

## MECHANICAL PROPERTIES OF SOLIDS

Rigid Body  $\Rightarrow$  A body is said to be rigid if it does not change its shape when external force is applied.

In reality there is no perfectly rigid body in the universe.

Deforming force  $\Rightarrow$  A force which changes the size or shape of a body is called deforming force.

Elasticity  $\Rightarrow$  If a body regains its original size and shape after removal of deforming force, it is said to be elastic body and this property is called elasticity.

Example  $\Rightarrow$  If we stretch a rubber band release it, it snap back to its original length.

Perfectly Elastic Body  $\Rightarrow$  If a body regains its original size and shape completely and immediately after removal of deforming force, it is called a perfectly elastic body.

Example  $\Rightarrow$  Quartz fibre is nearly perfectly elastic.

Plasticity  $\Rightarrow$  If a body does not regains its original size and shape even after the removal of deforming force, it is said to be ~~perfectly~~ plastic body and this property is called plasticity.

Perfectly Plastic body  $\Rightarrow$  If a body does not regains its shape even to very small extent even after removal of deforming force. Example  $\Rightarrow$  Paraffin wax.

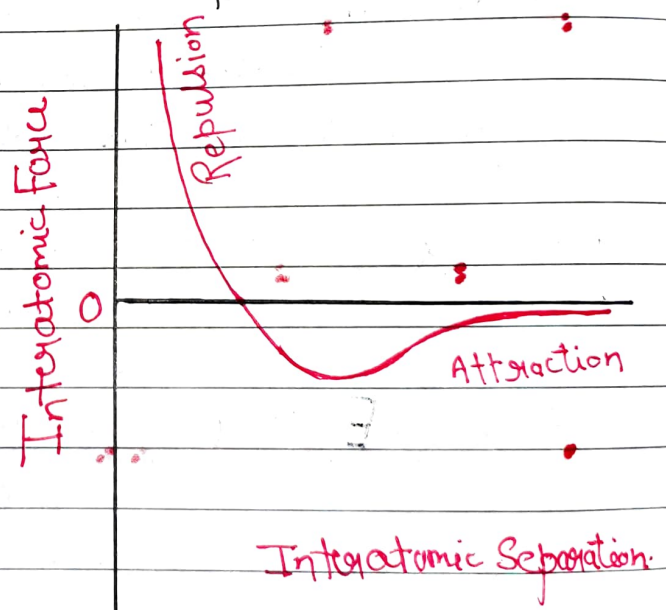
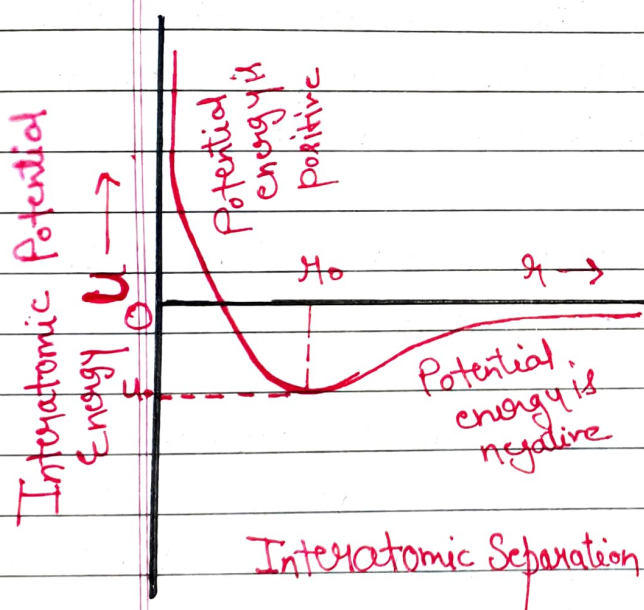
When potential energy is negative force will be attractive.

## ELASTIC BEHAVIOR IN TERMS OF INTERATOMIC FORCES

The atoms in a solid are held together by interatomic forces.

When the interatomic separation  $r$  is large, the potential energy of the atom is negative and the interatomic force is attractive.

At some particular separation  $r_0$ , the potential energy becomes minimum and interatomic force becomes zero. This separation  $r_0$  is called normal or equilibrium separation.



When separation reduces below  $r_0$  the potential energy increases sharply and the interatomic forces become repulsive.

Normally, the atoms occupy the positions of minimum potential energy called the position of stable equilibrium.

When a force of tensile or compressive nature is applied on a body its atoms are pulled apart or pushed closer to

distance is greater than or less than  $r_0$ .

When the deforming force is removed, the interatomic forces of attraction or repulsion restore the atom to their original positions.

Stress  $\Rightarrow$  The internal restoring force developed per unit cross-sectional area of a body when deforming force is applied on the body.

$$\text{Stress} = \frac{\text{Applied Force}}{\text{Area}} = \frac{F}{A}$$

Restoring force is equal and opposite to external deforming force.

SI unit of stress =  $\text{Nm}^{-2}$

Dimensional formula =  $[ML^{-1}T^{-2}]$

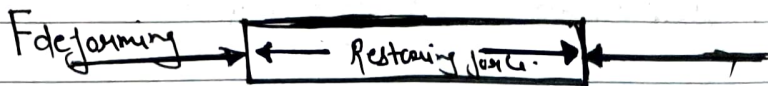
## Types OF Stress

(i) Tensile Stress  $\Rightarrow$  It is the restoring force set up per unit cross-sectional area of a body when the length of the body increases in the direction of the deforming force.

It is also known as longitudinal stress.

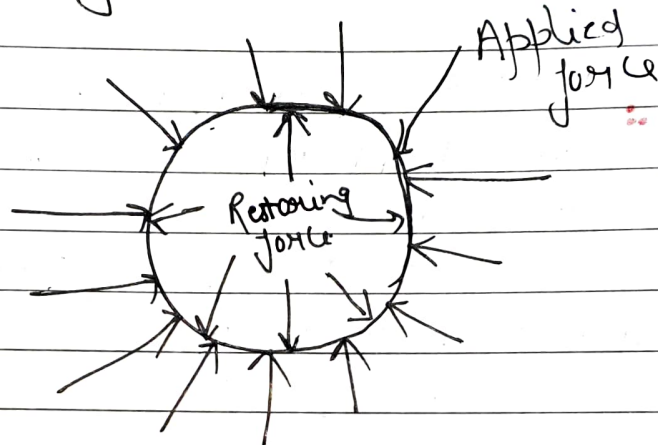


(ii) Compressive Stress  $\Rightarrow$  It is the restoring force developed per unit cross-sectional area of a body when its length decreases under deforming force.



Hydrostatic stress  $\Rightarrow$  If a body is subjected to a uniform force from all side then the corresponding stress is called hydrostatic stress.

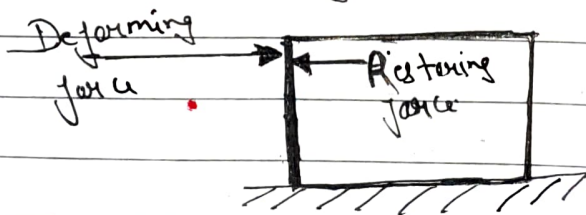
(P)



Tangential or Shear stress  $\Rightarrow$  When a deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body.

 $(\sigma_s)$ 

The tangential restoring force developed per unit area is called tangential stress.



Strain  $\Rightarrow$  It is the ratio of change in dimension to the original dimension when deforming force is applied over the body.

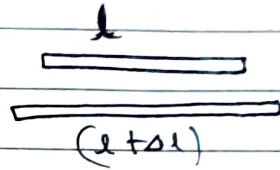
$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain is having no unit and dimension.

## TYPES OF STRAIN

- (i) Longitudinal Strain  $\Rightarrow$  It is defined as the increase in length per unit length when the body is deformed by external forces.

$$\text{Longitudinal strain} = \frac{\Delta l}{l}$$

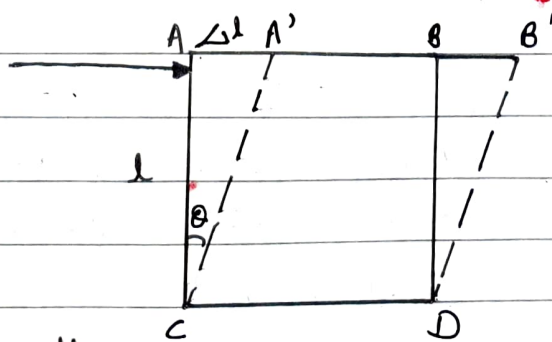


- (ii) Volumetric Strain  $\Rightarrow$  It is defined as the change in volume per unit original volume.

$$\text{Volumetric strain} = \frac{\Delta V}{V}$$

- (iii) Shear Strain  $\Rightarrow$  It is defined as the angle  $\theta$  through which a face originally perpendicular to the fixed sides gets turned on applying tangential deforming force.

$$\text{Shear strain} = \theta = \tan \theta$$



$$\tan \theta = \frac{\Delta l}{l}$$

Since angle is small

$$\tan \theta \sim \theta$$

$$\theta = \frac{\Delta l}{l}$$

Young Modulus and Bulk modulus of a perfectly rigid body is infinity

Elastic Limit  $\Rightarrow$  The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit.

### HOOKE'S LAW

According to Hooke's law within the elastic limit, the stress is directly proportional to the strain.

Stress  $\propto$  Strain

Stress = Constant  $\times$  Strain

$$\text{Stress} = k \text{ strain}$$

$k =$  Modulus of Elasticity

Modulus of Elasticity  $\Rightarrow$  The ratio of stress to the strain is called modulus of elasticity.

### TYPES OF MODULUS OF ELASTICITY

(i) Young's Modulus of Elasticity  $\Rightarrow$  The ratio of normal stress to the longitudinal strain within elastic limit is called Young's modulus of elasticity.

Young Modulus of Elasticity =  $\frac{\text{Normal Stress}}{\text{longitudinal strain}}$

$$Y = \frac{\sigma}{\epsilon}$$

$$Y = \frac{F/A}{\frac{\Delta l}{l}} = \frac{Fl}{A\Delta l}$$

$$Y = \frac{Fl}{A\Delta l}$$

Normal stress is either compressive or shear stress.

\* Young Modulus will have unit same as stress.

\* Greater the value of Young Modulus greater will be the strength of material and material will be more elastic.

\* Steel is more elastic than copper.

(ii) Bulk Modulus Of Elasticity  $\Rightarrow$  It is defined as the ratio of normal stress to the volumetric strain.

Bulk Modulus Of Elasticity =  $\frac{\text{Normal Stress (Hydraulic Stress)}}{\text{Volumetric Strain}}$

$$B = \frac{-P}{\Delta V/V}$$

$$B = \frac{-PV}{\Delta V}$$

The negative sign shows with increase in pressure volume will decrease.

Compressibility  $\Rightarrow$  The reciprocal of Bulk modulus of elasticity is called compressibility.

$$K = \frac{1}{B} =$$

\* Bulk Modulus of elasticity for solid is higher than gas and liquid.

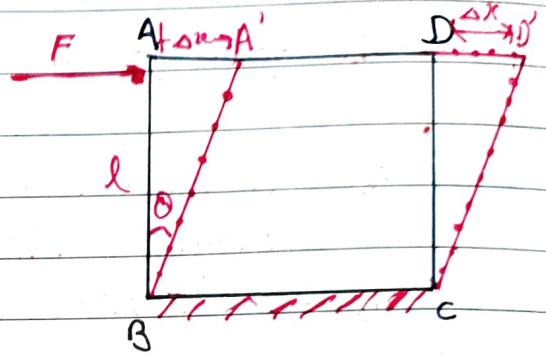
### (iii) Modulus Of Rigidity Or Shear Modulus [G]

The ratio of shearing stress to the shearing strain is called modulus of rigidity.

$$G = \frac{\text{Shearing stress } (\sigma_s)}{\text{Shearing strain } (\theta)}$$

$$G = \frac{F/A}{\frac{\Delta x}{L}} = \frac{FL}{A\Delta x}$$

$$G = \frac{FL}{A\Delta x}$$



\* Shear Modulus of Rigidity is generally less than Young modulus of elasticity.

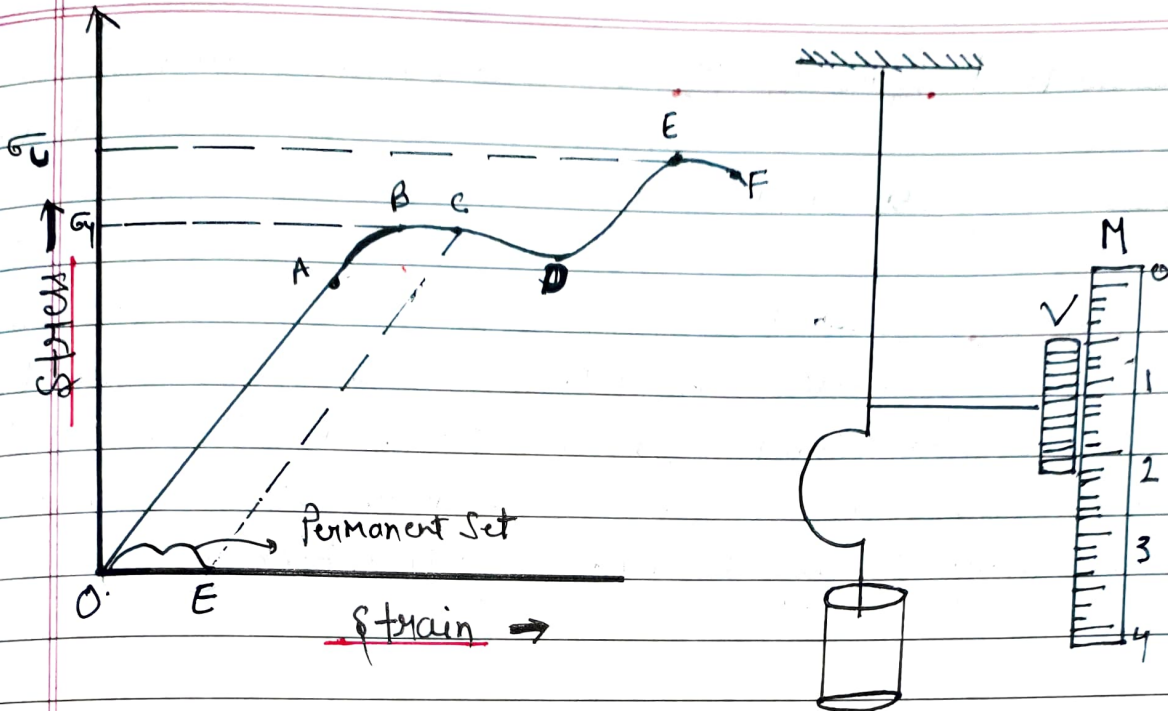
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### Stress-Strain Curve For a metallic Wire

Suspend a wire of uniform cross-sectional vertically from a rigid support through one end and attach a hanger at the other end of the wire which can slide over a ~~main scale~~. Attach a vernier scale V to the lower end of the wire which can slide over main scale M.

Put the different known weight in the hanger and note down the corresponding extension produced in the wire. Calculate the stress and strain for various observations.

E = ultimate strength point  
 F = Breaking point  
 B = Yield point



From the above graph we can observe following points

- (1) From O to A ⇒ (i) This part of the graph is a straight line that is Hooke's law is followed.  
 (ii) The body will return to its initial state when the stress is removed.  
 (iii) The point is called proportional limit.
- (2) From A to B ⇒ (i) The graph is not straight line hence Hooke's law is not followed.  
 (ii) The body will return back to its initial state when stress is removed.  
 (iii) The OB region of the graph is elastic region and point B is called elastic limit or yield point.  
 (iv) The stress at point B is called yield stress ( $\sigma_y$ )
- (3) After Point B ⇒ (i) Strain increases more rapidly than stress.  
 (ii) Even if load is removed at some point C the wire does not return to its original state.  
 (iii) Even after removing the ~~stress~~ <sup>stress</sup> completely residual ~~strain~~ strain equal to OE is left in the wire.

4 ~~If the load~~ Beyond C  $\Rightarrow$  Beyond C, the wire will start showing increase in strain without any increase in ~~strain~~ stress.

5 Beyond D  $\Rightarrow$  Beyond D graph is a curved line DEF which shows even if the wire is unloaded a little at point E, the thinning of the wire starts and the necks and waists are developed at few weaker portion of the wire and finally the wire breaks which is shown by point F.

### Elastic After Effect

The temporary delay in regaining the original configuration by an elastic body after the removal of a deforming force is called elastic after effect.

### Elastic Fatigue

It is the property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming force.

Poisson's Ratio  $\Rightarrow$  The ratio of lateral strain to the longitudinal strain is called poisson's ratio.

$$\sigma = \frac{\Delta R}{R} \div \frac{\Delta l}{l}$$

$$\sigma = \frac{-\Delta B \times l}{R \Delta l}$$

## ELASTIC POTENTIAL ENERGY OF STRETCHED WIRE

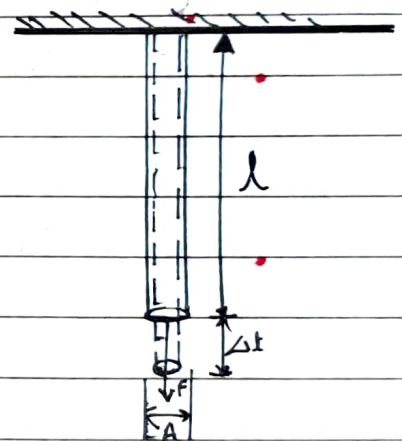
Suppose a force  $F$  applied on a wire of length  $l$  increases its length by  $\Delta l$ . Initial internal restoring force is zero.

Average internal force for an increase in the length  $\Delta l$  of wire

$$= \frac{0 + F}{2} = \frac{F}{2}$$

Work done on the wire = Average force  $\times$  increase in length of wire

$$W = \frac{F}{2} \times \Delta l$$



This work done is stored as elastic potential energy

$$U = \frac{1}{2} F \times \Delta l$$

Multiplying and dividing numerator and denominator by  $A \times l$

$$U = \frac{1}{2} \times \frac{F \times \Delta l (A \times l)}{A \times l}$$

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Volume}$$

$$\frac{U}{\text{Volume}} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$u = \frac{U}{\text{Volume}}$$

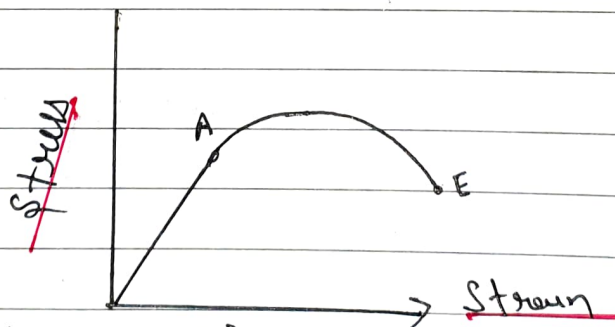
$$u = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$\text{stress} = \gamma \text{ strain}$$

$$u = \frac{1}{2} \times \gamma (\text{strain})^2$$

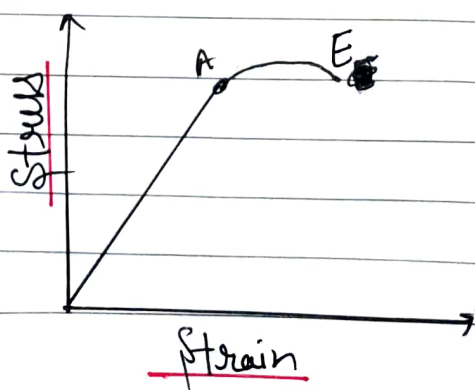
## CLASSIFICATION OF MATERIAL ON THE BASIS OF STRESS-STRAIN CURVE

Ductile Material  $\Rightarrow$  The material which have large plastic range of extension are called ductile material.



Brittle Material  $\Rightarrow$  The material which have very small range of plastic extension are called

Brittle material:



Elastomers  $\Rightarrow$  The materials which can be elastically stretched to large value of strain are called elastomers.

