



ASSIGNMENT 3

Subject : Mathematics

Class XII

October, 26

Chapter differential equations

Ques1 determine the order and degree of differential equation

$$(1) x \frac{dy}{dx} + \frac{2}{\frac{dy}{dx}} = y^2 \quad (2) \sqrt{a+x} \frac{dy}{dx} + x = 0 \quad (3) \frac{d^2y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{3}{2}}$$

$$(4) y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \quad (5) y - px = \sqrt{a^2 p^2 + b^2} \quad \text{where } p = \frac{dy}{dx}$$

Ques2 verify that $y = Ax + \frac{B}{x}$ is a solution of differential equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$

Ques3 verify that $y = e^{m \sin^{-1} x}$ is a solution of differential equation $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$

Ques4 verify that $y^2 = 4a(x + a)$ is a solution of differ. Equation $y \left\{ 1 - \left(\frac{dy}{dx} \right)^2 \right\} = 2x \frac{dy}{dx}$

Ques5 solve the differential equation

$$(1) \frac{dy}{dx} = \log(x+1) \quad (2) \frac{dy}{dx} = 1 - x + y - xy \quad (3) \frac{dy}{dx} = e^{x+y} + x^2 e^y \quad (4) y - x \frac{dy}{dx} = a \left(y^2 + \frac{dy}{dx} \right)$$

Ques 6 solve the differ. Equation

$$(1) (x\sqrt{x^2 + y^2} - y^2)dx + xydy = 0$$

$$(2) (x \cos \frac{y}{x})(ydx + xdy) = (y \sin \frac{y}{x})(xdy - ydx)$$

$$(3) \frac{dy}{dx} = \frac{y(2y-x)}{x(2y+x)} \quad \text{given that } y(0)=1$$

Ques 7 solve

$$(1) (1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x} \quad (2) (1-x^2) \frac{dy}{dx} + xy = ax \quad (3) (x-y^2) \frac{dy}{dx} + y = 0 \quad (4) (x+y+1) \frac{dy}{dx} = 1$$

Ques 8 Find the general solution of the differential equation $x \log x \cdot \frac{dy}{dx} + y = \frac{2}{x} \cdot \log x$

Ques 9 Find the particular solution of the differential equation satisfying the given conditions:

$$\frac{dy}{dx} = y \tan x, \quad \text{given that } y = 1 \text{ when } x = 0.$$

Ques 10 Find the particular solution of the differential equation satisfying the given condition:

$$x^2 dy + (xy + y^2)dx = 0; \quad y = 1 \text{ when } x = 1$$

Ques 11 Solve the following differential equation: $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}; |x| \neq 1$

Ques 12 Solve the following differential equation: $\sqrt{1 + x^2 + y^2 + x^2 y^2} + xy \frac{dy}{dx} = 0$

Chapter vectors

Ques1 Show that the points with position vectors $\vec{a} - 2\vec{b} + 3\vec{c} - 2\vec{a} + 3\vec{b} + 2\vec{c}$, and $-8\vec{a} + 13\vec{b}$ are Collinear ■

Ques2 if the position vector $\vec{a}$ of a point (12,n) is such that $|\vec{a}|=13$ find n

Ques3 find a vector of magnitude 5 units which is parallel to $2\hat{i} - \hat{j}$

Ques4 if $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$ find $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$

Ques5 for any vector $\vec{r}$, prove that $\vec{r} = (\vec{r} \cdot \hat{i})\hat{i} + (\vec{r} \cdot \hat{j})\hat{j} + (\vec{r} \cdot \hat{k})\hat{k}$

Ques6 if $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ then prove that $\sin \frac{\theta}{2} = \frac{1}{2} |\hat{a} - \hat{b}|$

Ques7 find $|\vec{a}|$ and $|\vec{b}|$ if $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 27$ and $|\vec{a}| = 2|\vec{b}|$

Ques8 if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 6$ find $|\vec{a} + \vec{b}|$ and $|\vec{a} - \vec{b}|$

Ques9 if $\hat{a}$ and $\hat{b}$ are unit vectors inclined to an angle θ then prove that $\cos \frac{\theta}{2} = \frac{1}{2} |\hat{a} + \hat{b}|$

Ques10 if $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$ then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal

Ques11 find a unit vector perpendicular to the plane ABC where A,B,C are A (3,-1,2) B(1,-1,-3) and C(4,-3,1) resp.

hint $\hat{n} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|}$

Ques12 Given $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$ find $|\vec{a} \times \vec{b}|$

Ques13 show that $(\vec{a} \times \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$

Ques14 if $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B, C of a ΔABC , show that area of ΔABC is $\frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ and deduce the condition for collinear

Ques15 for any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ show that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$

Ques16 If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $(\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$

Q17 Let $\vec{a}, \vec{b}, \vec{c}$ be the unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$

Ques18 If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

Chapter 3 Dimensional Geometry

Ques1. determine the point in XY –plane which is equidistant from three points A(2,0,3) B(0,3,2) C(0,0,1)

Ques2. find the ratio in which the line joining the points (1,2,3) and (-3,4,-5) is divided by xy-plane .find the co-ordinate of point of division .

Ques3. A vector $\vec{OP}$ is inclined to ox at 45° and OY at 60° find the angle at which $\vec{OP}$ is inclined at OZ

Ques4. the Cartesian equations of a line are $6x-2=3y+1=2z-2$. find its direction ratios and also find vector equation of a line

Ques5. if the points A(-1,3,2) B(-4,2,-2) c(5,5, γ) are collinear . find the value of γ

□

Ques6 find the equation of the line passing through the points A(-1,3,-2) and perpendicular to

lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

Ques7 find foot of perpendicular from the point(0,2,3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. also find the Length of perpendicular

Ques8 Find the distance of the point (-2, 4, -5) from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

Ques 9 Find the angle between the lines whose direction cosines are given by the equations: $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

Ques 10 Find the co-ordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, -1, 3) and C (2, -3, -1).

Ques 11 Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Ques 12 Find the distance of a point P (a, b, c) from x-axis

Ques13 If a line makes angles α, β, γ with the positive direction of the coordinate axes, then find the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$

Ques14 If a line makes an angle of $\frac{\pi}{4}$ with each of y & z axis, then find the angle which it makes with x-axis

Ques15 Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also find their point of intersection

Ques16 Prove that the line $x = py + q, z = ry + s$ & $x = p'y + q', z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$