**Motion**

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**Linear Motion**

**Some Basics – Rest and Motion**


To identify whether a body is at rest or in motion, we must first define a **frame of reference** or a **reference point**.

*A frame of reference is a set of geometrical axes in space with respect to which the position, **velocity**, **acceleration**, etc., of a body is determined.*

*A reference point is a fixed point in space with respect to which the relative position or distance of an object is ascertained.*

The man shown in the figure is in motion with respect to the tree, street lamp, house, and car.

*A body which is not moving with respect to a frame of reference or a reference point, with progressing time, is said to be at rest.*

*A body which is moving with respect to a frame of reference or a reference point, with progressing time, is said to be in motion.*

**Rest and Motion – Relative Quantities**

Everything in the universe is in motion. Think of the place where we live. We see many things around us to be at rest. Now, think of the planet Earth and the sun. Earth moves around the sun; so, we and the things around us on Earth also move around the sun. This implies that we are not at rest but in constant motion. Thus, standing on Earth, if we consider the planet to be at rest, then we see the trees, electric poles, buildings, mountains, etc., also to be at rest. However, if we consider the sun to be fixed, then Earth and everything on it is in motion around the sun. As a matter of fact, there is no perfectly static thing in the universe. Earth moves around the sun; the sun moves around its **barycentre**; the galaxy rotates and revolves; and everything in the universe moves.

Now, let us consider **absolute rest or motion**. Absolute rest or motion is physically impossible for a body. To be at absolute rest, a body must be at rest with respect to every object in the universe. However, such a frame of reference has never been detected. Famous experiments like the Michelson–Morley experiment have sought to do so without any success. Consequently, science does not accept the concept of absolute rest or motion.

Since there can be no absolute rest or motion, we can say that the states of rest and motion are relative in the universe.

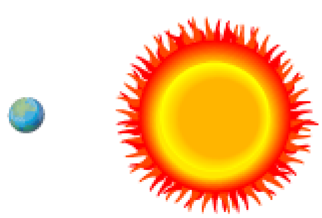
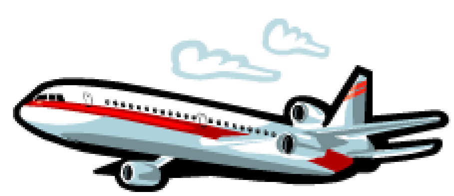
We cannot perceive Earth's motion around the sun because: *All the objects on Earth, including the atmosphere, are in motion along with the planet. So, it is not possible to perceive Earth's motion while staying on it*.

**Whiz Kid**

**What do you understand by the term 'system of reference'?**

It is a system of measurement for locating the positions of bodies in space with respect to an observer (or origin). In daily life, we become conscious of the motion of an object with respect to ourselves and other stationary objects. If the object maintains its position with respect to us, we say that the object is at rest; otherwise, we say that the object is moving.

In our daily observations, we assume Earth to be stationary. Therefore, we also assume that the object whose position does not change on Earth is also stationary. The fact, however, is otherwise. All objects on Earth move around the sun along with the planet at a very high speed—at a speed faster than the fastest aeroplane that mankind has ever developed. Earth moves around the sun at a speed of about 30 km/s (≈ 30000 m/s ≈ 100000 km/h), i.e., about hundred times faster than an aeroplane and about thousand times faster than an average car.

| | |
|---|---|
| <br>**Earth moves around the sun at a very fast speed.** | <br>**An aeroplane flies at a speed that is hundred times slower than Earth's speed.** |

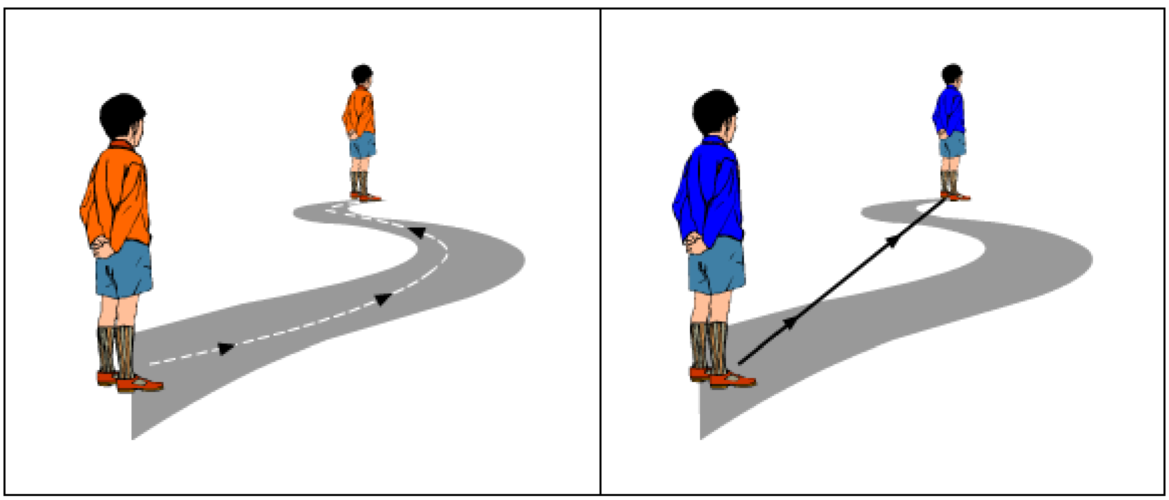
So, if we assume our origin to be on Earth, such that our system of reference moves along with it, then we can presume an object whose position does not change on Earth to be stationary. However, if our system of reference is outside Earth, then the planet and all its objects are observed to be in motion.

**Concepts of Distance and Displacement**

Whenever a body moves, it covers a **distance.** The straight line that joins the initial and final positions of the body is called its **displacement**.

*Distance is the length of the path travelled by a body while moving from an initial position to a final position.* It is a scalar quantity. Its SI unit is metre (m).

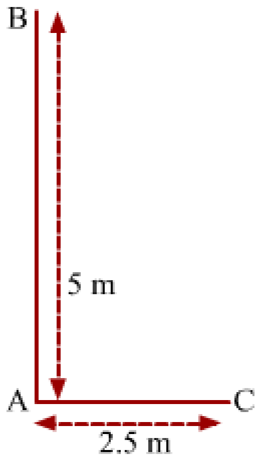
*Displacement is the shortest distance between the initial and final positions of the body.* It is a vector quantity. Its SI unit is also metre (m).



In displacement, the direction of motion is always directed from the initial position toward the final position.

For a straight-line motion, the distance travelled and the displacement are equal in magnitude.

**Concepts of Distance and Displacement**



In the science class, the teacher walks back and forth while discussing a problem in physics. He walks 5 m toward the students, turns around and then returns to his initial point. Then he walks 2.5 m toward his left and stops to answer a query from a student. What is the total distance covered by the teacher and his displacement from the point where he turns around?

The teacher walks from A to B, turns around and then walks back to A. Then, he walks from A to C.

So, total distance covered = AB + BA + AC = 5 m + 5 m + 2.5 m = 12.5 m

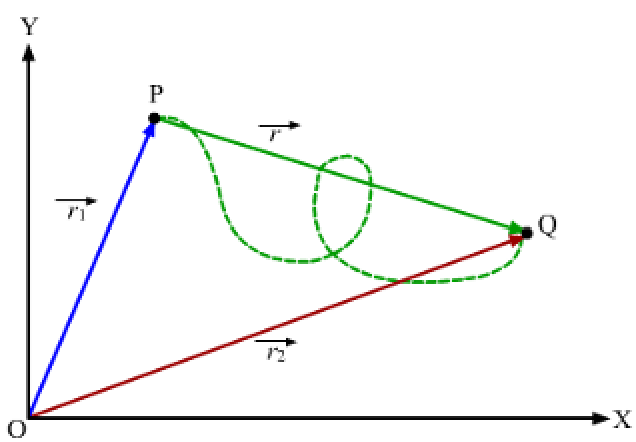
The teacher turns around at B. So, we need his displacement from B to C. BC is the hypotenuse of the right triangle BAC.

So: $BC^2 = AB^2 + AC^2$

=> $BC^2 = 5^2 + 2.5^2$ => $BC^2 = 31.25$ => $\therefore$ BC = 5.6 m

**Know More**

**Displacement and Position Vector**



The concept of displacement can be mathematically understood with the help of the concept of position vector.

Draw a reference frame with two axes on a paper and consider an ant to be moving from point P to point Q.

Position vector of point P = $\overrightarrow{OP}$

Coordinates of point P = $(x_1, y_1)$

Position vector of point P, in terms of the coordinates, is given as:

$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j} = \vec{r_1}$

Where, $\hat{i}$ = unit vector along X-axis

$\hat{j}$ = unit vector along Y-axis

Position vector of point Q = $\overrightarrow{OQ}$

Coordinates of point Q = $(x_2, y_2)$

Position vector of point Q, in terms of the coordinates, is given as:

$\overrightarrow{OQ} = x_2\hat{i} + y_2\hat{j} = \vec{r_2}$

Displacement of the ant is given by the change in position vector.

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$\vec{r} = \vec{r_2} - \vec{r_1}$

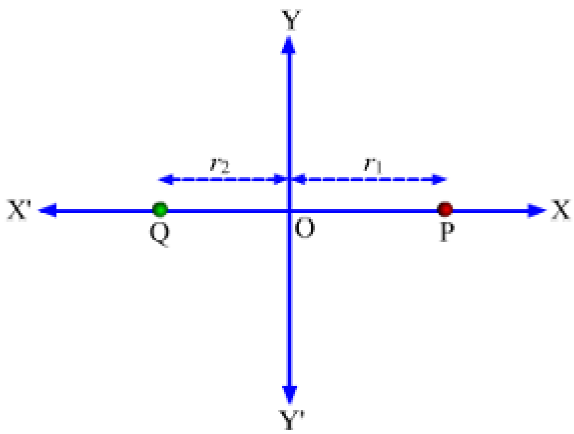
**Distance and Displacement – Differences**

| Distance | Displacement |
|---|---|
| 1. Distance is the length of the path travelled. | 1. Displacement is the straight-line distance between the initial and final positions. |
| 2. Distance is a scalar quantity. It has only magnitude. | 2. Displacement is a vector quantity. It has both magnitude and direction. |
| 3. Distance is always positive. Its minimum value can be zero. | 3. Displacement can be positive, negative or zero. |

| | |
|---|---|
| 4. Distances can be added algebraically to find the total distance travelled. | 4. The net displacement is a vector addition of individual displacements. |
| 5. Whenever there is a linear motion, there is a distance travelled. | 5. A body may undergo a linear motion without any displacement. |

**Quick Question**

**Question**: Displacement can be both positive and negative, but distance can be positive only. Why?




**Solution**: Displacement is a vector quantity. A vector quantity has both magnitude and direction. Magnitude is always a positive quantity, but direction can be positive or negative depending on the reference point or coordinate system. Suppose two bodies are moving opposite to each other with respect to a reference point. In this case, the displacement of one is taken as negative and that of the other is taken as positive, with respect to the reference point.

Let us consider a coordinate system, with X and Y as the axes and O as the origin. The part of the X-axis toward the right of the origin is taken as positive. The part of the X-axis toward the left of the origin is taken as negative.

Suppose we have two particles at points P and Q, displaced from their initial position O (as shown in the diagram). $r_1$ and $r_2$ are the respective distances of P and Q from O.

The displacement of P is along the positive direction of X-axis; so, $\overrightarrow{OP} = r_1\hat{i}$

The displacement of Q is along the negative direction of X-axis; so, $\overrightarrow{OQ} = -r_2\hat{i}$

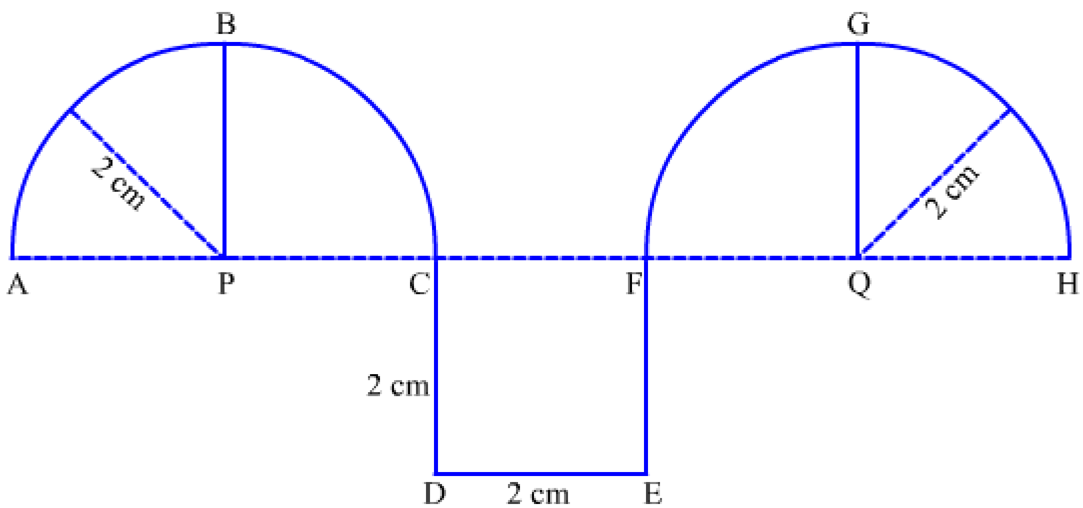
Thus, the sense of direction makes **displacement** both positive and negative.

**Distance**, on the other hand, is a scalar quantity; so, it has only magnitude. To calculate distance, we simply measure the length from a reference point (like O in the figure). The sense of direction is not necessary. Thus, distance is always positive.

**Solved Examples**

**Easy**

**Example 1**:**Raju walks on a track with junctions A, B, C, D, E, F, G and H.**




**What are the magnitudes of the distance travelled and displacement as Raju moves from:**

**(i) A to B: Distance = _____ cm ; Displacement = _____ cm**

**(ii) A to C: Distance = _____ cm ; Displacement = _____ cm**

**(iii) A to D: Distance = _____ cm ; Displacement = _____ cm**

**Solution**:

(i) $AP = PB = 2\text{ cm}$

$$\text{Distance} = \frac{1}{4} \times \text{Circumference of the circle with radius AP}$$
$$= \frac{1}{4} \times 2 \times \frac{22}{7} \times 2 = \frac{22}{7}\text{ cm}$$

$$\text{Displacement} = AB = \sqrt{AP^2 + PB^2} = 2\sqrt{2}\text{ cm}$$

(ii) $$\text{Distance} = \frac{1}{2} \times \text{Circumference of the circle with radius AP}$$
$$= \frac{1}{2} \times 2 \times \frac{22}{7} \times 2 = \frac{44}{7}\text{ cm}$$

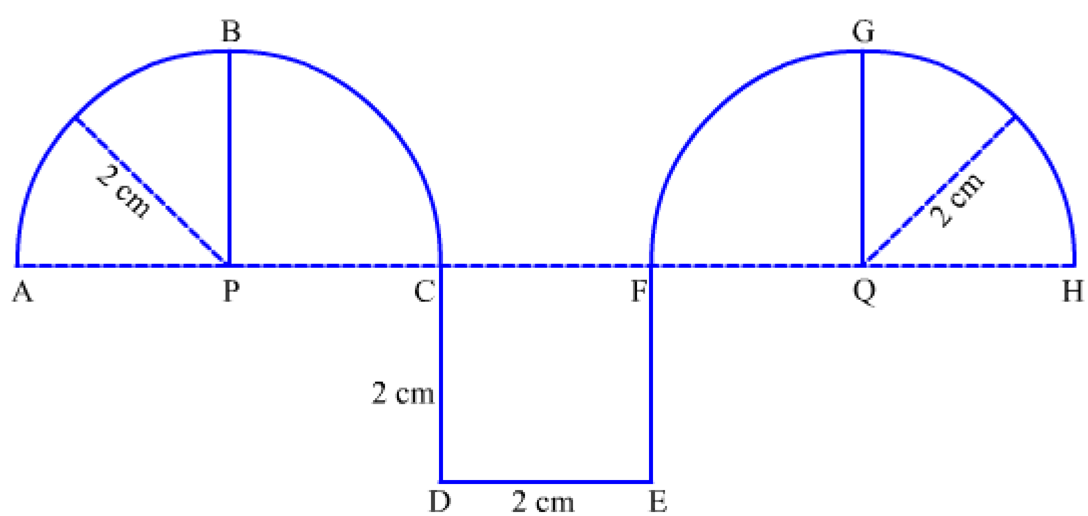
$$\text{Displacement} = AC = AP + PC = 4\text{ cm}$$

(iii) $$\text{Distance} = \left(\frac{1}{2} \times \text{Circumference of the circle with radius AP}\right) + CD$$
$$= \left(\frac{1}{2} \times 2 \times \frac{22}{7} \times 2\right) + 2 = \frac{58}{7}\text{ cm}$$

$$\text{Displacement} = AD = \sqrt{AC^2 + CD^2} = \sqrt{4^2 + 2^2} = \sqrt{20}\text{ cm}$$

**Medium**

**Example 2**: **Raju walks on a track with junctions A, B, C, D, E, F, G and H.**




**What are the magnitudes of the distance travelled and displacement as Raju moves from:**

**(i) B to E: Distance = ______ cm ; Displacement = ______ cm**

**(ii) B to G: Distance = ______ cm ; Displacement = ______ cm**

**(iii) B to H: Distance = ______ cm ; Displacement = ______ cm**

**Solution**:

$$\text{(i) Distance} = \left(\frac{1}{4}\times\text{Circumference of the circle with radius BP}\right)+\text{CD}+\text{DE}$$

$$=\left(\frac{1}{4}\times 2\times\frac{22}{7}\times 2\right)+2+2$$

$$=\frac{50}{7}\text{ cm}$$

$$\text{Displacement} = \text{BE} = \sqrt{(2+2)^2+(2+2)^2} = 4\sqrt{2}\text{ cm}$$

$$\text{(ii) Distance} = \left(\frac{1}{4}\times\text{Circumference of the circle with radius BP}\right)+\text{CD}+\text{DE}+\text{EF}$$

$$+\left(\frac{1}{4}\times\text{Circumference of the circle with radius QG}\right)$$

$$=\left(\frac{1}{4}\times 2\times\frac{22}{7}\times 2\right)+2+2+2+\left(\frac{1}{4}\times 2\times\frac{22}{7}\times 2\right)$$

$$=\frac{86}{7}\text{ cm}$$

$$\text{Displacement} = \text{BG} = 2+2+2 = 6\text{ cm}$$

$$\text{(iii) Distance} = \left(\frac{1}{4}\times\text{Circumference of the circle with radius BP}\right)+\text{CD}+\text{DE}+\text{EF}$$

$$+\left(\frac{1}{2}\times\text{Circumference of the circle with radius QG}\right)$$

$$=\left(\frac{1}{4}\times 2\times\frac{22}{7}\times 2\right)+2+2+2+\left(\frac{1}{2}\times 2\times\frac{22}{7}\times 2\right)$$

$$=\frac{108}{7}\text{ cm}$$

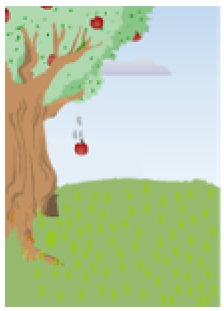
$$\text{Displacement} = \text{BH} = \sqrt{2^2+(2+2+2+2)^2} = \sqrt{2^2+8^2} = 2\sqrt{17}\text{ cm}$$

**Hard**

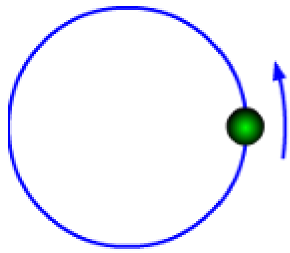
**Example 3**: **Raju walks on a track with junctions A, B, C, D, E, F, G and H.**

**Types of Motion**

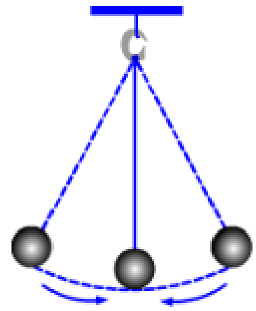
Motion can be broadly classified into three main categories: translatory motion, rotational motion and periodic motion



**Falling of an apple**



**A bead on a circular track**



**An oscillating bob**

*Translatory motion is the motion of a particle in a straight line.* A bus travelling on a straight road and an apple falling from a tree are examples of this kind of motion.

*Rotational motion refers to the motion of a body around a fixed axis.* A spinning top, a bead moving on a circular track and Earth's rotation are examples of this kind of motion.

The combination of translatory and rotational motion is called **rolling motion.** The motion of the wheels of a car moving along the road is an example of rolling motion.

*Periodic motion refers to the motion that is repeated in a regular interval of time.* An oscillating spring and the motion of a planet around the sun illustrate this type of motion.

**Know More**The simplest kind of periodic motion (to and fro or back and forth movement) about a mean position, under the influence of a restoring force that is directly proportional to the displacement from the mean position, is called a **Simple Harmonic Motion** (SHM).

**Types of Motion**

**Linear Motion:** The word 'linear' means 'straight' and the word 'motion' means 'change in position with respect to a frame of reference'. So, *a body moving in a straight line with respect to a frame of reference is said to be in linear motion.* An example of this is the motion of an ant on a straight wire.

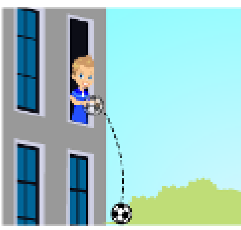
**Points to remember regarding linear motion**:

- In linear motion, the object must move in a straight line.
- The motion of the object along the straight line may not be uniform.

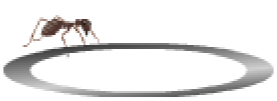
**Uniform motion**: If a body covers equal distances along a straight line in regular intervals of time, then the motion is said to be uniform. Examples:

1. A ball pushed in free space will continue to move uniformly, covering equal distances in equal intervals of time along a straight path.
2. If an ant covers equal distances in equal intervals of time along a straight wire, its motion is uniform.

**Non-uniform motion**:If a body covers unequal distances in regular intervals of time, then the motion is said to be non-uniform.Examples:

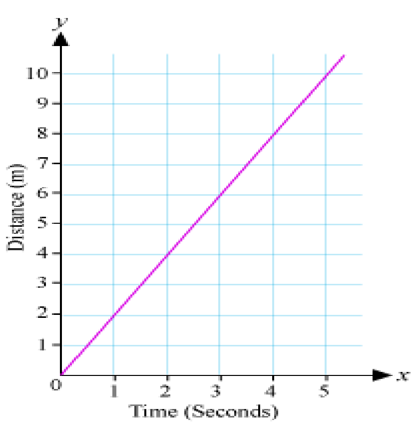


The ball takes a curved path when thrown. Its direction of motion changes with time. Also, it covers unequal distances in regular intervals of time. So, its motion is non-uniform.



The ant is moving on a circular wire. It is travelling equal distances in equal intervals of time, but its direction of motion is not constant. So, its motion is non-uniform.

**Types of Motion**



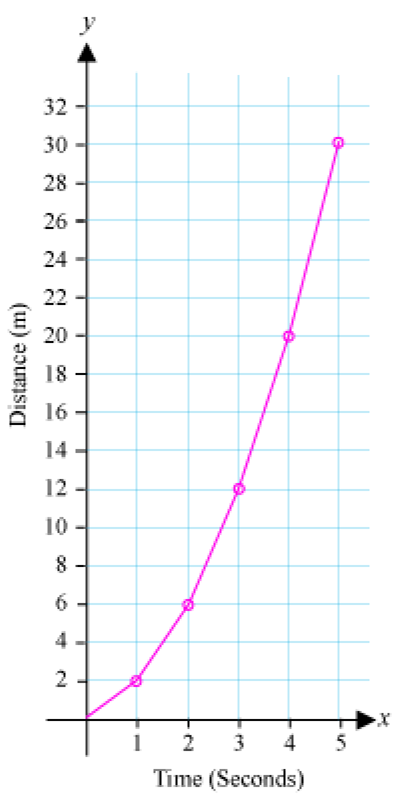
The distance−time graph of a body moving uniformly is a straight line.

Suppose a girl is riding a bicycle. The given table lists the data related to her motion.

| **Time (in seconds)** | **Distance (in metres)** |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |
| 5 | 10 |

On plotting these points, the distance−time graph turns out to be a straight line (as shown in the figure). Hence, the girl's motion is uniform.

**Types of Motion**




Let us assume that a car is moving in such a manner that the distance covered by it in every consecutive second increases by 2 m. The given table lists the data related to the motion of the car:

| **Time (in seconds)** | **Distance (in metres)** |
|---|---|

| | |
|---|---|
| 1 | 2 |
| 2 | 6 |
| 3 | 12 |
| 4 | 20 |
| 5 | 30 |

**What can you say about this motion? Is it uniform or non-uniform?**

On plotting these points, the distance−time graph turns out to be a curved line (as shown in the figure). Hence, the motion of the car is non-uniform.

Thus, we can conclude that the distance versus time graph of a body moving in non-uniform motion can never be a straight line.

**Whiz Kid**

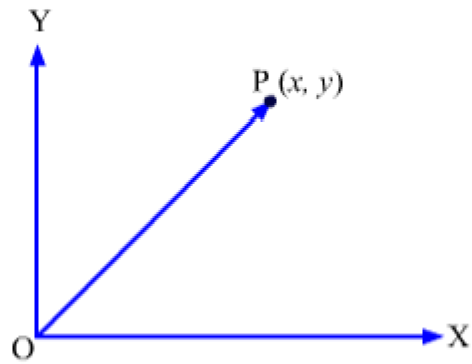
The table below shows the distances travelled by four cars after each minute from the start of a race.

| **Time (in min)** | **Car A (in m)** | **Car B (in m)** | **Car C (in m)** | **Car D (in m)** |
|---|---|---|---|---|
| 1 | 1020 | 1030 | 1050 | 1040 |
| 2 | 2040 | 2040 | 2090 | 2080 |
| 3 | 3060 | 3070 | 3120 | 3120 |
| 4 | 4080 | 4090 | 4130 | 4160 |
| 5 | 5100 | 5120 | 5150 | 5200 |

Can you determine which cars are travelling with uniform motion and which are travelling with non-uniform motion?

**Know More**

**Concept of Position Vector:** *The vector that represents the position of a point with respect to the origin of a frame of reference is called the position vector.* In the given figure, OX and OY are the axes of the frame of reference, with O as the origin.




Where $\hat{i}$ = unit vector along X-axis
$\hat{j}$ = unit vector along Y-axis

Position vector of point P = $\overrightarrow{OP}$

Coordinates of point P = $(x, y)$

Position vector of point P, in terms of the coordinates, is given as:

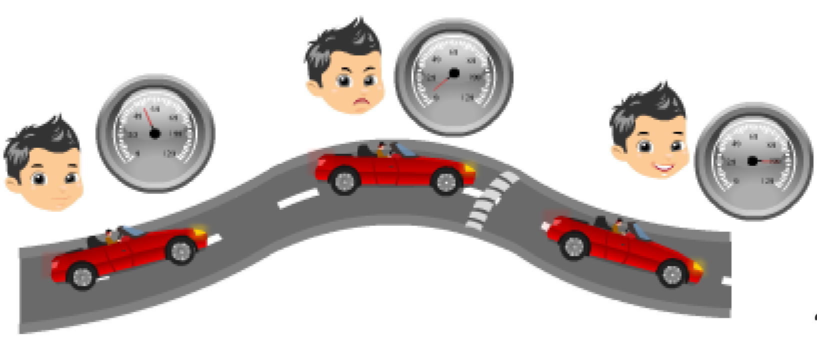
$\overrightarrow{OP} = x\hat{i} + y\hat{j}$

The magnitude of the position vector, which is also the distance between point P and origin O, is given as:

$OP = \sqrt{x^2 + y^2}$

**Speed and velocity**

**Speed: An Overview**



'Speed up!' is perhaps what you think when you are driving with your parents on a traffic-free road. You note how the reading in the speedodometer has changed since you started the journey. You stare at the speedometer to get an idea of the speed at which you are progressing. But the reading on it hardly ever stabilizes! It keeps rising and falling instead. Why do you think this happens? What if you want to find out at the end of your journey just how quickly you completed it? Will the speedometer help you to do that? Also, what about the velocity of the car? How is it different from its speed? Go through this lesson and find the answers to these queries.

We use the term ‘speed’ in our daily life to describe how fast a body moves.
The **distance** covered with progressing time gives us the value of speed.

*Speed is defined as the rate of distance covered by a body.*

Mathematically, speed is given as:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

**Average Speed**

A body travelling from one location to another might stop, slow down, speed up or move at a constant speed.

*The average speed of a body is defined as the total distance travelled divided by the total time taken.*

Mathematically, average speed is given as:

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Let us assume a body travels at a speed $v_1$ for time $t_1$, at a speed $v_2$ for time $t_2$, at a speed $v_3$ for time $t_3$ and so on, up to a speed $v_n$ for time $t_n$.

Distance travelled in time $t_1 = v_1 t_1$
Distance travelled in time $t_2 = v_2 t_2$
Distance travelled in time $t_3 = v_3 t_3$
..........................................................
Distance travelled in time $t_n = v_n t_n$
So, total distance travelled, $S = v_1 t_1 + v_2 t_2 + v_3 t_3 + \ldots\ldots + v_n t_n$
And, total time taken, $T = t_1 + t_2 + t_3 + \ldots\ldots + t_n$

$$\text{Therefore, average speed} = \frac{S}{T} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \ldots\ldots + v_n t_n}{t_1 + t_2 + t_3 + \ldots\ldots + t_n}$$

**Instantaneous Speed**

While travelling in a car in heavy traffic, you might have noticed the fluctuation of the speedometer. The speedometer gives the speed of the car at every instant of the journey.

*The speed of an object at any instant of its journey is called instantaneous speed.*


**Study Tip**:

If the units of distance and time are known in kilometre and hour respectively, then the unit of speed obtained will be km/h.

If the units of distance and time are known in metre and second respectively, then the unit of speed obtained will be m/s.

**Did You Know?**

The maximum speed that can be attained by a body is the speed of light. The speed of light is
$3 \times 10^8$ m/s. If you are travelling at this speed, then it will take you just about eight and a half minutes to reach the sun from Earth.

**Know More**

| **Average speed** | **Instantaneous speed** |
|---|---|
| 1. Average speed is the total distance covered divided by the total time taken. | 1. Instantaneous speed is the speed at any instant during the course of motion. |
| 2. Average speed is always a non-zero positive value whenever there is a motion. | 2. Instantaneous speed can be zero at any instant during the course of motion. |

In our daily life, whenever we use the term ‘speed’, we generally mean ‘instantaneous speed’.

**Velocity and Average Velocity**

When we include the direction of motion with speed, we are talking of the physical quantity called velocity. Thus, velocity is speed with direction. ***Velocity is defined as the rate of change of displacement*** . It is a vector quantity.

A body moving from one point to another may change its velocity a number of times, but it will have an average velocity of its journey. ***Average velocity of a body is defined as the net displacement divided by the total time of travel.*** **It is a vector quantity.** Its SI unit is m/s and it can be positive, negative or zero.

Let us assume a body travels at a velocity $\vec{v}_1$ for time $t_1$, at a velocity $\vec{v}_2$ for time $t_2$, at a velocity $\vec{v}_3$ for time $t_3$ and so on, up to a velocity $\vec{v}_n$ for time $t_n$.

[The velocities are all parallel to each other]

Displacement in time $t_1 = \vec{v}_1 t_1$

Displacement in time $t_2 = \vec{v}_2 t_2$

Displacement in time $t_3 = \vec{v}_3 t_3$

..........................................................

Displacement in time $t_n = \vec{v}_n t_n$

So, net displacement, $\vec{S} = \vec{v}_1 t_1 + \vec{v}_2 t_2 + \vec{v}_3 t_3 + \ldots\ldots + \vec{v}_n t_n$

And, total time taken, $T = t_1 + t_2 + t_3 + \ldots\ldots + t_n$

$$\text{Average velocity } = \frac{\vec{S}}{T} = \frac{\vec{v_1}t_1 + \vec{v_2}t_1 + \vec{v_3}t_3 + \ldots.. + \vec{v_n}t_n}{t_1 + t_2 + t_3 + \ldots.. + t_n}$$

**Solved Examples**

**Easy**

**Example 1**:**A car moves the first 20 km of a journey at a speed of 40 km/hr, and the next 20 km at a speed of 60 km/hr. What is the average speed of the car?**

**Solution**:

We have:

Total distance covered, $S = 20 + 20 = 40$ km

Speed during the first 20 km = 40 km/h

Time taken ($t_1$) to cover the above distance $= \frac{20}{40} = \frac{1}{2}$ h

Speed during the next 20 km = 60 km/h

Time taken ($t_2$) to cover the above distance $= \frac{20}{60} = \frac{1}{3}$ h

Total time taken, $T = t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ h

$\therefore$ Average speed of the car $= \frac{S}{T} = \frac{40}{\frac{5}{6}} = 48$ km/h

**Example 2**: **A toy car moves in a straight line, away from a child playing with it. It moves 116 m in 14 s. Due to some internal circuitry problem, the car turns abruptly and moves halfway back in 4.8 s.**

**(i) What is the average speed of the toy car?**

**(ii) What is average velocity of the toy car away from the child?**

**Solution**:

(i) Total distance travelled by the toy car $= 116 + \frac{116}{2} = 174$ m

Total time taken to travel the above distance $= 14 + 4.8 = 18.8$ s

$\therefore$ Average speed of the toy car $= \frac{174}{18.8} = 9.26$ m/s

(ii) Total displacement of the toy car $= 116 - \frac{116}{2} = 58$ m (away from the child)

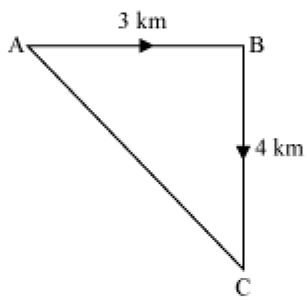
Total time taken to travel $= 14 + 4.8 = 18.8$ s

$\therefore$ Average velocity of the toy car $= \frac{58}{18.8} = 3.09$ m/s (away from the child)

**Medium**

**Example 3**: **A car moves 3 km east and then 4 km south. The total time taken by the car is 30 min. What is the velocity of the car in m/s? What is its speed?**

**Solution**:




The path taken by the car is depicted in the figure.

The displacement of the car is along AC. Since ABC is a right triangle, we can use Pythagoras theorem to find AC.

$AC^2 = AB^2 + BC^2$

Here, AB = 3 km

BC = 4 km

Hence, $AC^2 = 3^2 + 4^2$

$\Rightarrow AC^2 = 9 + 16$

$\Rightarrow AC^2 = 25$

$\Rightarrow \therefore AC = 5$ km

Time taken by the car = 30 min = (30 × 60) s

The velocity of the car is given as:

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$
$$= \frac{5}{30 \times 60} \text{ km/s} = \frac{5 \times 1000}{30 \times 60} \text{ m/s} = 2.78 \text{ m/s}$$

Therefore, the velocity of the car is 2.78 m/s toward southeast.

Total distance travelled by the car = 3 + 4 = 7 km

The speed of the car is given as:

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{7}{30 \times 60} \text{ km/s} = \frac{7 \times 1000}{30 \times 60} \text{ m/s} = 3.89 \text{ m/s}$$

**Example 4: A private plane travels 1937.5 mi at a speed of 493.75 mi/h. It encounters a tailwind that increases its speed to 618.75 mi/h for the next 1750 mi. (1 mi = 1.6 km)**

**(i) What is the total time of the trip?**

**(ii) What is the average speed of the trip?**

**Solution**:

We know that 1 mile = 1.6 km.

Note: Calculations are being done after converting all 'miles' to 'kilmetres'.

(i) Time taken to travel the first 3100 km $= \frac{3100}{790} = 3.92$ h

Time taken to travel the next 2800 km $= \frac{2800}{990} = 2.82$ h

$\therefore$ Total time of travel $= 3.92 + 2.82 = 6.74$ h

(ii) Average speed $= \frac{3100 + 2800}{6.74} = 875.37$ km/h

**Hard**

**Example 5: Rohit drives home from his friend's home at a constant speed of 95 km/h for 130 km. As it begins to rain heavily, he is forced to slow down to 65 km/h. He arrives home after driving for 3 hours and 20 minutes.**

**(i) How far is Rohit's home from that of his friend?**

**(ii) What is Rohit's average speed?**

**Solution**:

Rohit travels a distance of 130 km at speed of 95 km/h.

Time taken ($t_1$) to cover the above distance $= \frac{130}{95} = \frac{26}{19}$ h

His total time of travel, $T = 3\text{h } 20 \text{ min} = 3 + \frac{1}{3}$ h $= \frac{10}{3}$ h

So, time taken ($t_2$) for the rest of his journey $= T - t_1 = \frac{10}{3} - \frac{26}{19} = \frac{112}{57}$ h

Then, distance travelled in time $t_2 = 65 \times \frac{112}{57} = \frac{7280}{57}$ km

(i) Total distance travelled $= 130 + \frac{7280}{57} = 257.7$ km

(ii) Average speed $= \frac{257.7}{\frac{10}{3}} = 77.3$ km/h

**Acceleration**

**Acceleration: an Overview**

'Please fasten your seat belt...' is what the cabin crew tells us when the plane is about to take off or land. In each of these situations, there is a change in the **velocity** of the plane and our body can sense this change. Consequently, we move backward (in case of a takeoff) or forward (in case of a landing).This is also the case when the driver of the school bus presses the accelerator or the brake. We tend to move forward or backward as our body senses the change in the velocity of the vehicle. Let us learn how this change is measured.

**Did You Know?**

The human body cannot sense velocity or speed, but it can sense acceleration. In other words, the human body acts as an accelerometer, and not as a speedometer.

If any vehicle is moving at a constant velocity, then the person or persons travelling in it do not have any resultant bodily movement. As a result, the motion of the vehicle cannot be sensed. However, if the velocity of the vehicle changes, then the travellers can sense this change and thereby sense the motion of the vehicle.

**Acceleration**

Whenever there is a change in velocity, there is acceleration. A motion is said to be accelerated when there is a change in speed or change in direction of motion or change in both speed and direction.

*Acceleration is defined as the rate of change of velocity.*

It is a vector quantity and its direction is given by the direction of the force causing the acceleration. Its SI unit is $m/s^2$. Mathematically, acceleration is given as:

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$


Suppose the velocity of a car is $u$ at time $t_1$. Later, at time $t_2$, its velocity becomes $v$.

Change in velocity = $(v - u)$, time interval = $t_2 - t_1$

$$\therefore \text{Acceleration, } a = \frac{v-u}{t_2 - t_1}$$

**Uniform Acceleration and Non-Uniform Acceleration**

Force is the cause of acceleration of a body. The direction of force gives the direction of the acceleration induced by it. Depending on the force, the acceleration may increase or decrease the velocity of a body. Acceleration can be positive, negative or zero. Depending on the rate of change of velocity, the acceleration may be constant or may be a variable quantity. So, we can have **uniform acceleration** or **non-uniform acceleration.**

*If the rate of change of velocity remains constant, then the acceleration is uniform.* Examples of uniform acceleration include a ball under free fall, a ball rolling on an inclined plane and a car accelerating on a straight, traffic-free road.

*If the rate of change of velocity changes with time, then the acceleration is non-uniform.*An example of non-uniform acceleration is a car accelerating on a straight road with traffic.

The acceleration is positive when the velocity of the moving body increases with time and in the direction of velocity.
When the acceleration is such that the velocity of the moving body decreases, it is called deceleration or retardation. Deceleration or retardation is the rate of decrease in velocity. In this case, the acceleration is negative and in the opposite direction of velocity.
The acceleration of a body is considered to be zero when the velocity of the moving body does not change.

**SI Unit of Acceleration**

We know that the SI units of velocity and time are m/s and s respectively.

**Let us determine the SI unit of acceleration.**

Acceleration is defined as follows:

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$\therefore \text{SI unit of acceleration} = \frac{\text{SI unit of velocity}}{\text{SI unit of time}}$$

$$= \frac{\text{m/s}}{\text{s}}$$

$$= \text{m/s}^2 \text{ or ms}^{-2}$$

**Did You Know?**

Suppose a body is moving in a circular path at a constant speed. Its velocity will change at every instant of time because of a change in its direction. Therefore, this body is said to be moving under acceleration.

**Equations of motion:**

Equations of motion are derived from the relation between the velocity, time, distance and the acceleration. These equations are very helpful in determining the values of these quantities.

$$a = \frac{v-u}{t} \text{ or, } v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as \text{ or, } v^2 - u^2 = 2as$$

where $s$ = distance, $u$ = initial velocity, $v$ = final velocity, $t$ = time and $a$ = acceleration

**Solved Examples**

**Easy**

**Example 1**: **A car accelerates to a speed of 95 km/h from rest in 6.2 s. What is the acceleration in**
**SI unit?**

**Solution**:

Initial velocity, $u = 0$

Final velocity, $v = 95 \text{ km/h} = 95 \times \frac{5}{18} = \frac{475}{18} \text{ m/s}$

Time taken, $t = 6.2 \text{ s}$

$$\therefore \text{Acceleration, } a = \frac{v-u}{t} = \frac{\frac{475}{18} - 0}{6.2} = 4.3 \text{ m/s}^2$$

**Medium**

**Example 2**: **A bike accelerates at 2.5 m/s² for 10 s. What should be the initial velocity of the bike if after 20 s from the start its velocity is 50 m/s?**

**Solution**:

Acceleration, $a = 2.5 \text{ m/s}^2$

Time, $t = 10$ s (After 10s, acceleration is zero.)

Initial velocity $= u$

Final velocity, $v = 50$ m/s

We have the relation:

$$a = \frac{v-u}{t}$$

$$\Rightarrow u = v - at$$

$$\Rightarrow u = 50 - 2.5 \times 10$$

$$\Rightarrow \therefore u = 25 \text{ m/s}$$

**Hard**

**Example 3**: **A man driving a car on a straight road travels at a constant speed for ten seconds till he reaches the traffic point. Here, he applies brakes for five seconds to reduce the speed of the car to one-fourth the initial value. After crossing the traffic point, he keeps driving at the same slow speed due to traffic and covers a quarter of a kilometre in twenty seconds.**

**(i) What is the initial velocity of the car before the application of the brakes?**

**(ii) What is the final velocity of the car after the application of the brakes?**

**(iii) What is the acceleration of the car?**

**(iv) What is the total distance covered by the car during constant velocities?**

**Solution**:

Let us take:

$u$ = Initial velocity of the car

$v$ = Final velocity of the car

$a$ = Acceleration of the car

It is given that the speed is reduced to one-fourth the original speed

$$\therefore v = \frac{u}{4} \qquad ...(1)$$

The car travels at the velocity $v$ after crossing the traffic point.

It covers $\frac{1}{4}$ km or 250 m in 20 s.

So, $v = \frac{250}{20} = 12.5 \text{ m/s}$

$(1) \Rightarrow u = 4 \times v = 4 \times 12.5 = 50 \text{ m/s}$

Therefore,

(i) Initial velocity the car = 50 m/s

(ii) Final velocity of the car = 12.5 m/s

(iii) Acceleration, $a = \frac{v-u}{t} = \frac{12.5-50}{5} = -7.5 \text{ m/s}^2$ (Negative acceleration)

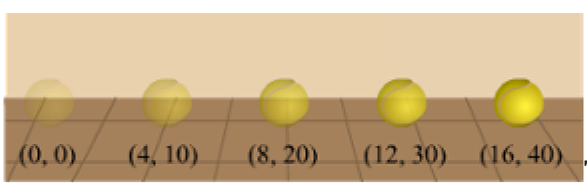
(iv) Distance travelled in the first 10 s of constant velocity $= 50 \times 10 = 500$ m

Distance travelled in the last 20 s of constant velocity $= 12.5 \times 20 = 250$ m

$\therefore$ Total distance travelled during the constant velocities $= 500 + 250 = 750$ m

**Distance-Time and Velocity-Time Graph**

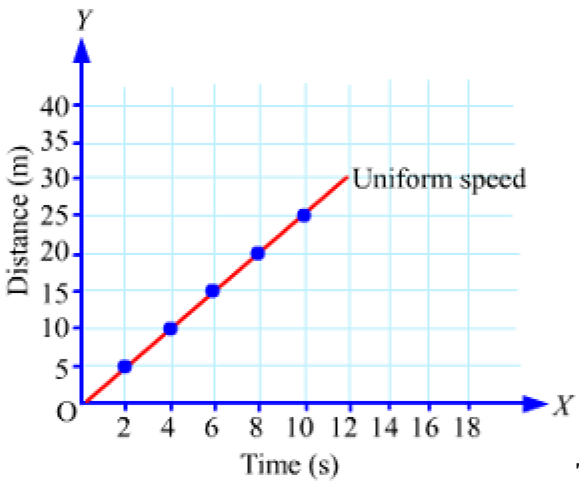
**Distance−Time Graph**




The given table lists the distances travelled by a ball in specific time intervals.

| **Time (s)** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Distance (m)** | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |




The positions of the ball at different time intervals are shown in the figure.

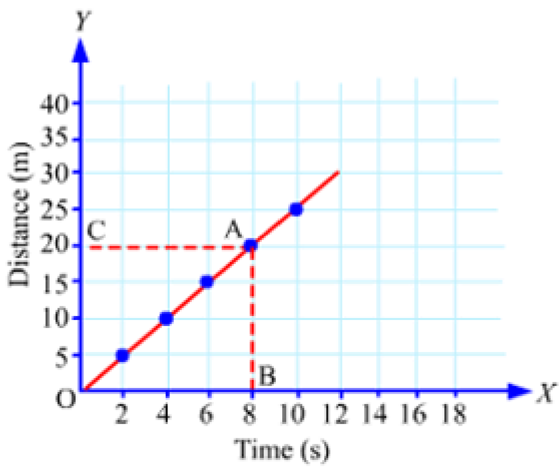
It is clear that the ball covers equal distances in equal time intervals. Hence, the motion of the ball is uniform.

We have plotted a distance−time graph to represent the nature of motion of the ball. The general convention while drawing such a graph is to take 'time' along the *x*-axis and 'distance' along the *y*-axis.

It is clear that for a uniform motion, the distance−time graph is a straight line.

**Distance−Time Graph Significance**

The distance−time graph of a moving body helps us ascertain whether the motion is uniform or non-uniform. It also helps us determine the speed of the




body. Let us consider the same distance–time graph of the ball.

| Time (s) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|---|---|---|---|
| **Distance (m)** | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |

We have marked the points A (8, 20), B (8, 0) and C (0, 20) on the graph.

$$\text{Slope} = \frac{AB}{BO} = \frac{AB}{AC}$$

$$\text{So, speed} = \frac{AB}{AC}$$

In the given case:

$$\text{Speed} = \frac{AB}{AC}$$

$$= \frac{(20-0)\text{ m}}{(8-0)\text{ s}}$$

$$= \frac{20}{8} = 2.5\text{ m/s}$$

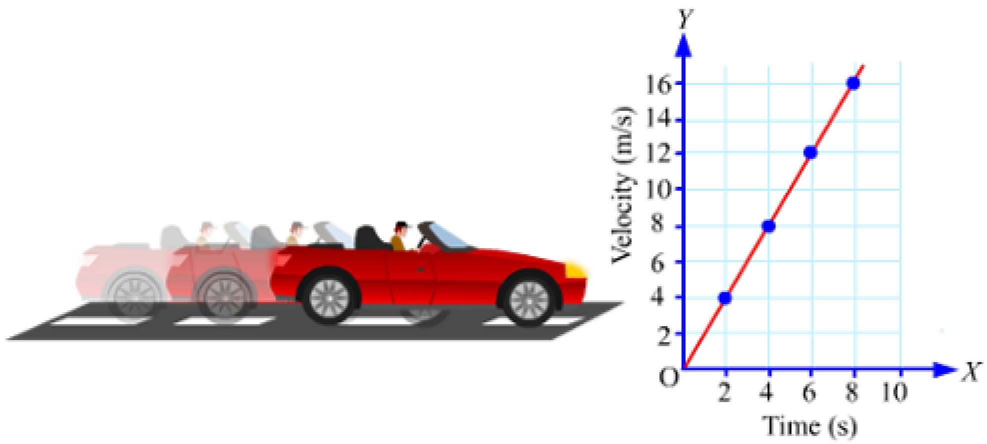
Thus, we have found that the ball is moving uniformly at a speed of 2.5 m/s.

**Velocity–Time Graph**

Graphical representation is the easiest and most convenient way to describe a motion. The graphical representation of the change in velocity of a moving body in equal time intervals is known as the velocity–time graph.

The following table lists the velocities of a car in specific time intervals. We have plotted a velocity–time graph using this information

| **Time (s)** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| **Velocity (m/s)** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |




The acceleration of the car can be computed from the slope of the velocity–time graph. In case of zero acceleration, the graph is a straight line parallel to the time axis.

We know that,

**Displacement = Velocity × Time**

So, the area under the velocity–time graph, on the time axis, gives the net displacement of the car in a given interval of time. For example, the displacement of the car in the first four seconds is given as:

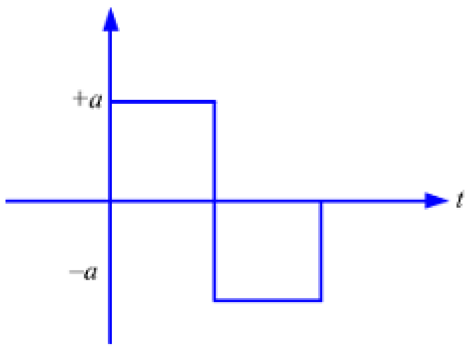
Displacement = Area under the velocity - time graph.

= $\frac{1}{2} \times Base \times height$

= $\frac{1}{2} \times 8 \times 4$

= 16 m

**Acceleration–Time Graph**




The acceleration−time graph gives us a clear idea about the acceleration and deceleration of a moving body.

The given graph shows a car accelerating at first, and then decelerating.

The straight line on the positive side of acceleration axis tells us that the car is accelerating uniformly.

The straight line on the negative side of acceleration axis tells us that the car is decelerating uniformly.

In case of zero acceleration, the graph coincides with the time axis.

**You must know that time can never be negative for any type of motion. Therefore, acceleration can only be negative when the velocity is decreasing.**

We know that: **Velocity = Acceleration × Time**

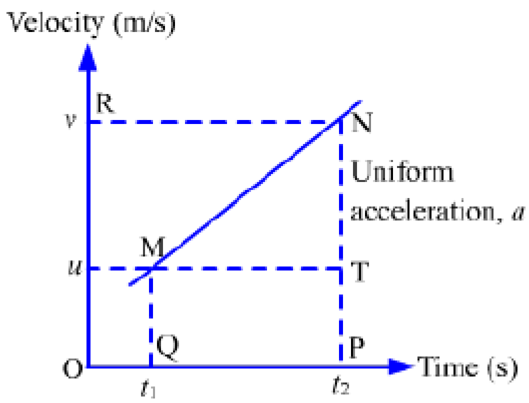
So, the area under the acceleration−time graph, on the time axis, gives the velocity of the moving object.

**First Equation of Motion**

**Velocity−Time Relation**

Suppose a body is moving under a **uniform acceleration** in a given time interval. We can relate the change in the **velocity** of the moving body with the acceleration and time taken by using the one-dimensional velocity-time equation.

The velocity-time equation can be used for obtaining the final velocity, after time *t*, of a uniformly accelerating body.




**Velocity–Time relation through the graphical method**

Suppose a body is moving in a straight line, with an initial velocity $u$ and under a uniform acceleration $a$. Its velocity becomes $v$ after time $t$. The motion of this body is represented by the given velocity-time graph.

We can obtain the velocity-time equation if the velocities of a body ($u$ and $v$) at times $t_1$ and $t_2$ are given, as shown in the velocity-time graph.

Initial velocity, $u$ = MQ

Final velocity, $v$ = NP

Time taken, $t$ = QP = $(t_2 - t_1)$

Acceleration, $a$ = Slope of line MN = NT/ MT = (NP - TP)/ (OP - OQ)

It is clear from the graph that TP = MQ

So, $a = (v - u)/ t_2 - t_1$

or, a $(t_2 - t_1) = v - u$ or, $v = u + a\,(t_2 - t_1)$

For initial time $t_1 = 0$, the equation reduces to: $v = u + a\,t_2$ or $v = u + at$ (as $t_2 = t$)

This is the **first equation of kinematics** and it is independent of the distance travelled. It is also known as the first equation of motion.

**Solved Examples**

**Easy**

**Example 1**: **On spotting a prey, a cheetah runs directly towards it with constant acceleration. The time taken by the cheetah is 50 s and its velocity, as it catches its prey, is 25 m/s. If we assume that the cheetah was initially at rest, then what is its acceleration?**

**Solution**:

It is given that:

Initial velocity ($u$) of the cheetah = 0

Its final velocity, $v$ = 25 m/s

Time taken ($t$) by it to catch its prey= 50 s

We can determine the acceleration ($a$) of the cheetah using the relation:

$$a = \frac{v-u}{t}$$
$$= \frac{25-0}{50} = \frac{25}{50} = 0.5 \text{ m/s}^2$$

**Medium**

**Example 2**: **A motorcyclist is travelling at a constant velocity of 10 m/s. In order to overtake a car, he accelerates at the rate of 0.2 m/s$^2$. If he overtakes the car in 60 seconds, then what is his velocity while overtaking?**

**Solution**:

It is given that:

Initial velocity ($u$) of the motorcyclist = 10 m/s

His acceleration, $a$ = 0.2 m/s$^2$

Time taken ($t$) by him to overtake the car = 60 s

Using the first equation of motion, we can compute the velocity ($v$) of the motorcyclist while overtaking the car.

$v = u + at$

$\Rightarrow v = 10 + 0.2 \times 60$

$\Rightarrow v = 10 + 12$

$\Rightarrow \therefore v = 22$ m/s

**Hard**

**Example 3**:

**A train is moving under a constant acceleration of 150 km/h$^2$. It attains a velocity of 125 km/h in half-hour. What is the initial velocity of the train in SI unit?**

**Solution**:

It is given that:

Final velocity ($v$) of the train= 125 km/h

Time taken ($t$) by it to attain the above velocity= 0.5 h

Its acceleration ($a$) = 150 km/h$^2$

Using the first equation of motion, we can compute the initial velocity ($u$) of the train.

$v = u + at$

$\Rightarrow u = v - at$

$\Rightarrow u = 125 - 150 \times 0.5$

$\Rightarrow u = 125 - 75$

$\Rightarrow \therefore u = 50$ km/h

$$\text{Since } 1\,\text{km/h} = \left(\frac{5}{18}\right)\text{m/s}$$

$$50\,\text{km/h} = 50\left(\frac{5}{18}\right)\text{m/s}$$

$$= 13.89\ \text{m/s}$$

Therefore, the initial velocity of the train is 13.89 m/s.

**Second Equation of Motion**

**Position–Time Relation**

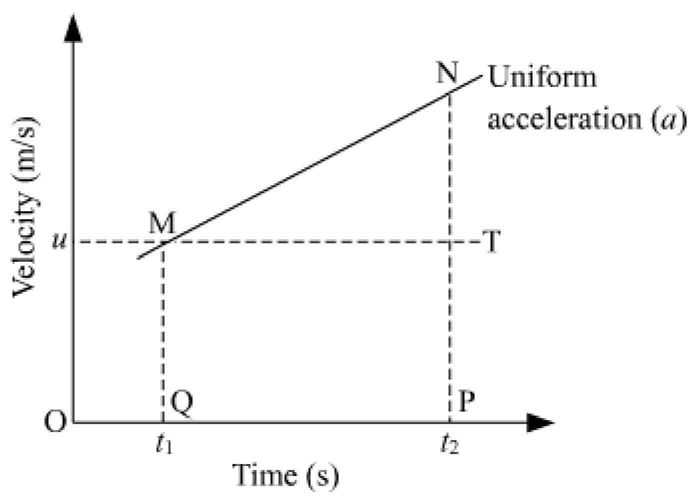
Suppose a body moving under a **uniform acceleration**

covers a certain **distance**

in a given time interval. We can relate the change in the **velocity**

of the moving body with the acceleration, distance covered and time taken by using the one-dimensional position–time equation.The position–time equation is used to obtain the distance travelled by a uniformly accelerating body in a given interval of time.

Suppose a body is moving in a straight line, with an initial velocity $u$ and under a uniform acceleration $a$. The distance covered by the moving body from time $t_1$ to time $t_2$ is represented in the given velocity–time graph.




It is clear from the graph that:

Initial velocity, $u$ = MQ = TP

Time, $t$ = PQ = $(t_2 - t_1)$ = MT

Change in velocity, NT = $a\,(t_2 - t_1)$

Distance, $s$ = Area of trapezium QMNP

= Area of rectangle QMTP + Area of triangle MTN

= (MQ × QP) + ($\frac{1}{2}$ × NT × MT)

$= [u \times (t_2 - t_1)] + \frac{1}{2} \times a\ (t_2 - t_1) \times (t_2 - t_1)$

$\therefore s = u\ (t_2 - t_1) + \frac{1}{2}\ a\ (t_2 - t_1)^2$

For $t_1 = 0$ and $t_2 = t$, the equation reduces to:

$$s = ut + \frac{1}{2}\ at^2$$

This is the **second equation of kinematics** or the second equation of motion.

**Average Velocity for Uniformly Accelerated Motion**

We can obtain the relation for average velocity using the velocity–time and position–time equations.

The velocity–time equation is given as:

$v = u + at$

The position–time equation is given as:

$s = ut + \frac{1}{2}\ at^2$

$\Rightarrow \frac{s}{t} = u + \frac{1}{2}\ at$

$\Rightarrow \frac{s}{t} = \frac{2u + at}{2}$

$\Rightarrow \frac{s}{t} = \frac{u + (u + at)}{2}$

$\Rightarrow \frac{s}{t} = \frac{u + v}{2}$ (Using $v = u + at$)

□**Average velocity ($v_{av}$) =** $\frac{s}{t}$

$= \frac{u+v}{2}$

**Did You Know?**

If the acceleration is zero, then the second equation of motion denotes the distance travelled as the product of the initial velocity and time.

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow s = ut + \frac{1}{2} \times 0 \times t^2$$
$$\Rightarrow s = ut$$

**Solved Examples**

**Easy**

**Example 1**: **A motorcyclist is travelling at a constant velocity of 10 m/s. He overtakes a car by accelerating at the rate of 0.2 m/s$^2$. If he overtakes the car in 60 s, then how much distance does he cover before overtaking the car?**

**Solution**:

It is given that:

Initial velocity ($u$) of the motorcyclist = 10 m/s

His acceleration, $a$ = 0.2 m/s$^2$

Time taken ($t$) by him to overtake the car = 60 s

Using the second equation of motion, we can compute the distance covered ($s$) by the motorcyclist before overtaking the car.

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow s = 10 \times 60 + \frac{1}{2} \times 0.2 \times 60^2$$
$$\Rightarrow s = 600 + \frac{1}{2} \times 0.2 \times 3600$$
$$\Rightarrow s = 600 + \frac{1}{2} \times 720$$
$$\Rightarrow s = 600 + 360$$
$$\Rightarrow \therefore s = 960 \text{ m}$$

**Medium**

**Example 2**: **A train moving at a speed of 180 km/h comes to a stop at a constant acceleration in 15 min after covering a distance of 25 km. What is its acceleration?**

**Solution**: It is given that:

Initial velocity ($u$) of the train = 180 km/h

Distance covered ($s$) by it = 25 km

Time taken ($t$) by it to cover the above distance = 15 min = 0.25 h

Using the second equation of motion, we can compute the acceleration ($a$) of the train.

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow 25 = 180 \times 0.25 + \frac{1}{2} \times a \times 0.25^2$$
$$\Rightarrow 25 = 45 + \frac{1}{2} \times a \times 0.0625$$
$$\Rightarrow 25 - 45 = 0.03125a$$
$$\Rightarrow a = \frac{-20}{0.03125}$$
$$\Rightarrow \therefore a = -640 \text{ km/h}^2$$

Hence, the train is retarding at a rate of 640 km/h$^2$. Note that since the speed of the train is decreasing, the acceleration comes out to be negative.

**Hard**

**Example 3**: **Brakes are applied on a car moving at a velocity of 72 km/h. It decelerates uniformly at the rate of 4 m/s$^2$ until it stops after 5 s. How far does the car go before it stops?**

**Solution**: It is given that:

Initial velocity ($u$) of the car = 72 km/h

$= 72 \times \left(\frac{5}{18}\right)$ m/s

= 20 m/s

Its acceleration, $a = -4$ m/s$^2$ (since the car decelerates)

Time taken ($t$) by it to stop = 5 s

Using the second equation of motion, we can compute the distance covered ($s$) by the car before stopping.

$s = ut + \frac{1}{2}at^2$

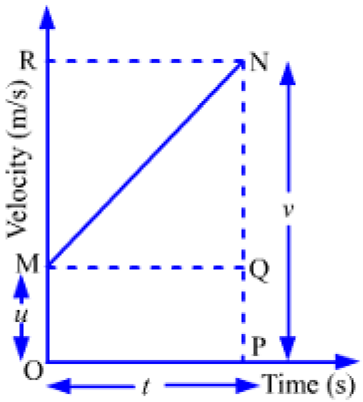
$\Rightarrow s = 20 \times 5 + \frac{1}{2} \times (-4) \times (5)^2$

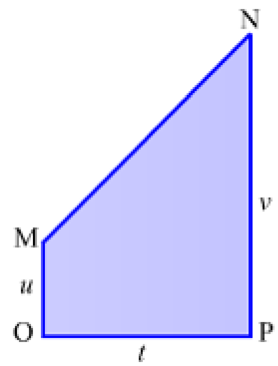
$\Rightarrow s = 100 - 50$

$\therefore s = 50$ m

**Third Equation of Motion**

**Position–Velocity Relation**




Suppose a body is moving in a straight line, with an initial velocity $u$ and under a uniform acceleration $a$. Its velocity becomes $v$ after time $t$ and it covers a distance $s$ in the given time interval. The motion of this body is represented in the given velocity–time graph.

It is clear from the graph that:

Initial velocity, $u$ = MO = QP

Final velocity, $v$ = OR = NP

The straight line MN represents the velocity–time curve.

Distance ($s$) covered by the body = Area of trapezium OMNP

$= \frac{1}{2} \times (\text{OM} + \text{PN}) \times \text{OP} \quad = \frac{1}{2} \times (u + v) \times t$

$\therefore s = \frac{1}{2}(u + v)\, t$ … (i)

Now, let us eliminate time $t$ from this equation.

The velocity-time equation is given as:

$v = u + at$

$\therefore t = \frac{v - u}{a}$ … (ii)

On substituting the value of $t$ from equation (ii) in equation (i), we obtain:

$s = \frac{1}{2} \times (u + v) \times \left(\frac{v-u}{a}\right)$

$$\Rightarrow s = \frac{(u+v)(v-u)}{2a}$$

$$\Rightarrow s = \frac{v^2 - u^2}{2a}$$

$$\therefore \boxed{v^2 - u^2 = 2as}$$

This is the **third equation of kinematics**. It is independent of time. It is also known as the third equation of motion.

**Deriving the Second Equation of Motion**

The third equation of motion is given as:

$v^2 = u^2 + 2as$ … (i)

The first equation of motion is given as:

$v = u + at$ … (ii)

On eliminating velocity $v$ from equation (i) with the help of equation (ii), we obtain:

$(u + at)^2 = u^2 + 2as$

$\Rightarrow u^2 + 2uat + a^2t^2 = u^2 + 2as$

$\Rightarrow 2uat + a^2t^2 = 2as$

$\Rightarrow s = \frac{1}{2a}(2uat + a^2t^2)$

$\Rightarrow s = ut + \frac{1}{2}at^2$ … (iii)

This is the second equation of motion.

**Solved Examples**

**Easy**

**Example 1**:**On applying the brakes, a cyclist travelling initially at 2 m/s comes to a halt at a constant retardation of 2 m/s$^2$. How much distance does the cyclist cover before coming to rest?**

**Solution**: It is given that:

Initial velocity ($u$) of the cyclist = 2 m/s

His final velocity, $v = 0$

His acceleration, $a = -2$ m/s$^2$ (since he is decelerating)

Using the third equation of motion, we can compute the distance covered ($s$) by the cyclist before stopping.

$$v^2 - u^2 = 2as$$
$$\Rightarrow v^2 = u^2 + 2as$$
$$\Rightarrow 0^2 = 2^2 - 2\times 2\times s$$
$$\Rightarrow 4 = 4s$$
$$\Rightarrow s = \frac{4}{4}$$
$$\Rightarrow \therefore s = 1 \text{ m}$$

**Medium**

**Example 2**: **A car covers 40 m in 8.5 s while applying brakes to a final speed of 2.8 m/s.**

**(i) What is the initial speed of the car?**

**(ii) What is its acceleration?**

**Solution**:

It is given that:
Final velocity ($v$) of the car = 2.8 m/s
Distance covered, $s = 40$ m
Time taken ($t$) to cover the above distance = 8.5 s

Let us take:
Initial velocity of the car = $u$
Acceleration of the car = $a$

We have the relation:

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow 40 = 8.5u + \frac{1}{2}a \times 8.5^2 \qquad ...(1)$$

We know that:

$$v = u + at$$

$$\Rightarrow u = v - at$$

$$\Rightarrow u = 2.8 - a \times 8.5 \qquad ...(2)$$

From (1) and (2) we get:

$$40 = 8.5(2.8 - a \times 8.5) + \frac{1}{2}a \times 8.5^2$$

$$\Rightarrow \therefore a = -0.45 \text{ m/s}^2$$

On substituting the value of $a$ in (2), we get:

$$u = 2.8 - (-0.45) \times 8.5$$

$$\Rightarrow \therefore u = 6.63 \text{ m/s}$$

Thus,
(i) The initial velocity of the car is 6.63 m/s.
(ii) The acceleration of the car is $-0.45$ m/s$^2$.

**Hard**

**Example 3**: **When the brakes are applied, a racing car stops within 0.0229 of a mile from a speed of**
**60 mi/h and within 0.0399 of a mile from a speed of 80 mi/h.**

**(i) What is the braking acceleration of the car for 60 mi/h to rest?**

**(ii) What is its braking acceleration for 80 mi/h to rest?**

**(iii) What is its braking acceleration for 80 mi/h to 60 mi/h?**

**Solution**:

(i) In the first case:
Inital velocity ($u$) of the car = 60 mi/h = 26.82 m/s
Its final velocity, $v = 0$
Distance covered, $s = 0.0229$ mile = 36.88 m
Let the acceleration of the car be $a$.

We know that:
$v^2 - u^2 = 2as$
$\Rightarrow v^2 = u^2 + 2as$
$\Rightarrow 0 = 26.82^2 + 2 \times a \times 36.88$
$\Rightarrow \therefore a = -9.75 \text{ m/s}^2$
Thus, the braking acceleration of the car for 60 mi/h to rest is $-9.75 \text{ m/s}^2$.

(ii) In the second case:
Initial velocity ($u$) of the car = 80 mi/h = 35.76 m/s
Its final velocity, $v = 0$
Distance covered, $s = 0.0399$ mile = 64.31 m
Again, let the acceleration of the car be $a$.

We know that:
$v^2 - u^2 = 2as$
$\Rightarrow v^2 = u^2 + 2as$
$\Rightarrow 0 = 35.76^2 + 2 \times a \times 64.31$
$\Rightarrow \therefore a = -9.94 \text{ m/s}^2$
Thus, the braking acceleration of the car for 80 mi/h to rest is $-9.94 \text{ m/s}^2$.

(iii) In the third case:

Initial velocity ($u$) of the car = 80 mi/h = 35.76 m/s

Its final velocity, $v$ = 60 mi/h = 26.82 m/s

Distance covered, $s$ = Distance covered in the second case − Distance covered in the first case

$$= 64.31 - 36.88 = 27.43 \text{ m}$$

Again, the acceleration is taken as $a$.

We know that:

$$v^2 - u^2 = 2as$$

$$\Rightarrow v^2 = u^2 + 2as$$

$$\Rightarrow 26.82^2 = 35.76^2 + 2 \times a \times 27.43$$

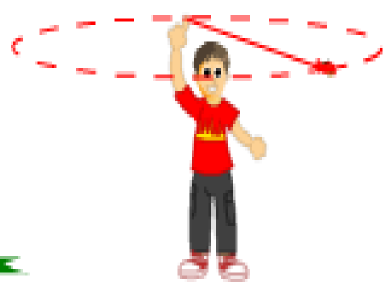
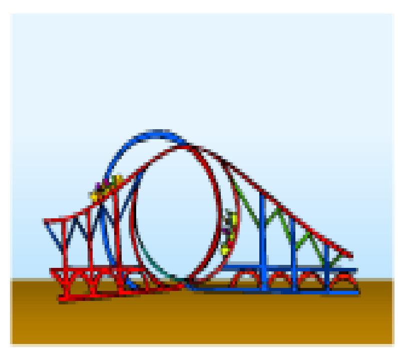
$$\Rightarrow \therefore a = -10.19 \text{ m/s}^2$$

Thus, the braking acceleration of the car for 80 mi/h to 60 mi/h is $-10.19 \text{ m/s}^2$.

## Uniform Circular Motion

### Uniform Circular Motion: An Overview

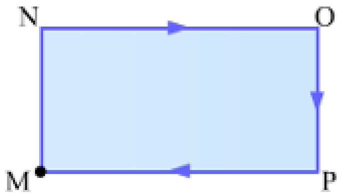
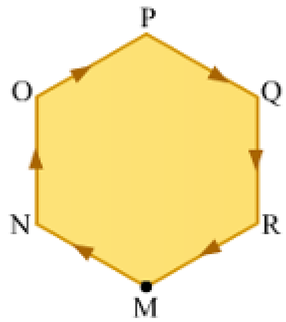
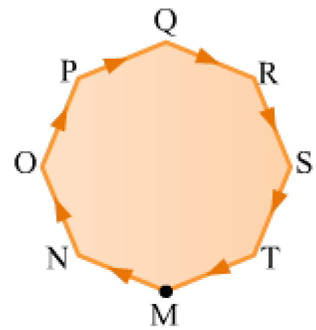
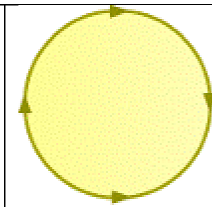
Circular motion is all around us. It is there in the English nursery rhyme 'Here We Go Round the Mulberry Bush'. It is there when you go round and round an endless circle on a merry-go-round. It is there when you rotate a string with a ball tied to it. How many real-life examples of this motion can you think of in a minute? Make a list.

Now, let us learn the physics of circular motion.

### Motion in a Closed Path and Circular Motion

When a body moves in a closed path, its final position is the same as its initial position. Let us consider some closed paths and an object moving on each of them at a constant speed.

|  |  |  |  |
| --- | --- | --- | --- |
| The object is moving at a constant speed on a rectangular path, along the edges MN, NO, OP and PM. During one round, the object changes its direction of motion four times at the corners N, O, P and M. | The object is moving at a constant speed on a hexagonal path, along the edges MN, NO, OP, PQ, QR and RM. During one round, the object changes its direction of motion six times at the corners N, O, P, Q, R and M. | The object is moving at a constant speed on an octagonal path, along the edges MN, NO, OP, PQ, QR, RS, ST and TM. During one round, the object changes its direction of motion eight times at the corners N, O, P, Q, R, S, T and M. | The object is moving at a constant speed on a circular path. Its direction of motion changes at every point on the path. The path can be considered as a polynomial with an infinite number of sides. |

**Uniform Circular Motion**

*When a body moves along a circular arc or a circle, it is said to be in circular motion.*

What is the kind of closed path that you trace every day, right from leaving the bed in the morning to going to bed at night?

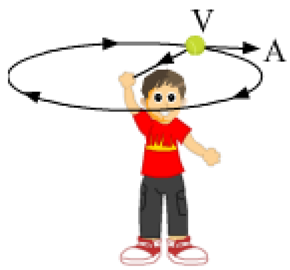
In **uniform motion**

, the velocity of the moving body remains constant. In circular motion, the **velocity**

can never be constant, but the **speed** of the moving body can be constant.

*A body moving in a circular path at a constant speed is said to be in uniform circular motion.*

Take a string with a ball tied at one end and rotate it. As you rotate the string, the ball traces a circular path in the air (as shown in the figure).




Let the length of the string be $r$. So, the radius of the circular track traced by the stone is $r$.
Let the time taken by the stone to complete one rotation be $T$.
Distance travelled in one rotation = Circumference of the circular track = $2\pi r$

Let the speed of rotation of the ball be *v*.

We know that:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow v = \frac{2\pi r}{T}$$

The time taken to complete one rotation is given as:

$$T = \frac{2\pi r}{v}$$

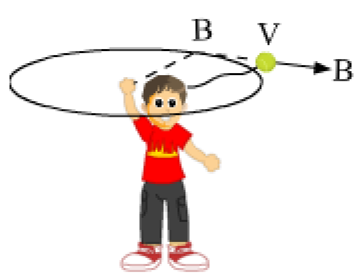
This is called the time period of revolution.

The number of complete revolutions per second is called frequency. It is measured in hertz or $s^{-1}$.

**Circular Motion: Velocity at Any Instant**

We know that the velocity of a body in circular motion changes at every instant. For one complete rotation, the average velocity is zero. This is because the displacement is zero. However, the body will have some finite instantaneous velocity at every instant.

*The velocity of a body in a circular path is always tangential to the path and is perpendicular to the radius of the circle.*



Consider once again the string with the ball tied to it. Suppose the boy loose grip of string. The stone with string will be free from boy's hand and leave the circular path tangentially. This happens because the tension in string—provided by the centripetal force required for the revolution—vanishes when the string flies off.

**Circular Motion: Velocity at Any Instant**

**What would happen if Earth were to stop rotating about its axis all of a sudden?**

At the equator, Earth rotates about its axis at the speed of 1674.4 km/h. We cannot feel this rotation because we, too, move with Earth at the same rate. Now, if Earth were to suddenly stopped spinning, everything on its surface at the equator would suddenly move at more than 1600 km/hour tangentially. The escape velocity of Earth is around 40000 km/h, so these things would not fly off into outer space; however, the speed is high enough to project the things to about 11 km from Earth's surface. This would result in serious damage.

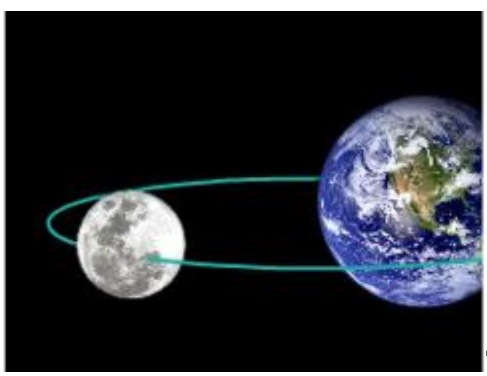
**When a body of mass *m* revolves with a uniform speed *v* on a circular path of radius *r*, the work done by the body in one complete rotation is zero. Why?**

Work is the scalar product of force and displacement. In one complete rotation, the displacement is zero. So, the work done by the force is also zero.

**Circular Motion: Forces**

Circular motion is an accelerated motion. The velocity of a body in circular motion changes at every instant. The force that keeps the body in the circular path is called the centripetal force.

*The force that acts radially inward and keeps the moving body in a circular path is called centripetal force.*



**The centripetal force acts on the body toward the centre. Why then does the body not leave its circular path and move toward the centre?**

The answer lies in the nature of motion of the body. Circular motion is an accelerated motion. For Newton's laws to be applicable in such motion, a fictitious force is introduced. A fictitious force is a force that doesn't exist in reality. This force is the centrifugal force. In circular motion, the centrifugal force is always directed opposite to the centripetal force and is equivalent to the magnitude of the centripetal force. Thus, the centripetal force is balanced by the centrifugal force. This is the reason why the moon doesn't fall down to Earth.

*Centrifugal force is the fictitious outward force experienced by a body in circular path.* The magnitude of this force is always equal to the centripetal force on the body and is always directed opposite to the centripetal force.

**What provides the centripetal force to the moon for it to revolve around Earth?**

The gravitational force of attraction between Earth and the moon provides the centripetal force required to keep the moon revolving around Earth.

**Did You Know?**

At the circus, the motorcyclist in the death pit or the well of death (*maut ka kuan*) does not fall from the wall because his high speeds along the circular path provide him with sufficient force to remain attached to the wall.

**Solved Examples**

**Easy**

**Example 1**: **A 150 g stone is tied at one end of a string of length 60 cm. It is made to revolve in a circular path at a constant speed. What is the centripetal acceleration of the stone if it makes two complete revolutions in a second?**

**Solution**:

The stone completes 2 revolutions in 1 s; so, time period, $T = \frac{1}{2} = 0.5$ s

Length of the string, $r = 60$ cm $= 0.6$ m

Speed of revolution of the stone, $v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 0.6}{0.5} = 7.54$ m/s

Centripetal force, $F_c = \frac{mv^2}{r} = ma_c$

Centripetal acceleration ($a_c$) of the stone is given as:

$$ma_c = \frac{mv^2}{r}$$

$$\Rightarrow a_c = \frac{v^2}{r}$$

$$\Rightarrow a_c = \frac{7.54^2}{0.6} = 94.7 \text{ m/s}^2$$

**Medium**

**Example 2: The radius of the nearly circular orbit of the moon around Earth is about 384000 km and the time period of revolution of the moon around Earth is 27.3 days. What is the centripetal acceleration of the moon toward Earth?**

**Solution**:

Radius of moon's orbit, $r = 3.84 \times 10^8$ m

Time period of moon's revolution, $T = 27.3$ days $= 27.3 \times 24 \times 60 \times 60 = 2.36 \times 10^6$ s

Speed of moon's revolution, $v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 3.84 \times 10^8}{2.36 \times 10^6} = 1021.83$ m/s

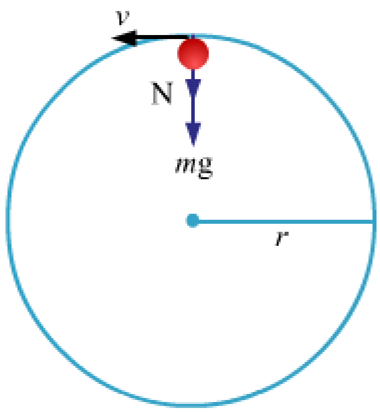
Centripetal acceleration of the moon is given as:

$$a_c = \frac{v^2}{r} = \frac{1021.83^2}{3.84 \times 10^8} = 0.0027 \text{ m/s}^2$$

**Hard**

**Example 3: At the circus, a motorcyclist rides in a circular track of radius *r*, in the vertical plane. What is the minimum velocity at the highest point of the track so that the motorcyclist can complete the track successfully?**

**Solution**:




We will apply Newton's second law of motion for the highest point.

$$N + mg = \frac{mv^2}{r}$$

Where, N is the normal reaction by the track

The condition for which the motorcyclist will just complete the track is: $N = 0$

This means that his speed at the topmost position is such that there is no normal reaction.

So,

$$mg = \frac{mv^2}{r}$$

$$\Rightarrow \therefore v = \sqrt{rg}$$

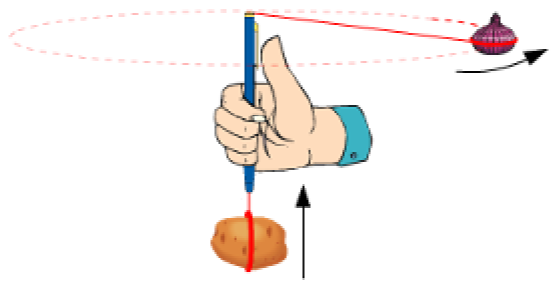
**Centripetal and Centrifugal Forces in Nature**

Do you know that Earth's bulging out at the equator is due to the centrifugal force?

At the equator, Earth's rotational speed is about 1674.4 km/h. The centrifugal force experienced by the land and water mass is the maximum at this region. The radially outward force experienced by the mass distribution in this region is large enough to affect Earth's shape. This is the reason for the equatorial bulge of Earth.


**Food for Thought**

Take an onion and a potato. Can you think of a way to lift the potato with the onion? Remember, the onion must be smaller than the potato.



**Explanation**: You will need a pen and a one-metre thread to do this activity. Take out the refill of the pen, insert the thread through one end of the pen and bring it out from its other end. Each end of the thread must be at least 30 cm from the pen. Attach the onion to one end of the thread and the potato to the other. Now, rotate the pen such that the onion starts revolving about it. The potato rises as you increase the speed of revolution of the onion.

The centripetal force required to move the onion in the circular path is provided by the tension in the thread, which is in turn induced by the gravitational force on the potato. When the speed of revolution of the onion is increased, the centripetal force necessary for this motion also increases. When the tension in the thread exceeds the weight of the potato, the potato rises.

Decreasing the speed of revolution decreases the centripetal force and, consequently, makes the potato go down.