this differential equation be?

(a) $\frac{d^3y}{dx^3} - \left(\frac{d^2y}{dx^2}\right)^2 = 0$ (b) $\tan\left(\frac{d^2y}{dx^2}\right) + \left(\frac{d^3y}{dx^3}\right)^2 = 0$

(c) $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3 = 0$ (d) $\left(\frac{d^3y}{dx^3}\right)^3 + \left(\frac{dy}{dx}\right)^2 = 0$

15. The graph drawn below depicts



(a) $y = \cos^{-1} x$ (b) $y = \text{cosec}^{-1} x$ (c) $y = \sin^{-1} x$ (d) $y = \cot^{-1} x$

16. $f(x) = |\sin x|$ is non-differentiable at

(a) $x = n\pi,\ n \in Z$ (b) $x = (2n \pm 1)\pi,\ n \in Z$

(c) $x = (2n \pm 1)\frac{\pi}{2},\ n \in Z$ (d) $x = \square - \left\{(2n \pm 1)\frac{\pi}{2}\right\},\ n \in Z$

17. If $[.]$ denotes the greatest integer function, then $f(x) = [x]$ is discontinuous at

(a) infinite points, in $2 < x < 5$ (b) only two points, in $2 < x < 5$

(c) only three points, in $2 < x < 5$ (d) no point, in $2 < x < 5$

18. The area under the curve $x^2 = y$ between the line $x = 0$ and $x = k$ is 9 square units. Which of the following could be the correct value of k?

(a) $\frac{3}{2}$ (b) 9 (c) 3 (d) $\frac{9}{2}$

*Followings are **Assertion-Reason based questions**.*
In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A) :** All the points in the feasible region of an L.P.P. (linear programming problem) are optimal solutions to the problem.
Reason (R) : Every point in the feasible region satisfies all the constraints of an L.P.P. (linear programming problem).

20. **Assertion (A) :** If the angle between $\vec{p}$ and $\vec{q}$ is obtuse, then $\vec{p}.\vec{q} < 0$.
Reason (R) : Value of $\cos\theta$ lies in $(-1, 0)$, when $90^\circ < \theta < 180^\circ$.

SECTION B

*(Question numbers 21 to 25 carry **2 marks** each.)*

21. If $4x + \frac{1}{x} = 4$, then find $\cos^{-1} x - \sin^{-1} 2x$.

22. The total cost (in ₹) of manufacturing 'n' earphone sets per day in a Karnataka based start-up Maxier Electronics Limited is given by $C(n) = 0.0001n^2 + 4n + 400$. Find $C(n = 100)$.
Also, find the marginal cost of manufacturing 10000 earphone sets.

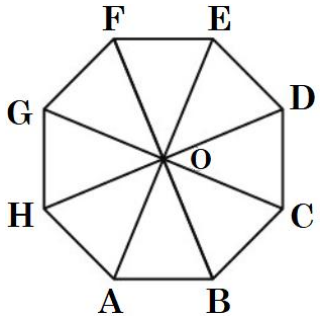
23. If $y = a\,x^{n+2} + \frac{b}{x^{n+1}}$, where $n \in N$ and $a, b \in \square$, then prove that $x^2 \frac{d^2y}{dx^2} = (n+1)(n+2)\,y$.

OR

Differentiate the function $y = \cos^{-1}\left(\frac{1-3^{2x}}{1+3^{2x}}\right)$ with respect to x.

24. A bird is sitting on an electric wire (assuming that the wire has no slack). If the equation of wire is given by $\frac{x+1}{-3} = y - 2 = z$ and the position of bird is at a point P such that the distance between P and Q(–1, 2, 0) is $6\sqrt{11}$ units, then find the position of bird (coordinates of point P).

25. Shown below is a regular octagon ABCDEFGH, with centre O.



Show that $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AD} - \overrightarrow{BC})$.

OR

A parallelogram ABCD is constructed such that its adjacent sides, AB and AD, are $3\vec{a} - 5\vec{b}$ and $\vec{a} - 2\vec{b}$ respectively. If $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = \sqrt{8}$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, find the length of diagonal BD.

SECTION C

*(Question numbers 26 to 31 carry **3 marks** each.)*

26. A cylindrical disk of radius R and height H is pressed by a hydraulic press. During the process, the radius and the height of the disk change such that the cylindrical shape is retained and the volume (V) of the disk remains

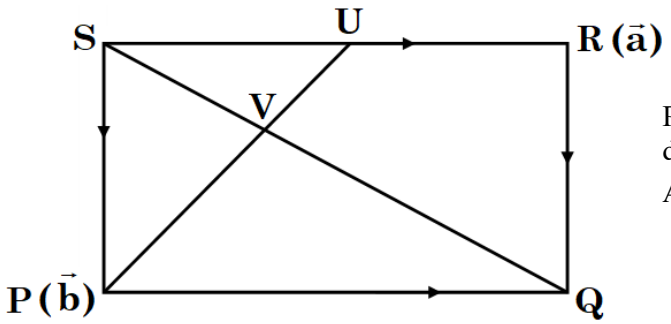
constant. What is the ratio of the rate of change of height to the rate of change of radius in terms of R?

27. Sumathi bought a luxury car for `y. The price $P(t)$ of the car depreciates as $\frac{dP(t)}{dt} = -a(T-t)$, after 't' years; where 'T' is the total life of the car (in years) and 'a' is an arbitrary constant.
Find the value of the car when the car has been used for 'T' years, in terms of 'y'.

OR

The first derivative of a function y with respect to x is given by $-\frac{1}{x^2(1+x^2)}$. Find the function, if it is given that $y = \frac{\pi}{4}$, when $x = 1$.

28. In the rectangle PQRS below, S is at the origin and the position vectors of the points R and P are $\vec{a}$ and $\vec{b}$ respectively.



Point U bisects SR. Use vectors to prove that V divides PU and QS in the same ratio.
Also find the ratio in which V divides QS.

29. Evaluate : $\int \sqrt{\sqrt{x^2+4}+x}\, dx$.

OR

Evaluate : $\int e^x \left\{\frac{x^3+x+1}{(x^2+1)^{3/2}}\right\} dx$.

30. Consider the following Linear Programming Problem.

Maximize $Z = x + 2y$

Subject to $2x + 3y \geq 6,\ 4x + y \geq 4;\ x,\ y \geq 0.$

Show graphically that the maximum value of Z will not occur.

31. A company conducts a mandatory health check-up for all the newly hired employees, to check for infections that could affect other employees. A blood infection affects roughly 5% of the population. The probability of a false positive report on the test for this infection is 4%, while the probability of a false negative report on the test is 3%. If a person tests positive for the infection, what is the probability that he is actually infected?

OR

Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. Draw the rough sketch of the curve $y = \sin 2x$; where $\frac{\pi}{12} \leq x \leq \frac{\pi}{6}$.

Using integration, find the area of the region bounded by the curve $y = \sin 2x$ from the ordinates $x = \frac{\pi}{12}$ to $x = \frac{\pi}{6}$ and the x-axis.

33. The curve $y = ax^2 + bx + c$; (where $a, b, c \in \mathbb{R}$ and $a \neq 0$) passes through the points $(-1, 0)$, $(2, 12)$ and $(3, 20)$. Use matrix method to determine the values of a, b and c by solving the system of linear equations

in a, b and c. Find the equation of the curve. If $y = ax^2 + bx + c = 0$, then write the real roots of quadratic equation (if possible).

34. The vertices of a ΔABC are A(1, 1, 0), B(1, 2, 1) and C(–2, 2, –1). Find the equations of the medians through A and B. Use the equations so obtained to find the coordinates of the centroid.

OR

Find the Cartesian equation of a line L_2 which is the mirror image of the line L_1 with respect to line L : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that line L_1 passes through the point P(1, 6, 3) and parallel to line L.

35. Discuss the continuity of $f(x) = \begin{cases} x\,e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $x = 0$.

Is it differentiable at the same point? Justify your answer.

OR

If $\sqrt{1+x^2} + \sqrt{1+y^2} = a(x-y)$, then show that $\frac{dy}{dx} = \sqrt{\frac{1+y^2}{1+x^2}}$.

SECTION E

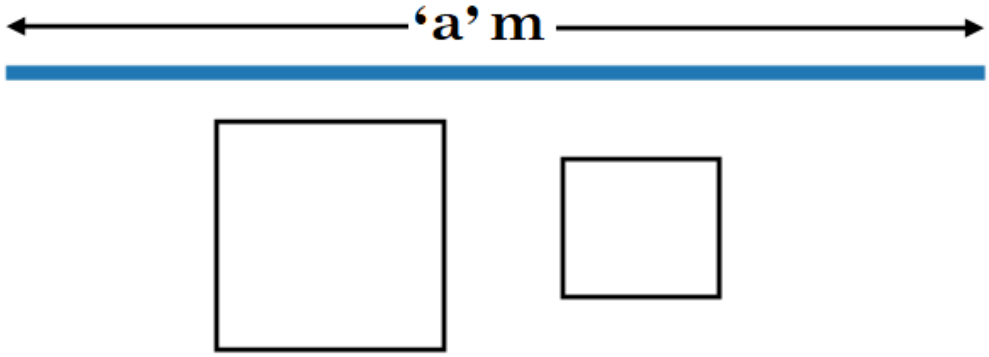
*(Question numbers 36 to 38 carry **4 marks** each.)*

*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** (i) and (ii) of **2 marks** each.*

36. CASE STUDY I :

Sumaiya cuts a metallic wire of length 'a' m into two pieces. She uses both pieces to create two squares of different side lengths.

Assume that the wire of length 'x' m is used to make the first square.



Based on the above information, answer the following questions.

(i) Express the side lengths of both squares in terms of 'a' and 'x' only.

(ii) Find an expression for the Combined area (A) of both squares in terms of 'x'.

(iii) Determine the side lengths of both the squares (in terms of 'a') for which the Combined area (A) will be minimum. Use derivatives.

OR

(iii) Using derivatives, find the minimum value of Combined area (A) of both squares in terms of 'a'.

37. CASE STUDY II :

Pratibha Vikas is an innovative program by the Government of Delhi, where cultural and literacy competitions are held between schools at cluster, block, district and state levels.

One of those competitions - Yogasana, is conducted under two categories : Middle school and High school. From South Delhi district, three students from middle school and two students from high school were selected for the state level.

Let $M = \{m_1, m_2, m_2\}$ and $H = \{h_1, h_2\}$, represent the set of students from middle school and high school respectively who got selected for the state level from that district.

A relation $R : M \to M$ is defined by $R = \{(x, y) : x \text{ and } y \text{ are students from the same category}\}$.

On the basis of the above information, answer the following questions.

(i) Check if the relation R is reflexive. Justify your answer.

(ii) Check if the relation R is symmetric. Justify your answer.

(iii) Check if the relation R is transitive. Is R an equivalence relation? Justify your answer.

OR

(iii) Let a function $f : M \to H$ is defined as $f = \{(m_1, h_1), (m_2, h_2), (m_3, h_2)\}$. Check whether the function f is one-one and onto. Justify your answer.

38. CASE STUDY III :

Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health, A_2 : People with average health, and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50% respectively.

Using the information given above, answer the following questions.

(i) A person was tested randomly. What is the probability that he/she has contracted the disease

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ?