THERMAL PROPERTIES OF MATTER

Heat ⇒ It is form energy that when given to a body produces randomness in the body.

Heat possesed by a body is the total thermal energy of the body which is sum of kinetic energy of all the individual molecule of the body due to translational, vibrational and rotational motion of the molecule.

- ✯ SI unit of Heat is Joule and C.g.S unit is Erg.

$$1 \text{ Erg} = 10^{-7} \text{ Joule}$$

- ✯ Heat is a scalar quantity.
- ✯ Heat energy is part of internal energy in transit.
- ✯ Heat energy is always transferred from high temprature to low temprature.

TEMPRATURE ⇒ Temprature is one of the seven fundamental quantity which decides the direction of flow of heat.

THERMOMETERY

The branch of physics that deals with the measurement of temprature is known as thermometery.

THERMOMETER ⇒ It is a device which is used to measure the temprature of body.

While designing thermometer we may take any of the following property that changes with change in temprature.

(i) Variation of volume of liquid with temprature
(ii) Variation of pressure or volume of gas with temprature
(iii) Variation of resistance of metal with temprature.
(iv) The variation of thermo emf with temprature of a junction in couple.

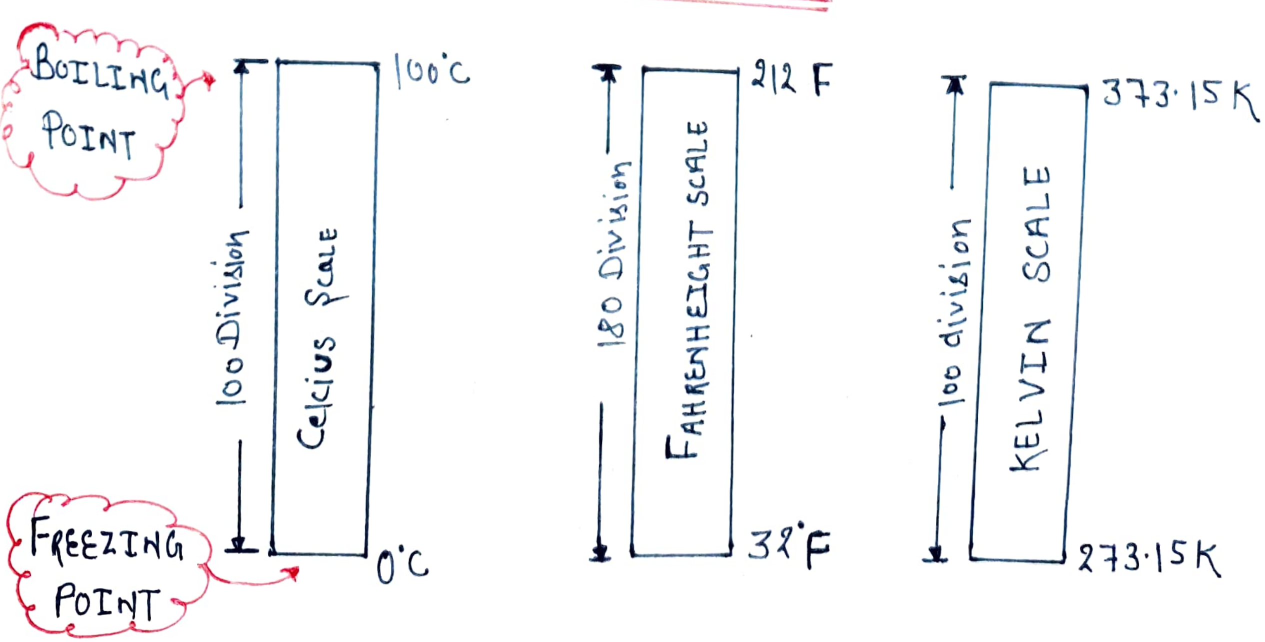
TEMPRATURE SCALE

Thermometer calibrated for a temprature scale is used to measure the value of given temprature on that scale.

In order to define any standard temprature scale, for the measurement of temprature we need two fixed reference point at which physical phenomena always occurs at same temprature.

These two Reference points are → FREEZING POINT / BOILING POINT → water

THERE ARE FOLLOWING SCALE



One degree celcius Change = One Kelvin Change = 1.8 Degree Farenheight Change

RELATIONSHIP BETWEEN DIFFERENT TEMPRATURE SCALE

1) Between °C and °F

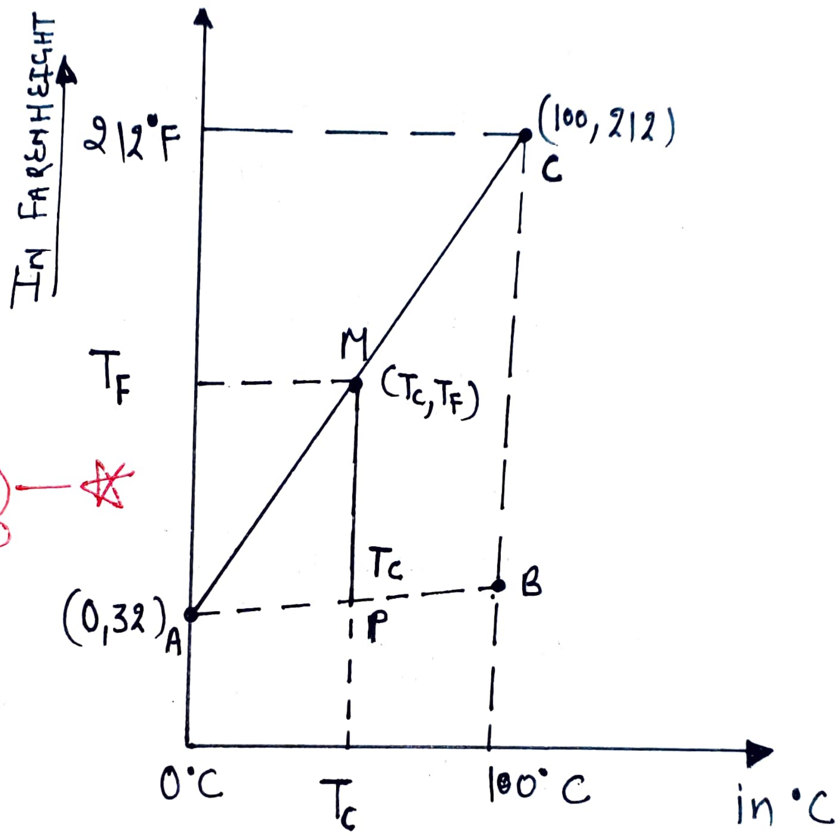
Since
$\Delta ABC \sim \Delta APM$

So

$$\frac{AP}{AB} = \frac{MP}{BC}$$

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} \quad ☆$$

$$\frac{T_C}{100} = \frac{T_F - 32}{180}$$



$$\frac{T_C}{5} = \frac{T_F - 32}{9}$$ → Relationship Fareheight and degree celcius

FROM ☆ equation we can Relate Any two scale.

$$\frac{\text{Temprature on scale (A)} - \text{FREEZING POINT}}{\text{Total No. Dirion [Difference b/w freezing and boiling point]}} = \frac{\text{Temperature on scale B} - \text{Freezing point}}{\text{Difference b/w freezing and boiling point}}$$

$$\frac{T_C - 0}{100^\circ - 0^\circ} = \frac{T_F - 32}{212 - 80} = \frac{T_K - 273.15}{373.15 - 273.15}$$

$$\frac{T_C}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

$$\frac{T_C}{100} = \frac{T_K - 273.15}{100}$$

$$\boxed{T_C = T_K - 273.15}$$ → Relationship b/w °C and kelvin

NUMERICAL ON TEMPRATURE SCALE

Q→1 Convert 30°C into kelvin and Farheight

Sol

$K = 273.15 + {}^\circ C$

$K = 273.15 + 30^\circ = 303.15^\circ K$

$$\boxed{K = 303.15^\circ K}$$

$$\frac{T_C}{5} = \frac{T_F - 32}{9}$$

$$\frac{9}{5} T_C + 32 = T_F$$

$$\frac{9}{5} \times 30 + 32 = T_F$$

$$\boxed{T_F = 86^\circ F}$$

Q→2 For Reumer's scale ice point is 0°R and steam point is 80°R. Find the value of 20°C in R.

Sol

$$\frac{T_R - \text{freezing point}}{\text{steam point} - \text{Freezing point}} = \frac{T_C - \text{freezing point}}{\text{steam point} - \text{Freezing point}}$$

$$\frac{T_R - 0}{80 - 0^\circ} = \frac{20^\circ - 0^\circ}{100 - 0^\circ} \Rightarrow T_R = \frac{2\cancel{0} \times 8\cancel{0}}{1\cancel{0}\cancel{0}}$$

$$T_R = 16^\circ R$$

Q→3 If temperature is from 30°C into 60°C is changed then what is the change on kelvin scale and Fareheight scale?

Sol

We know:

$\Delta 1^\circ C = \Delta 1^\circ K$

so if 30°C change then Kelvin will also change by 30

For 30° change
change in F
$= 30 \times 1.8$
54

OR

100°C change = 180°F change

$1^\circ C$ " $= \frac{180}{100}$ °F

30° " " $= \frac{180}{100} \times 30^\circ F$ change

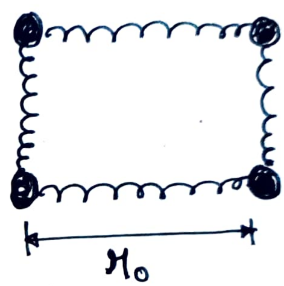
$= 54^\circ F$

THERMAL EXPANSION

Whenever we supply heat to a substance then expansion in shape of substance will take place.

Reason Behind Expansion Of Substance.

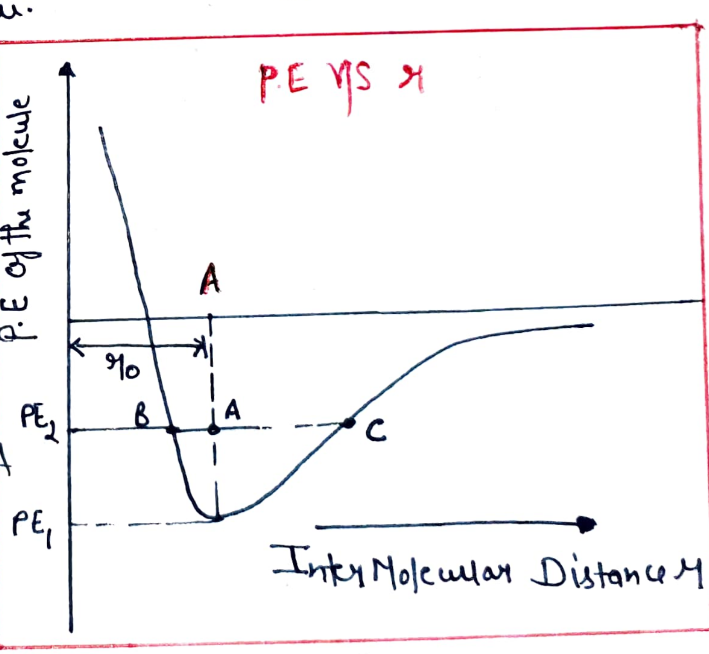
- Let us consider a substance in which molecules are at distance (r_0) when the energy was minimum ($P.E_1$). and molecules are at highly stable condition



- When we supply energy to the substance then energy of the molecules will increase to $P.E_2$. At this energy the distance b/w two molecules may have two value.

- When we supply energy then molecules may come closer to each other then they will move away from each other.

- The particles of molecule will start oscillating from mean position but with different displacement on both side.



- We can see from graph that compression distance (AB) is less than expansion distance [AC]. So on an average we can see that substance will expand.

Types Of Thermal Expansion

1. Linear Expansion
2. Area Expansion
3. Volume Expansion

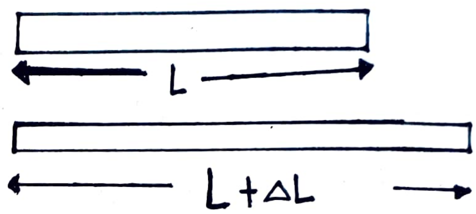
Linear Expansion

When a Rod of length 'L' is heated such that its change in temperature ΔT then its length changes by ΔL.

Experimentally it was found that

$\Delta L \propto L$ — (1)

$\Delta L \propto \Delta T$ — (2)



Combining (1) and (2) equation

$\Delta L \propto L \Delta T$

$\Delta L = \alpha L \Delta T$

α = Coefficient of Linear Expansion

$$\alpha = \frac{\Delta L}{L \Delta T}$$

if we take $L = 1$ $\Delta T = 1$ unit then

$$\alpha = \frac{\Delta L}{1}$$

So "Coefficient of thermal expansion is defined as small change in length per unit original length per unit change in temperature".

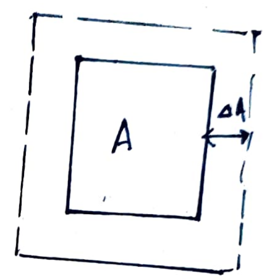
AREA EXPANSION

Let us consider a solid having surface Area A at temperature T When heat is supplied to the solid such that its temperature Changed by ΔT and its Area Changed by ΔA

The change in area was experimentally found to vary as follows

$\Delta A \propto A$ — (1)

$\Delta A \propto \Delta T$ — (2)



Combining equation (1) and (2)

$\Delta A \propto A \Delta T$

$$\boxed{\Delta A = \beta A \Delta T}$$

$\boxed{\beta = \text{Coefficient of area expansion}}$

$$\beta = \frac{\Delta A}{A \Delta T}$$

if we take $A = 1$, $\Delta T = 1$

Then

$\beta = \Delta A$

Coefficient of area expansion is defined as "change in surface area per unit original area per unit change in volume."

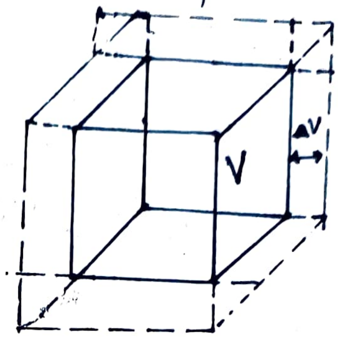
VOLUME EXPANSION [3-D Expansion]

Let us consider a solid having volume 'V' at temperature T. When the heat is supplied to the solid then temperature of the solid changed by ΔT and volume of solid changed by 'ΔV'

experimentally it was found that volume expansion varies as follows

$\Delta V \propto V$ — (1)

$\Delta V \propto \Delta T$ — (2)



Combining (1) and (2)

$\Delta V \propto V \Delta T$

$\Delta V = \gamma V \Delta T$

γ = Coefficient of volume Expansion

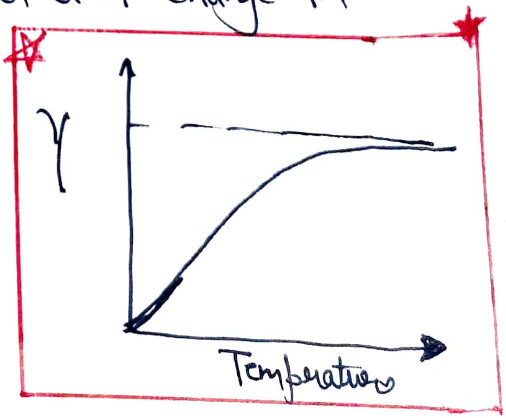
$$\gamma = \frac{\Delta V}{V \Delta T}$$

if $V = 1$ and $\Delta T = 1$

then

$\gamma = \Delta V$

Coefficient of volume expansion defined as "small change in volume per unit original volume per unit change in temprature."



- ✯ At low temprature γ increases with temprature
- ✯ At high temprature γ is independent to temprature and only depends upon Nature of material

Relationship between α, β and γ

Let us consider a cube having edge L, area of each face of cube A and volume of cube is V.

Now when heat is supplied to the cube and temperature of cube changes to ΔT then

New length of cube $L' = L + \alpha L \Delta T$

New Area of face of cube $A' = A + \beta A \Delta T$

New volume of cube $= V' = V + \gamma V \Delta T$

(I) Relationship b/w α and β

$$A' = A + \beta A \Delta T = (L + \Delta L)^2$$

$$A + \beta A \Delta T = (L + \alpha L \Delta T)^2$$

$L^2 = A$

$$A(1 + \beta \Delta T) = L^2 (1 + \alpha \Delta T)^2$$

$$A(1 + \beta \Delta T) = A[1 + \alpha^2 \Delta T^2 + 2\alpha \Delta T]$$

$$1 + \beta \Delta T = 1 + 2\alpha \Delta T$$

$$\beta \Delta T = 2\alpha \Delta T$$

{ $\alpha^2 \Delta T^2$ is very small term so it can be neglected }

$$\beta = 2\alpha \Rightarrow \alpha = \frac{\beta}{2} \quad \text{—(1)}$$

(II) Relationship b/w α and γ

$$V' = V + \gamma V \Delta T = (L + \Delta L)^3$$

$$V(1 + \gamma \Delta T) = (L + \alpha L \Delta T)^3$$

{ $L^3 = V$

$$V(1 + \gamma \Delta T) = L^3 (1 + \alpha \Delta T)^3$$

$$V(1 + \gamma \Delta T) = V[1 + \alpha^3 \Delta T^3 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2]$$

$$1+\gamma\Delta T = 1+3\alpha\Delta T$$

{ $3\alpha^2\Delta T^2$, $\alpha^3\Delta T^3$ → These terms are very small so They can be neglected }

$$\gamma\Delta T = 3\alpha\Delta T$$

$$\gamma = 3\alpha$$

$$\alpha = \frac{\gamma}{3} \quad \text{---} \ (2)$$

from equation (1) and (2)

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

NUMERICAL

Q. 1 For an ideal gas find the value of γ at constant pressure when temperature is changed.

Sol

$$PV = nRT$$

$$P\,dV = nR\,dT$$

$$PV\gamma\,dT = nR\,dT \qquad \{\Delta V = \gamma V\,dT$$

$$\gamma = \frac{nR}{PV}$$

$$\gamma = \frac{nR}{nRT}$$

$$\gamma = \frac{1}{T}$$

P

Numerical

EXPANSION OF BIMETALLIC STRIP

Let Us consider two strip of two different Material Rivitted together

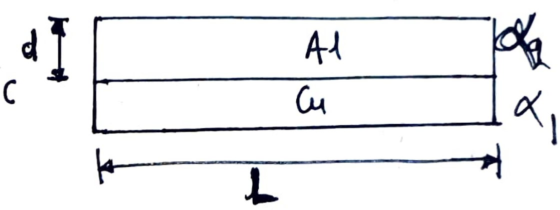
	Length	thickness	Coefficient of linear expansion
Copper strip →	L	d	α_1
Aluminium strip →	L	d	α_2

If $\alpha_2 > \alpha_1$ and the strip combination is heated so that temperature changes by ΔT so change in length of the two strips take place

$L_1 = L(1+\alpha_1\Delta T)$ $\qquad\qquad$ $L_2 = L(1+\alpha_2\Delta T)$

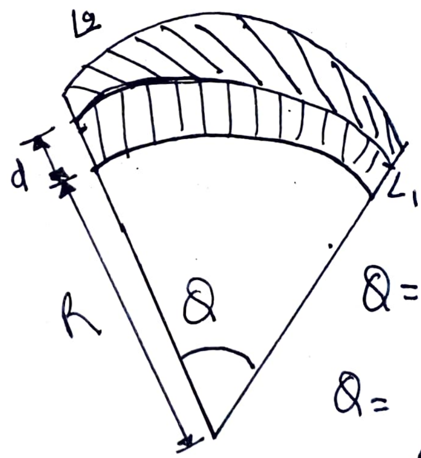
since $L_2 > L_1$ because $\alpha_2 > \alpha_1$

the strip will bend to form an arc of Radius R



from equation (1) and (2)

$$\frac{L_2}{R+d} = \frac{L_1}{R}$$



$\theta = \frac{\text{Length of arc}}{\text{Radius}}$

$\theta = \frac{L_1}{R}$ —(1)

$\theta = \frac{L_2}{R+d}$ —(2)

$$RL_2 = L_1R + L_1 d$$

$$R[L_2 - L_1] = L_1 d$$

$$R = \frac{L_1 d}{L_2 - L_1}$$

$$R = \frac{L(1+\alpha_1\Delta T)d}{L(1+\alpha_2 T) - L(1+\alpha_1\Delta T)}$$

$$R = \frac{L(1+\alpha_2\Delta T)d}{L[1+\alpha_2\Delta T - 1 - \alpha_1\Delta T]}$$

$$\Rightarrow R = \frac{(1+\alpha_2\Delta T)d}{[\alpha_2-\alpha_1]\Delta T}$$

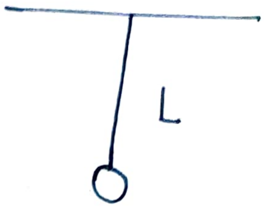
Numerical

FAULT IN TIME PERIOD OF PENDULUM CLOCK

Let us consider a pendulum clock in which bob is hanged by a metallic strip of length L having coefficient of linear expansion α at temperature θ

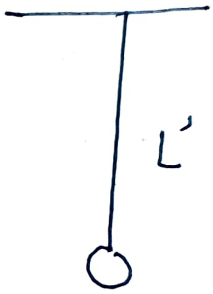
So Time period of pendulum at temperature θ

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{—(1)}$$



Now when the temperature of pendulum is changed then length of pendulum changes to L'

$$L' = L(1+\alpha\Delta\theta)$$



$\Delta\theta$ = Change in temp

So New time period

$$T' = 2\pi\sqrt{\frac{L'}{g}} \quad \text{—(2)}$$

Divide (2) by (1)

$$\frac{T'}{T} = \left(\frac{L'}{L}\right)^{1/2}$$

$$T' = T\left[\frac{L(1+\alpha\Delta\theta)}{L}\right]^{1/2}$$

$$T' = T(1+\alpha\Delta\theta)^{1/2}$$

expanding bionomially

$(1+x)^n = 1+nx$

$$T' = T\left(1+\frac{1}{2}\alpha\Delta\theta\right)$$

Where T = Original Time period

$\Delta\theta$ = Change in temperature

T' = New time period.

Transfer Of Heat

Heat is a form of energy which can be transferred from one part of the system to another part depending upon the difference of temperature b/w them.

Methods Of Heat Transfer

I. Conduction II. Convection III. Radiation.

Variable State

➡ If the molecules of rod are absorbing some portion of heat supplied to it and remaining heat is transferred to other molecule then it is known as variable state

➡ In this different part of rod will have different temperature with respect to time and distance

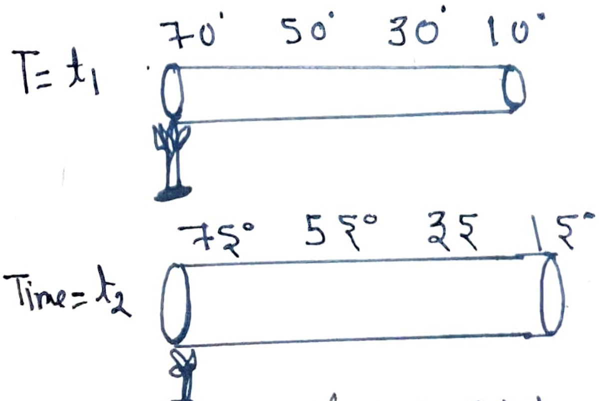
➡ Heat passing through each cross-section will not be same.

Steady State

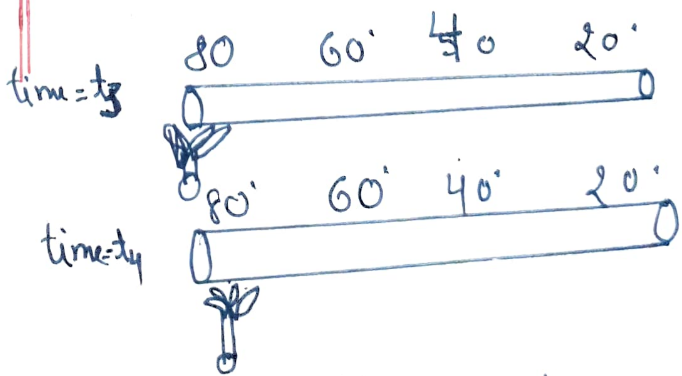
If the molecules of the rod are transfering all the heat to the other molecule and they do not absorb any amount of heat is known as steady state.

In this different part of rod will have constant temperature ~~with~~ and temperature will not change with time.

In this heat passing through each cross-section will be same.



Variable State



Steady State

∴ ∴ proportional to temperature difference

Equation Of Steady State

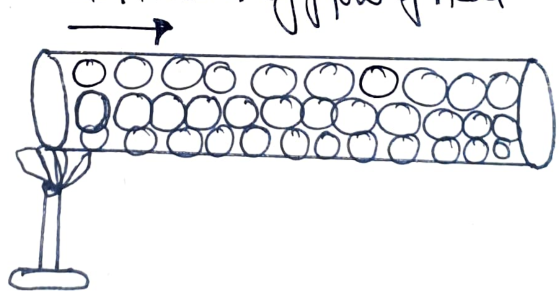
$$\left(\frac{dT}{dt}\right)_{x=\text{Constant}} = 0$$

This mean that temprature of rod at a given distance is Constant. i.e temprature does not change with time.

CONDUCTIVITY

It is a mode of heat transfer from one part of the body to another, from particle to particle in the direction of fall of temprature without any actual movement of the heated particle.

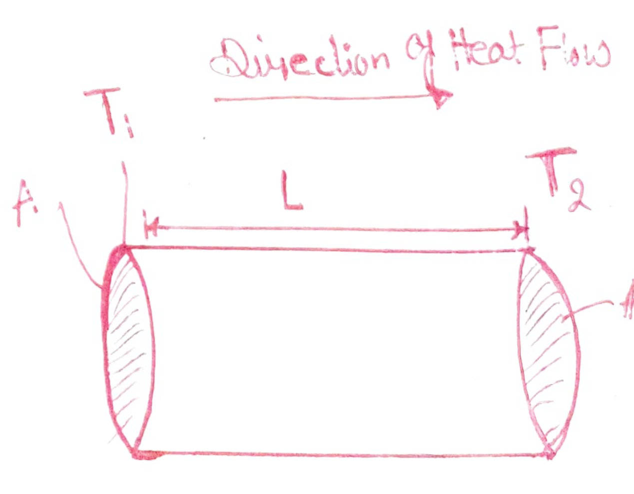
Direction of flow of heat.



Inthis heat is transfered because of vibration of molecule.

THERMAL CONDUCTIVITY

Let us consider a rod having length 'L' and cross-sectional area A and the temprature at its two end is T_1 and T_2 respectively.



$T_1 > T_2$

In steady state the heat flowing through rod in time t is found to

(1) It is directly proportional to area of cross-section

$$Q \propto A \quad \text{—} (1)$$

(II) It is directly proportional to temperature difference

$Q \propto T_1 - T_2$

$Q \propto \Delta T$ — (2) $\{\Delta T = T_1 - T_2\}$

(III) It is inversely proportional to the length b/w two ends of rod

$Q \propto \frac{1}{L}$ — (3)

(IV) $Q \propto t$ — (4) It is directly proportional to time.

Combining (1), (2), (3) and (4) equation

$$Q \propto \frac{A \Delta T}{L} \Rightarrow Q = \frac{K A \Delta T t}{L}$$

$$Q = \frac{K A \Delta T t}{L}$$

Where K = Coefficient of thermal Conductivity

SI unit of K = Joule $s^{-1} m^{-1} K^{-1}$

$W m^{-1} K^{-1}$

C.g.s unit = Cal. $s^{-1} cm^{-1} {}^\circ C^{-1}$

Heat Current (H)

It is the rate of transfer of Heat.

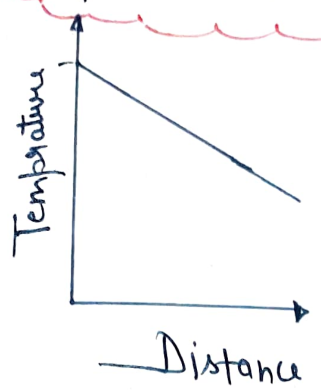
$$H = \frac{Q}{t} = \frac{K A \Delta T t}{L t}$$

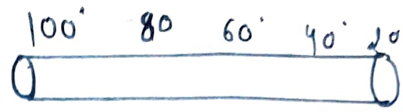
$$H = \frac{K A \Delta T}{L}$$

Temprature Gradient

It is defined as change in temprature with respect to change in distance. $\text{Temprature Gradient} = \frac{dT}{dx}$

$\frac{dT}{dx} = \text{Slope}$





So slope of the graph b/w temprature and distance give temprature gradient.

Analogy

Heat

Heat Current → H

Heat flows due temprature difference ΔT

Flow of Heat is apposed by Thermal Resistance.

$$H = \frac{KA\Delta T}{l}$$

$$H = \frac{\Delta T}{\frac{l}{KA}}$$

$$H = \frac{\Delta T}{R_{th}}$$

R_{th} = Thermal Resistance

Electricity

Current – I

Current flows due ~~to~~ ~~temp~~ potential difference V

The flow of current is apposed by Resistance R

$$V = IR$$

$$I = \frac{V}{R}$$

$H \longrightarrow I$

$\Delta T \longrightarrow V$

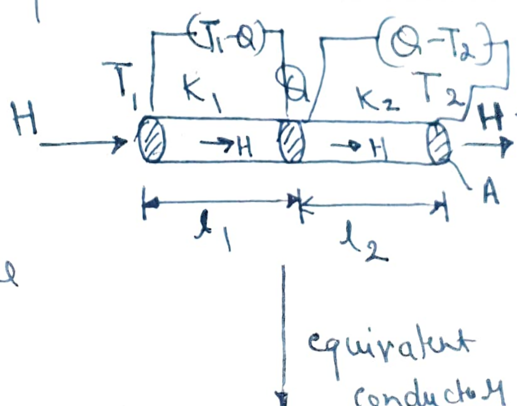
$R_{Th} \longrightarrow R$

$R_{th} = \frac{l}{KA}$ R_{th} = thermal Resistance.

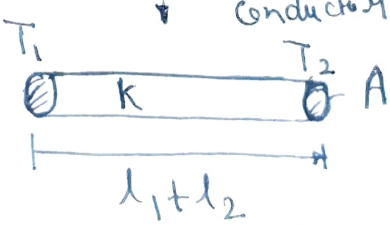
Important FORMULA → $H = \frac{\Delta T}{R_{Th}}$ $R_{Th} = \frac{l}{KA}$ $Q = \frac{KA\Delta T t}{L}$

Equivalent Thermal Conductivity

In series combination same heat will flow through conductor but temperature difference will be different



$T_1 - T_2 = (T_1 - \theta) + (\theta - T_2)$ — *



$H = \frac{T_1 - T_2}{R_{TH}}$

$T_1 - T_2 = H(R_{TH})$ — (1)

$T_1 - \theta = H(R_{TH1})$ — (2)

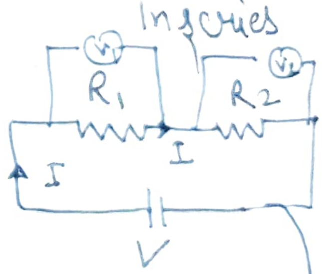
$\theta - T_2 = H(R_{TH2})$ — (3)

Putting the value of (1) (2) and (3) in *

$H(R_{TH}) = H(R_{TH1}) + H(R_{TH_1})$

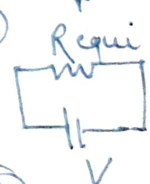
$\boxed{R_{TH} = R_{TH1} + R_{TH_2}}$

Equivalent Resistance in series



$V = V_1 + V_2$ — *

$V = I R_{eq}$ — (1)



$V_1 = IR_1$ — (2)

$V_2 = IR_2$ — (3)

Put the value of (1), (2) and (3) in * equation

$IR_{eq} = IR_1 + IR_2$

$\boxed{R_{eq} = R_1 + R_2}$

$R_{TH} = \frac{l}{KA}$ $\quad R_{TH_1} = \frac{l_1}{K_1 A}$ $\quad R_{TH_2} = \frac{l_2}{K_2 A}$

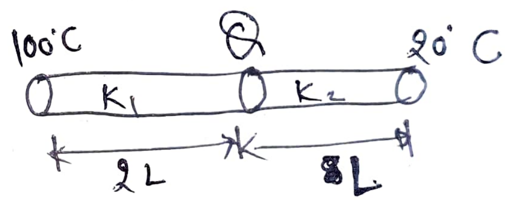
$$\frac{L}{KA} = \frac{l_1}{KA} + \frac{l_2}{K_2 A} \Rightarrow \boxed{\frac{L}{K} = \frac{l_1}{k_1} + \frac{l_2}{K_2}}$$

where = L = Total length of conductor,
K = equivalent thermal conductivity

Q→ Find equivalent thermal conductivity for the following.

Sol

$$\frac{L}{K} = \frac{L_1}{K_1} + \frac{L_2}{K_2}$$



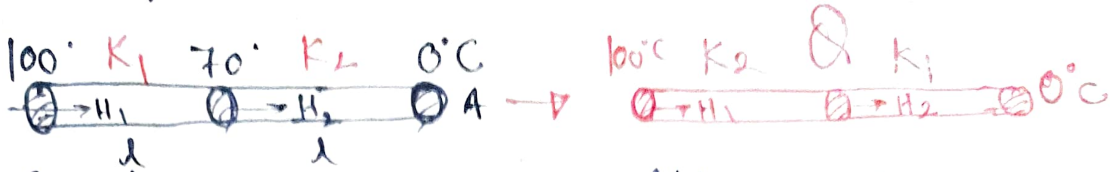
$$\frac{2L+L}{K} = \frac{2L}{K_1} + \frac{L}{K_2}$$

$$\frac{3L}{K} = L\left(\frac{2}{K_1} + \frac{1}{K_2}\right)$$

$$\frac{3}{K} = \left(\frac{2K_2+K_1}{K_1K_2}\right)$$

$$\frac{K}{3} = \frac{K_1+K_2}{2K_2+K_1} \Longrightarrow \boxed{K = \frac{3(K_1+K_2)}{2K_2+K_1}}$$

Q→ Find the temperature of the junction when position of conductor is reversed.



Sol

Before Reversing

$$H_1 = H_2$$

$$\frac{\Delta T}{R_1} = \frac{\Delta T}{R_2}$$

$$\frac{100-70}{\frac{l}{K_1}} = \frac{70-0°C}{\frac{l}{K_2}}$$

$$\frac{30K_1}{l} = \frac{70K_2}{l}$$

$$\frac{K_1}{K_2} = \frac{70}{30} = \frac{7}{3}$$

$$\frac{K_1}{K_2} = \frac{7}{3}$$

After Reversing.

$$H_1 = H_2$$

$$\frac{\Delta T}{R_1} = \frac{\Delta T}{R_2}$$

$$\frac{100-\theta}{\frac{l}{K_2}} = \frac{\theta-0°}{\frac{l}{K_1}}$$

$$K_2(100-\theta) = K_1(\theta-0°)$$

$$100-\theta = \frac{K_1}{K_2}\theta$$

$$100-\theta = \frac{7}{3}\theta$$

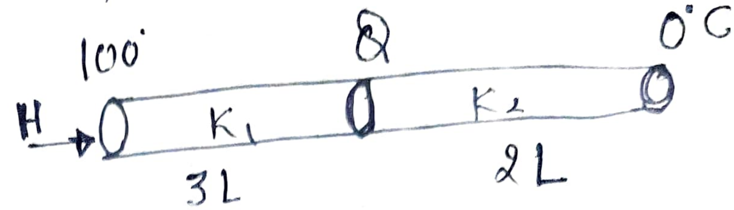
$$100 = \frac{7}{3}\theta+\theta$$

$$10\cancel{0} = \frac{1\cancel{0}\theta}{3}$$

$$\theta = 30°C$$

Q→ Find the junction temperature if $K_1 = 2K_2$

Sol Let the junction temperature be θ



Since the heat through both conductor is same

$$H = \frac{\Delta T}{R_1} = \frac{\Delta T}{R_2}$$

$$\frac{100-\theta}{R_1} = \frac{\theta - 0°}{R_2}$$

$$R_2(100-\theta) = R_1\theta$$

$$\frac{l_2}{K_2}(100-\theta) = \frac{l_1}{K_1}\theta$$

$$\frac{2L}{K_2}(100-\theta) = \frac{3L}{K_1}\theta$$

$$\frac{2}{K_2}(100-\theta) = \frac{3}{2K_2}\theta$$

$$100-\theta = \frac{3}{4}\theta$$

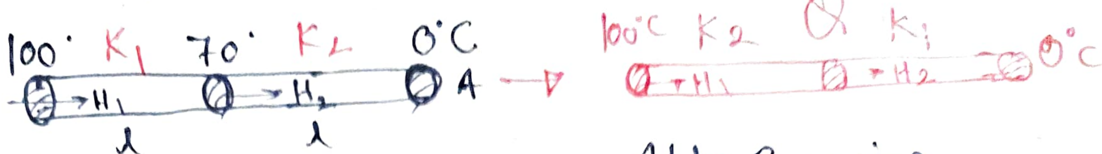
$$100 = \frac{3\theta + \theta}{4}$$

$$100 = \frac{7\theta}{4}$$

$$\theta = \frac{400}{7} = 57.1$$

$$\theta = 57.1°C$$

Q→ Find the temperature of the junction when position of conductor is reversed.



Sol Before Reversing

$H_1 = H_2$

$$\frac{\Delta T}{R_1} = \frac{\Delta T}{R_2}$$

$$\frac{100-70}{\frac{l}{K_1}} = \frac{70-0°C}{\frac{l}{K_2}}$$

$$\frac{30K_1}{l} = \frac{70K_2}{l}$$

$$\frac{K_1}{K_2} = \frac{70}{30} = \frac{7}{3}$$

$$\frac{K_1}{K_2} = \frac{7}{3}$$

After Reversing.

$H_1 = H_2$

$$\frac{\Delta T}{R_1} = \frac{\Delta T}{R_2}$$

$$\frac{100-Q}{\frac{l}{K_2}} = \frac{Q-0°}{\frac{l}{K_1}}$$

$$K_2(100-Q) = K_1(Q-0°)$$

$$100-Q = \frac{K_1}{K_2}Q$$

$$100-Q = \frac{7}{3}Q$$

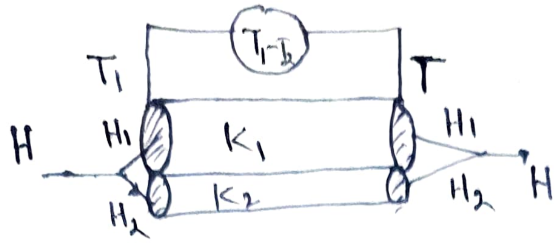
$$100 = \frac{7}{3}Q + Q$$

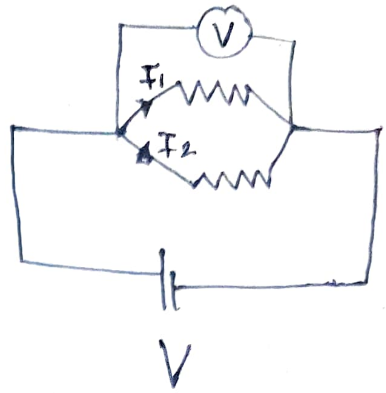
$$10\cancel{0} = \frac{1\cancel{0}Q}{3}$$

$$Q = 30°C$$

Equivalent Thermal Conductivity

In Parallel Combination





$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

Using Analogy with electrical Circuit

Equivalent thermal Conductivity in parallel Combination

$$\frac{1}{R_{P_{TH}}} = \frac{1}{R_{TH(1)}} + \frac{1}{R_{TH(2)}} \quad \text{---} (*)$$

$$R_{P_{TH}} = \frac{l}{KA} \quad \text{---} (1) \qquad R_{TH(1)} = \frac{l}{K_1 A_1} \quad \text{---} (2)$$

$$R_{TH(2)} = \frac{l}{K_2 A_2} \quad \text{---} (3)$$

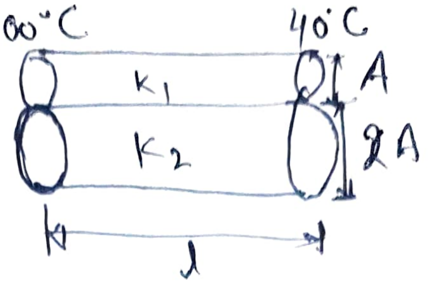
Substituting the value of (1), (2) and (3) in (*)

$$\frac{KA}{l} = \frac{K_1 A_1}{l} + \frac{K_2 A_2}{l}$$

$$K = \frac{K_1 A_1 + K_2 A_2}{A}$$

Question Based Upon Parallel Combination

Q→ Find Equivalent thermal Resistance and equivalent Thermal Conductivity



Sol

$R_1 = \frac{l}{K_1 A}$

$R_2 = \frac{l}{K_2 2A}$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ = $\frac{1}{R} = \frac{2K_2A}{l} + \frac{K_1A}{l}$

$\frac{1}{R} = \frac{2K_2A + K_1A}{l}$

$R_{eq} = \frac{l}{A(2K_2 + K_1)}$ → Equivalent thermal Resistance

Equivalent thermal Conductivity

$\cancel{H} = \cancel{\frac{\Delta T}{R}} = \cancel{\frac{\Delta T K A}{l}}$

$\cancel{\frac{KA\Delta T}{l}} = \cancel{\frac{\Delta T}{R_{eq}}}$

$R_{eq} = \frac{l}{K_{eq}\ \text{total area}}$

$\frac{l}{A(2K_2+K_1)} = \frac{l}{K_{eq}}$

$A(2K_2+K_1) = K_{eq}$

$K_{eq} = A(2K_2+K_1)$

$R_{eq} = \frac{l}{A(2K_2+K_1)}$