VECTOR

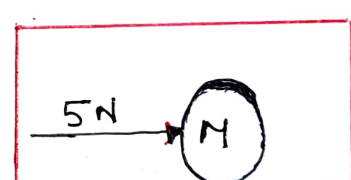
Physical Quantity ⇒ It is a property of material that can be quantified by measurement.

Physical Quantity

Scalar Quantity	Vector Quantity
Maginitude	Magnitude + Direction
Example ⇒ Work, speed distance	Example ⇒ Force, displacement Velocity, momentum

Need Of Vector ➜ There are certain physical quantity which can not be complitely described without direction.

For Example



I. Suppose you are given that a force of 5N acts on a body the can you tell the direction of motion of the body, answer in No. You can not tell the direction of motion of body.

II. But if you are given that a force of 5N acts on a body is North direction then you can tell the direction of motion of the body.

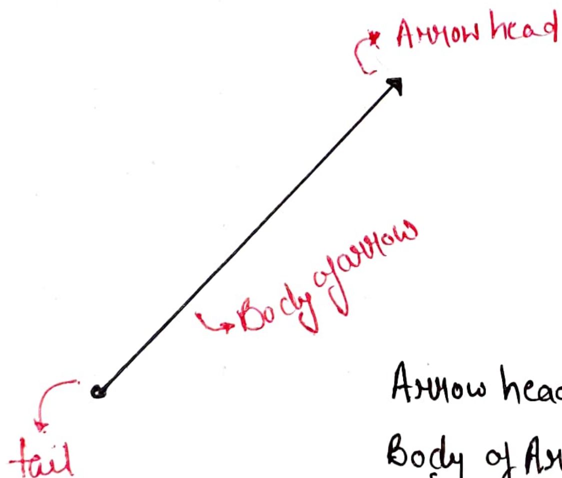
"So from above example we can see that direction of force is required to specify the impact of force so force is a vector quantity."

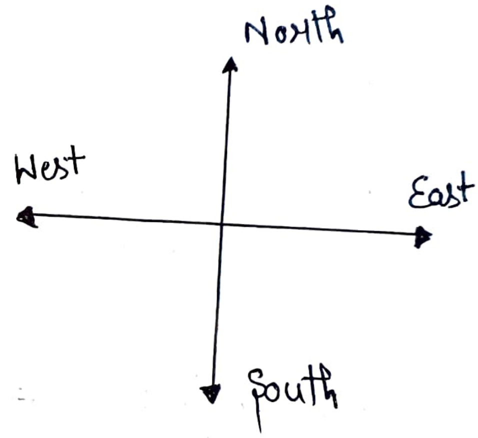
REPRESENTATION OF VECTOR

I **Geometrical Method** ⇒ In this method a vector is represented by the arrow pointed in the direction of given direction.

For Example Represent 60N force acting in North-east direction

Scale 1 cm = 10N





Arrow head → Direction of vector
Body of Arrow → Magnitude of arrow
Tail → Starting point

II **Vector Notation** ⇒ Generally a vector can also be represented by a Bold alphabet with arrow over it.

$\vec{B}$ $\vec{A}$ $\vec{R}$

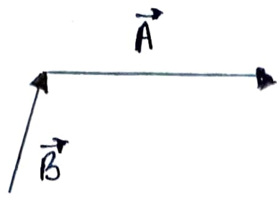
ANGLE B/W TWO VECTOR

To find the angle b/w two vectors both the vectors are drawn from one point in such manner that arrow of both the vectors are outward from that point. Now the smaller angle b/w them is called angle b/w two vector

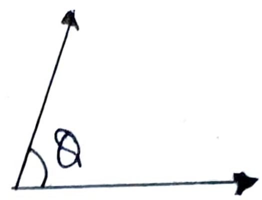
FIND THE ANGLE B/W TWO VECTOR

Q→ I

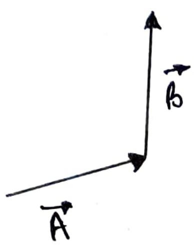
Question

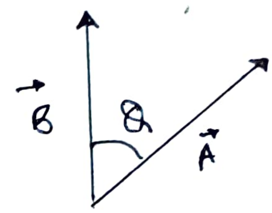


Sol



II

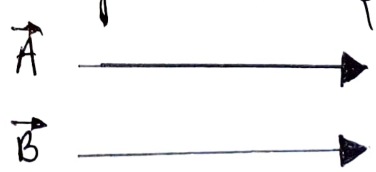




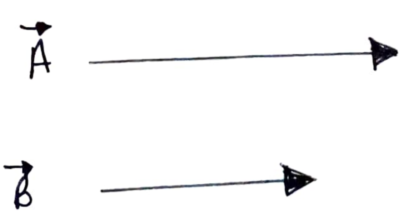
Angle b/w two vector is always the smallest vector when two vectors are joined together from tail

TYPE OF VECTOR

I. Equal Vector ⇒ Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal vector if both have same direction and magnitude



II. Parallel vector ⇒ Two vector $\vec{A}$ and $\vec{B}$ are said to be parallel if they have same direction.



two equal vector can be parallel vector but two parallel vector can not be equal vector

Antiparallel Vector ⇒ Two vector $\vec{A}$ and $\vec{B}$ are said to be antiparallel vector if they have opposite direction.

Collinear Vector ⇒ When two vectors act along same line.

Zero Vector ⇒ A vector having zero magnitude but arbitary direction.

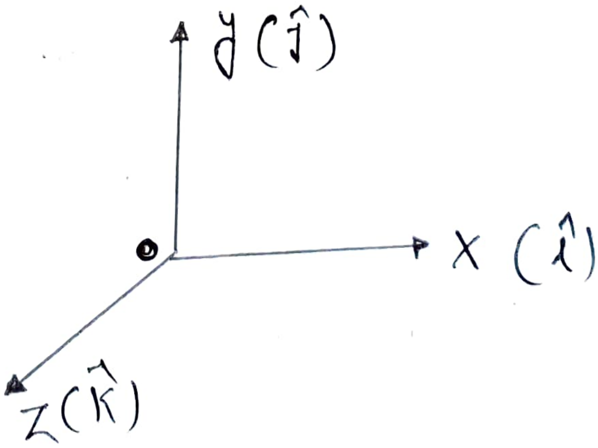
$$\vec{O}$$

Orthogonal Unit Vector ⇒ $\hat{i}$, $\hat{j}$ and $\hat{k}$ are called orthogonal unit vector. These vectors are perpendicular to each other and have unit magnitude.

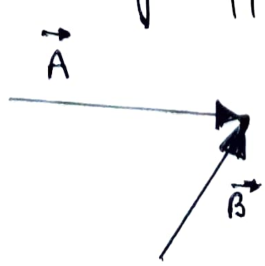
$$\hat{i} = \frac{\vec{x}}{x} = \frac{\text{Vector}}{\text{Magnitude}}$$

$$\hat{j} = \frac{\vec{y}}{y}$$

$$\hat{k} = \frac{\vec{z}}{z}$$



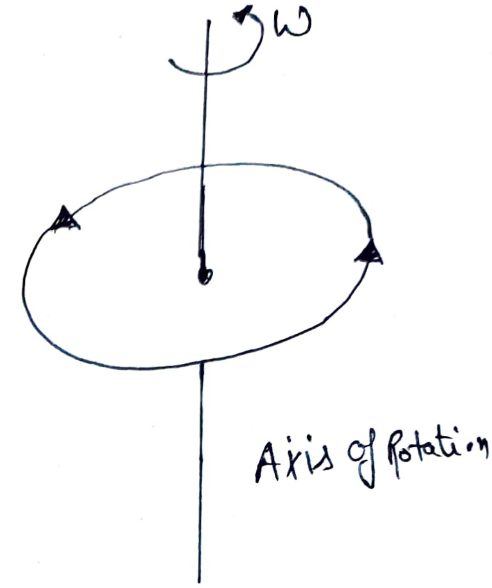
Polar Vector ⇒ These are the vectors which has same point of application



Axial Vector ⇒ These vector represents rotational effect. And their direction is always along the axis of rotation.

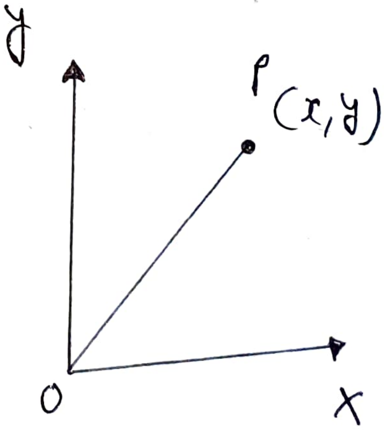
Example → Angular Velocity
Torque, Angular momentum

Direction of Axial vector is given by right hand screw rule.



Position Vector ⇒ To locate the position of any point P in a plane or space, generally a fixed point of reference called the origin is taken. The vector OP is called the position vector of Point P.

$$\vec{OP} = x\hat{i} + y\hat{j}$$



Q→ Find the position vector of a point Z whose coordinates are (2, 3, 6)

Sol $\vec{OZ} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Q→ Find the position Vector whose coordinates are Q (2, −6, 2) and P (−3, 2, −1)

Sol $\vec{OQ} = 2\hat{i} - 6\hat{j} + 2\hat{k}$ $\qquad \vec{OP} = -3\hat{i} + 2\hat{j} - \hat{k}$

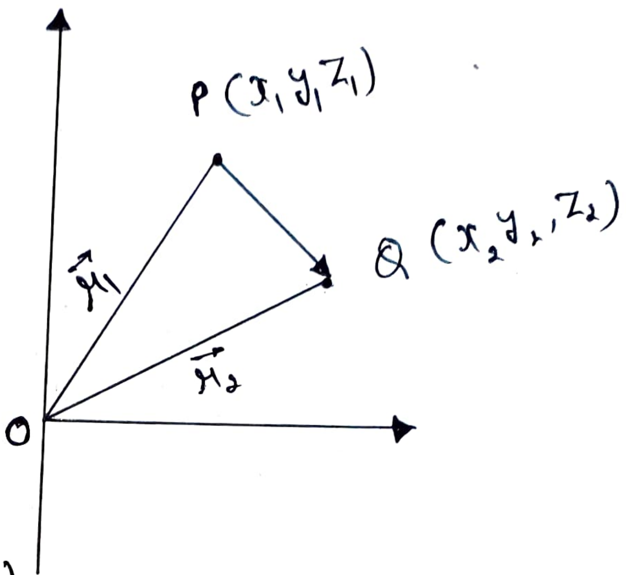
Displacement Vector → It is a vector which gives the displacement b/w two point.

Position Vector of Point P

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

Position Vector of Point Q

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$



Displacement Vector

$\vec{PQ} = \vec{OP} - \vec{OQ}$

$\vec{PQ} = \vec{r}_1 - \vec{r}_2 = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) - (x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$

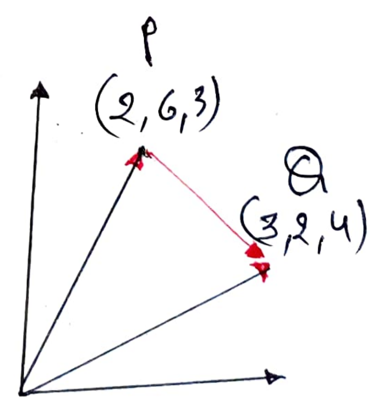
$$\vec{PQ} = (x_1 - x_2)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Q → ~~Let~~ Two points P and Q having coordinate (2, 6, 3) and (3, 2, 4). Find displacement vector PQ

Sol

$\vec{OP} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$OQ = 3\hat{i} + 2\hat{j} + 4\hat{k}$



$\vec{PQ} = \vec{OP} - \vec{OQ} = -\hat{i} + 4\hat{j} - \hat{k}$

MODULUS OF VECTOR

Modulus of vector means magnitude of vector.

Modulus of vector $\vec{A}$ is represented by $|\vec{A}|$ OR A

Example $\vec{A}$ = 3 Newton North

Magnitude OR Modulus = 3 Newton

Q Find Modulus of following Vector

$\vec{R} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ $\vec{Q} = x\hat{i} + y\hat{j} + z\hat{k}$

Sol $|\vec{R}| = R = \sqrt{(2)^2 + (3)^2 + (4)^2}$ | $|\vec{Q}| = Q = \sqrt{x^2 + y^2 + z^2}$

$R = \sqrt{4 + 9 + 16}$

$R = \sqrt{29}$

UNIT VECTOR

UNINT Vector of a vecter is a vector having unit magnitude and direction of given vector.

$$\text{Unit Vector} = \frac{\text{Vector}}{\text{Modulus Vector)}}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Q. Find Unit Vector $\vec{A} = 2\hat{i} + 2\hat{j} + 6\hat{k}$

Sol $$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 2\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (2)^2 + (6)^2}}$$

$$= \frac{2\hat{i} + 2\hat{j} + 6\hat{k}}{\sqrt{44}}$$

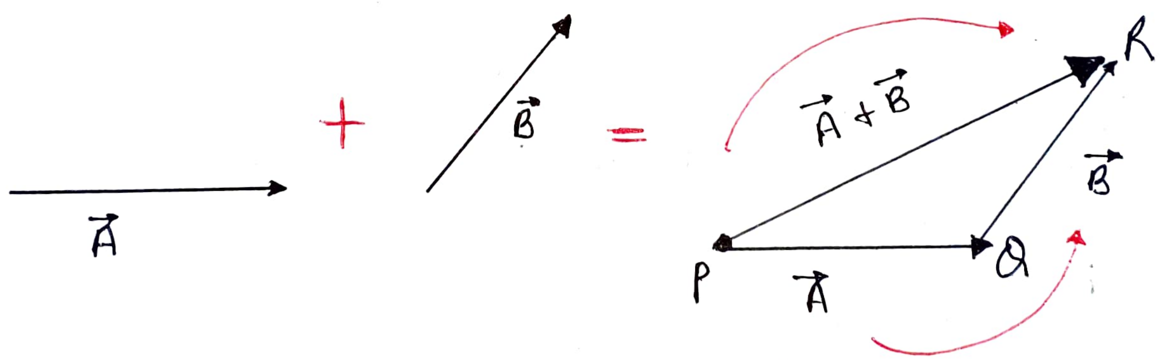
$$\hat{A} = \frac{2}{\sqrt{44}}\hat{i} + \frac{2}{\sqrt{44}}\hat{j} + \frac{6}{\sqrt{44}}\hat{k}$$

Addition Of Vector

- Triangle law of Vector Addition
- Parallelogram law of vector addition
- Polygon laws of vector addition

Tringle Law Of Vector Addition

If two non zero vectors are represented by two sides of a triangle taken in same oder then resultant vector both in terms of direction and magnitude is given by the third side of the triangle drawn in opposite order



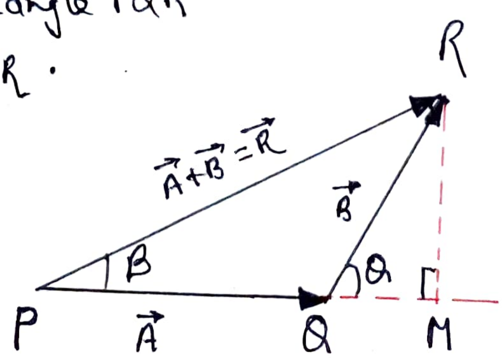
$PQ = \vec{A}$

$QR = \vec{B}$

$\vec{PR} = \vec{A} + \vec{B}$

Mathematical Form Of Triangle Law Of Addition

Let us consider two vector $\vec{A}$ and $\vec{B}$ are represented by two sides of triangle PQR and the resultant is given by side PR. The angle b/w two vector $\vec{A}$ and $\vec{B}$ is θ.



Const → Extend side PQ to M and draw RM ⊥ PM

CALCULATION FOR MAGNITUDE

In $\triangle$ PRM using Pythagorus theorem

$$(PR)^2 = (PM)^2 + (RM)^2$$

$PR = R$ $\quad$ $PM = PQ + QM$

$$(R)^2 = (PQ + QM)^2 + (RM)^2 \text{ --- } \star$$

Now in $\triangle$ RQM

$$\sin\theta = \frac{RM}{QR} \qquad \cos\theta = \frac{QM}{QR} \qquad \boxed{PQ = A} \text{ --- (3)}$$

$$RM = QR\sin\theta \qquad QM = QR\cos\theta$$

$$\boxed{RM = B\sin\theta} \text{ --- (1)} \qquad \boxed{QM = B\cos\theta} \text{ --- (2)}$$

Putting the value of (1), (2), and (3) in equation $\star$

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R = \sqrt{A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta}$$

$$R = \sqrt{A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

CALCULATION FOR ANGLE OF Resultant [Direction]

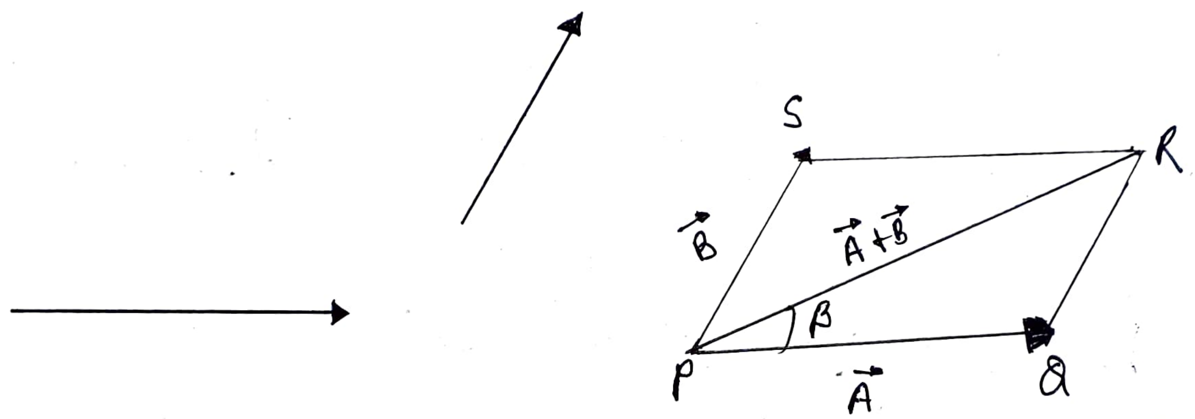
In $\triangle$ PRM

$$\tan\beta = \frac{RM}{QM} = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\beta = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

PARALLELOGRAM LAW OF VECTOR ADDITION

If two non zero vectors are represented by two adjacent sides of a parallelogram then the resultant is given by the diagonal of the of the parallelogram passing through the point of intersection of the two vector.



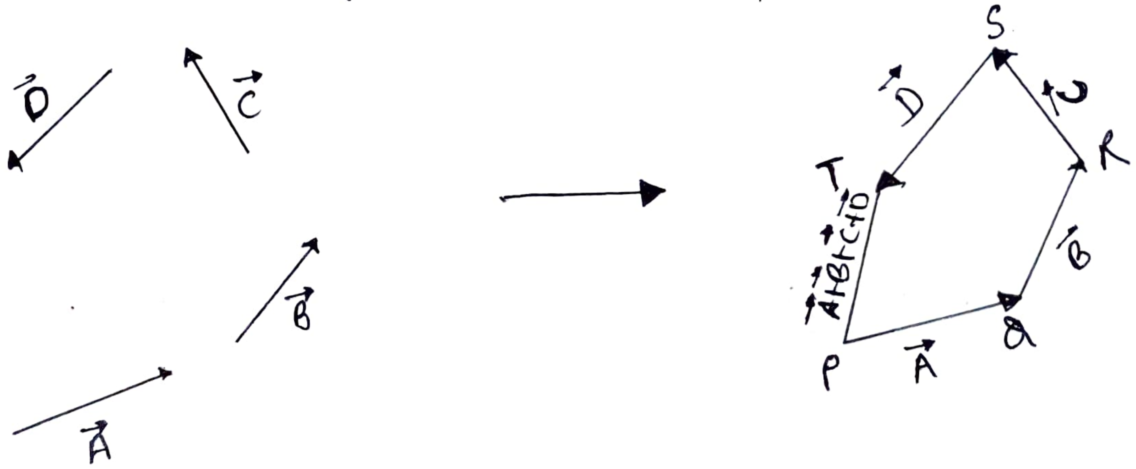
Mathematical form of parallelogram is simillar to triangle law of vector addition.

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$\tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$$

POLYGON LAW OF VECTOR ADDITION

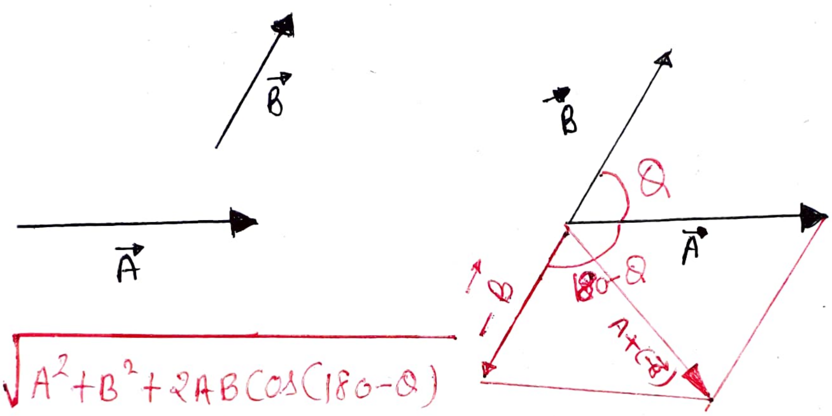
If a number of non zero vectors are represented by (n-1) sides of n sided polygon then the resultant is given by closing side or n side of polygon taken in opposite order.



Subtraction Of Two Vector

Suppose two vector $\vec{A}$ and $\vec{B}$ and we have to subtract $\vec{B}$ from $\vec{A}$. For doing so we will draw $-\vec{B}$ and then we will add them together.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



$$R = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$$

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

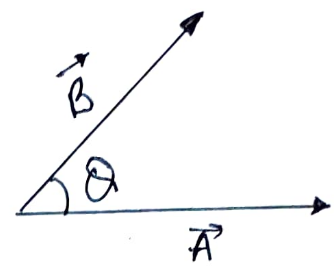
Multiplication Of Vector

Dot Product	Cross product
Scalar Product	Vector Product
$\vec{A} \cdot \vec{B} = AB\cos\theta$	$\vec{A} \times \vec{B} = AB\sin\theta$

Dot Product OR Scalar Product

Dot product of two vectors is defined as the product of the magnitude of two vector with cosine of angle b/w them.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$



Properties Of Dot Product

I. It is always scalar

II. It is Commutative

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

III. It is distributive

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

(IV) Scalar ~~vector~~ product of vector $\vec{A}$ with itself is ~~called~~ A^2

Dot product of Orthogonal vector

$$\hat{i} \cdot \hat{i} = 1 \times 1 \cos 0^\circ = 1$$

$$\hat{i} \cdot \hat{j} = 1 \cdot 1 \cos 90^\circ = 0$$

$\hat{i} \cdot \hat{i} = 1$
$\hat{j} \cdot \hat{j} = 1$
$\hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = 0$
$\hat{j} \cdot \hat{k} = 0$
$\hat{i} \cdot \hat{k} = 0$

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$
$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$
$$\vec{r}_1 \cdot \vec{r}_2 = x_1x_2 + y_1y_2 + z_1z_2$$

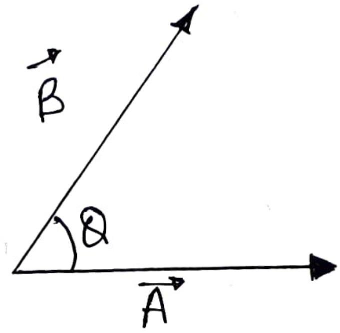
Example

$$\vec{r}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$
$$\vec{r}_2 = 3\hat{i} + 2\hat{j} + 2\hat{k}$$
$$\vec{r}_1 \cdot \vec{r}_2 = 6 + 6 + 8$$
$$\vec{r}_1 \cdot \vec{r}_2 = 20$$

CROSS PRODUCT

Cross product of two vector is defined as a vector having a magnitude equal to the product of the magnitude of two vectors with sine of angle b/w them and direction perpendicular to the plane containing two vector.

$\vec{A} \times \vec{B} = AB\sin\theta$ — Magnitude



Direction of $\vec{A} \times \vec{B}$ = It will be given by right hand screw rule and the screw is rotated from A to B.

So direction of $\vec{A} \times \vec{B}$ will be perpendicular to the plane and outside the plane of the paper.

PROPERTIES OF CROSS PRODUCT

1. Cross product is always vector
2. It does not follow law of commutative

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

3. Cross product of vector $\vec{A}$ with itself will give you zero

$$\vec{A} \times \vec{A} = 0$$

Cross Product of Orthogonal unit Vector

$\hat{i} \times \hat{i} = 1 \times 1 \sin 0 = 0$

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = 1 \times 1 \sin 90^\circ = 1$

Direction will be $\hat{k}$

so

$\hat{i} \times \hat{j} = 1\hat{k}$

$\hat{i} \times \hat{j} = \hat{k}$

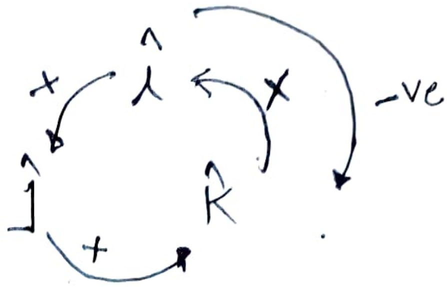
$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$



Anticlockwise +ve

Clockwise = -ve

Q Find $\vec{r_1} \times \vec{r_2}$

$\vec{r_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{r_2} = \hat{i} + \hat{j} - \hat{k}$

$$\vec{r_1} \times \vec{r_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

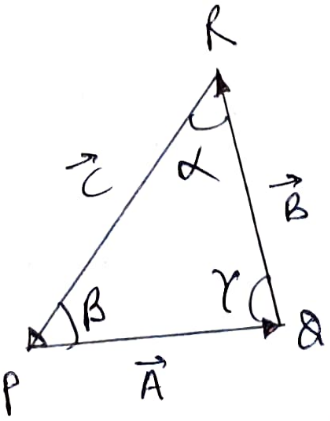
$$\vec{r_1} \times \vec{r_2} = (-3-4)\hat{i} + (-2-4)\hat{j} + (2-3)\hat{k}$$

$$\vec{r_1} \times \vec{r_2} = -7\hat{i} + 6\hat{j} - \hat{k}$$

Law Of Sine OR LAMI's Theorem

According to LAMI's theorem

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$



This theorem is used to find the magnitude of unknown Vector

This theorem mostly used in Laws of Motion when equilibrium is there

PROOF OF LAW OF SINE

$$(\vec{a}+\vec{b}+\vec{c}) = 0$$

$$\vec{c}\times(\vec{a}+\vec{b}+\vec{c}) = \vec{0}\times\vec{c}$$ {Doing cross multiplication by c on both side}

$$\vec{c}\times\vec{a}+\vec{c}\times\vec{b}+\vec{c}\times\vec{c} = 0$$

$$ca\sin(180-\beta)+cb\sin(180-\alpha)+0=0$$

$$-ca\sin\beta + cb\sin\alpha = 0$$

(inward) (outward)

$$cb\sin\alpha = ca\sin\beta$$

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \quad -(1)$$

similiarly

$$\vec{b}(\vec{a}+\vec{b}+\vec{c}) = 0\times\vec{b}$$

we will get

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \quad -(2)$$

from equation (1) and (2)

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$