

MECHANICAL PROPERTIES OF FLUIDS

Fluid \Rightarrow A fluid is a substance that can flow.

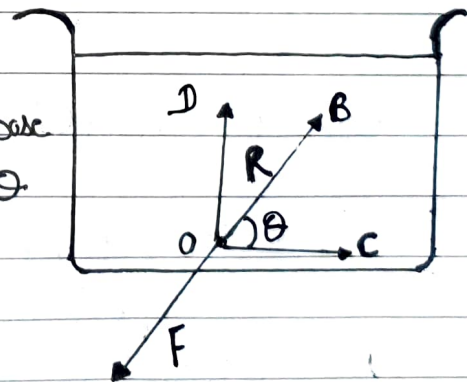
* Both liquids and gases are collectively known as fluid.

Fluid Statics \Rightarrow The branch of physics that deals with the study of fluid at rest.

Fluid dynamics \Rightarrow The branch of physics that deals with fluids in motion.

A fluid always exerts force perpendicular to the surface of the container at every point when the fluid is at rest.

Proof \Rightarrow Consider a liquid contained in the vessel in the equilibrium state at rest. Suppose the liquid exerts a force F at angle θ to the surface of vessel.



Now according to Newton's third law of motion the container will exert a reaction R .

Now this reaction will have two components

- (i) Tangential Component $OC = R \cos \theta$
- (ii) Normal Component $OD = R \sin \theta$

The $R \cos \theta$ component will cause the liquid to flow along OC but the liquid is at rest so

Question

① Find out the pressure exerted by liquid Column

② Find the height of atmosphere of earth.

classmate

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$$R \cos \theta = 0$$

$$R \neq 0$$

$$\cos \theta = 0$$

$$\boxed{\theta = 90^\circ}$$

This proves that a liquid always exerts force at right angle.

Pressure \Rightarrow Thrust acting per unit area over a surface is called pressure.

$$\boxed{P = \frac{F}{A}}$$

SI unit = Pascal (Nm^{-2})

Other units of pressure are

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

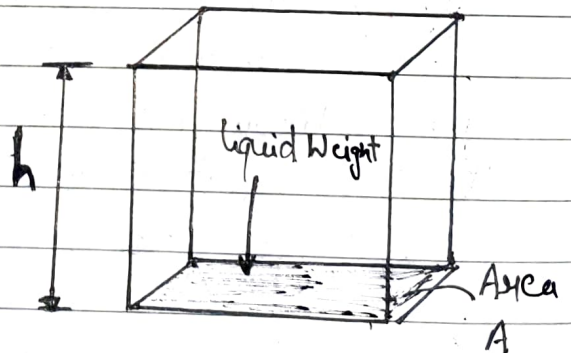
Pressure Exerted by a liquid Column \Rightarrow Consider a vessel of height h having cross-sectional area A is filled with a liquid of density ρ .

Weight of the liquid column =

$$= \text{Mass of liquid} \times g$$

$$= \text{Volume} \times \text{density} \times g$$

$$\boxed{W = Ah \times \rho g}$$



$$P = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{W}{A} = \frac{Ah\rho g}{A}$$

$$\boxed{P = \rho gh}$$

Variation Of liquid Pressure With depth

Imagine a cylindrical element of liquid of cross-sectional Area A and height h . Let P_1 and P_2 be the pressure at point ① and ② respectively.

As the cylinder is at rest, the horizontal forces must be zero

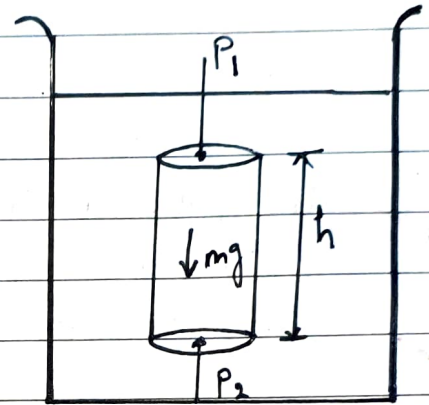
Downward force = Upward force

$$F_1 + \text{Weight of the liquid} = F_2$$

$$P_1 A + \rho A h g = P_2 A$$

$$P_1 + \rho g h = P_2$$

$$P_2 - P_1 = \rho g h$$



if we shift the point ① to the liquid surface then P_1 can be replaced by P_a and P_2 by P

$$P - P_a = \rho g h$$

$$P = P_a + \rho g h$$

Gauge Pressure \Rightarrow The excess pressure at depth h due to gravitation is called Gauge pressure.

$$P - P_a = \rho g h$$

Gauge Pressure.

Pascal's law \Rightarrow (i) ~~The pressure exerted at any point on an enclosed liquid is transmitted equally in all direction~~

(ii) ~~A liquid at rest exerts equal pressure in all the direction when effect of gravity is ignored.~~

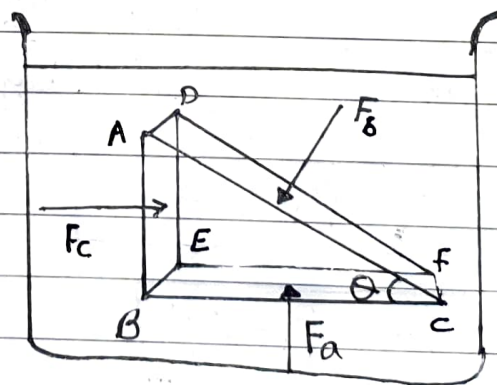
Proof of pascal's law \Rightarrow Pascal law can be proved by using two principles ..

(i) ~~The force on any layer ~~of~~ of a liquid at rest is normal to the layer.~~

(ii) ~~Newton's first law of motion.~~

Consider a small element ABC-DEF in the form of right angled prism in the interior of the fluid. The element is so small that it can be assumed to be at same depth. Thus for effect of gravity can be ignored.

Let F_a , F_b and F_c be the normal forces acting on the faces BEFC, ADFC and ADEB respectively of this element.

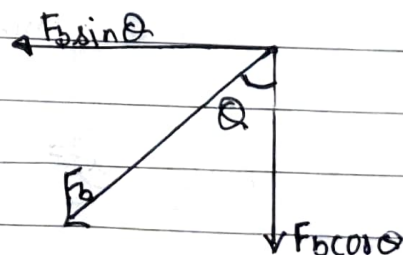


Let A_a , A_b and A_c are area of respective faces. As the liquid element is at rest then the total forces acting on the element must balance.

Along horizontal direction

$$F_b \sin \theta = F_c \quad \text{--- (1)}$$

$$F_b \cos \theta = F_a \quad \text{--- (2)}$$



From the above equation geometry of the figure

$$A_b \sin \theta = A_c \quad \text{--- (3)}$$

$$A_b \cos \theta = A_a \quad \text{--- (4)}$$

Divide equation (1) by (3)

$$\frac{F_b \sin \theta}{A_b \sin \theta} = \frac{F_c}{A_c}$$

$$P_b = P_c \quad \text{--- (5)}$$

Divide equation (2) by (4)

$$\frac{F_b \cos \theta}{A_b \cos \theta} = \frac{F_a}{A_a}$$

$$P_b = P_a \quad \text{--- (6)}$$

from (5) and (6) we can see that pressure at all the point is same.

Application of pascal law

(1)

Hydraulic lift

According to Pascal law

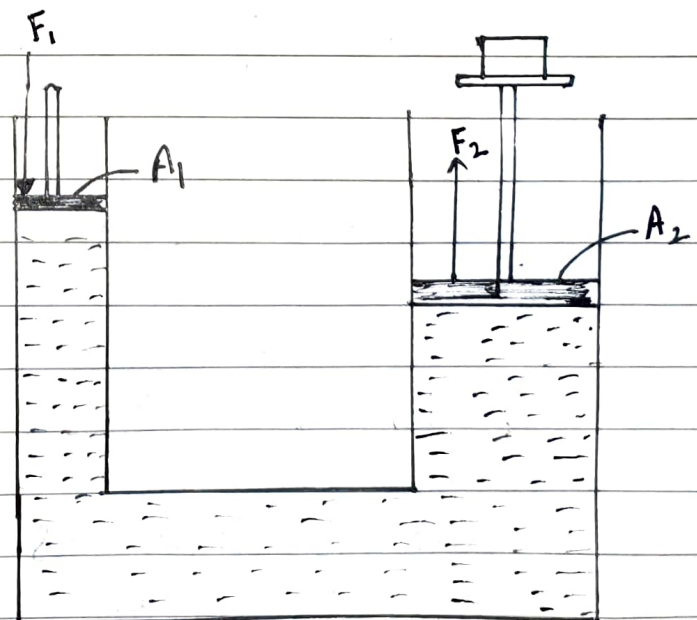
$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{F_1 \times (A_2)}{A_1}$$

since $A_2 > A_1$

$$F_2 > F_1$$



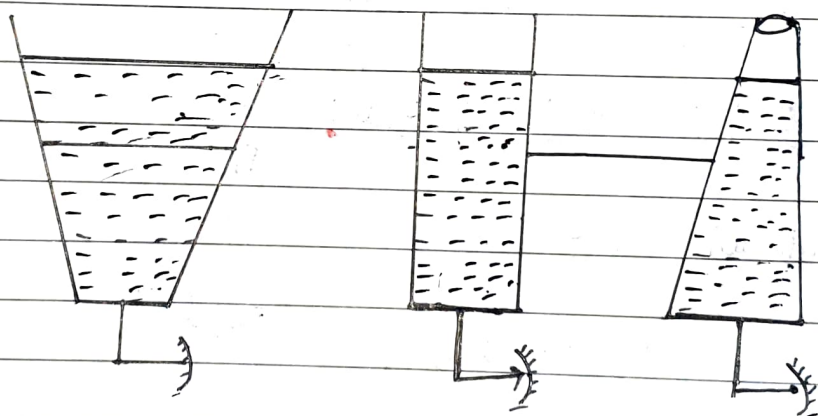
other applications of pascal law is

(i) Hydraulic Brakes

(ii) Hydraulic Lift

Hydrostatic Paradox

According to Pascal's experiment he showed that the pressure exerted by liquid column depends upon the height of the liquid column not on the shape of the vessel.



Archimedes' Principle \Rightarrow According to this when a body is fully or partially immersed in a fluid, it experiences an upward thrust which is equal to the weight of the liquid displaced.

The upthrust acts through the centre of gravity of displaced fluid.

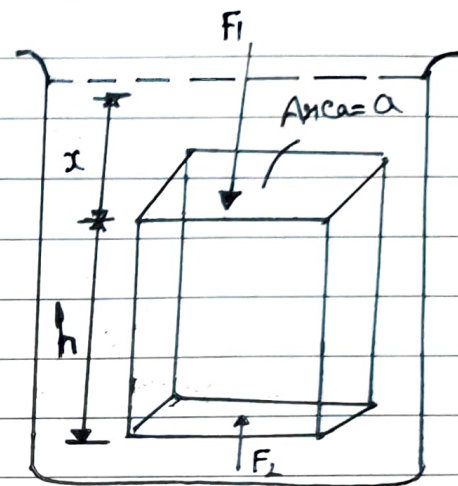
Proof Of Archimedes' Principle \Rightarrow let us consider a body of height h lying inside a liquid of density ρ at a depth x below the free surface of the liquid

Pressure at the upper face of the body

$$P_1 = x\rho g$$

Pressure at the lower face of the body

$$P_2 = (x+h)\rho g$$



Thrust acting on the upper face of the body

$$F_1 = x\rho g a$$

Thrust acting on the lower part of the body

$$F_2 = (x+h)\rho g a$$

Upthrust force $F = F_2 - F_1$

$$F = (x+h)\rho g a - x\rho g a$$

$$F = \rho g a h = \rho V g \quad \left\{ \begin{array}{l} V = a h \\ \rho V = m \end{array} \right.$$

$$\boxed{F = mg}$$

Apparent Weight Of body Immersed In liquid

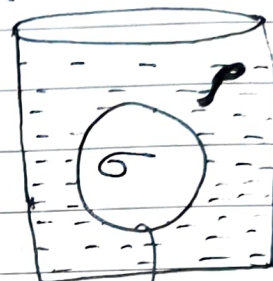
Apparent Weight = Actual weight - Upthrust force

$$W_{app} = W - U$$

$$W_{app} = V\sigma g - V\rho g$$

$$W_{app} = V\sigma g \left(1 - \frac{V\rho g}{V\sigma g}\right)$$

$$W_{app} = W \left(1 - \frac{\rho}{\sigma}\right)$$



$V =$ Volume of body.

Law of floatation

$$W > U$$

$$W = U$$

$$W < U$$

$$\sigma > \rho$$

Body will sink

$$\sigma = \rho$$

Body will float fully immersed

$$\sigma < \rho$$

Body will float

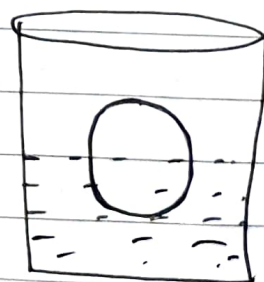
Fractional Submerged Volume of floating body \Rightarrow Let V be

of body and v' is the volume of liquid displaced by body

Weight of the body = Weight of liquid displaced

$$V\sigma g = v'\rho g$$

$\frac{v'}{V}$	$\frac{\rho}{\sigma}$
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Equilibrium Of Floating Bodies \Rightarrow A floating body will be in equilibrium if

- (i) Weight of the liquid displaced must be equal to the weight of the body.
- (ii) The centre of gravity of the body and the centre of buoyancy must lie on the same vertical line.

Viscosity

It is the property of fluid by virtue of which an internal force of friction comes into play when fluid is in motion which opposes the relative motion b/w its different layers.

According to Newton, a force of viscosity F acting tangentially b/w two layers is

- (i) Proportional to the area A of the layers in contact

$$F \propto A \quad \text{--- (1)}$$

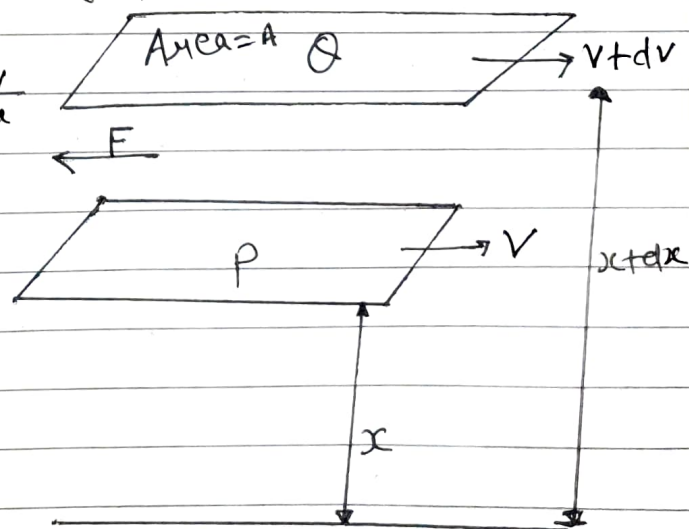
- (ii) Proportional to the velocity gradient $\frac{dv}{dx}$ b/w the two layers

$$F \propto \frac{dv}{dx} \quad \text{--- (2)}$$

Combining (1) and (2)

$$F \propto A \frac{dv}{dx}$$

$$F = -\eta A \frac{dv}{dx}$$



where η = coefficient of viscosity.

Coefficient of Viscosity

$$\text{When } A = 1 \quad \frac{dv}{dx} = 1$$

$$\text{Then } F = \eta$$

So coefficient of viscosity of a fluid may be defined as tangential viscous force required to maintain a unit velocity gradient b/w its two parallel layers each of unit area.

$$\eta = [ML^{-1}T^{-1}]$$

Unit of η = Poise

- (i) Viscosity of liquid decreases with increase in temperature
- (ii) Viscosity of gas increases with increase in temperature because viscosity of gas is due to diffusion of molecule from one moving layer over other moving layer and rate of diffusion is directly proportional to the temp.
- (iii) Viscosity of liquid except water increases with increase in ~~temperature~~ pressure.
- (iv) Viscosity of gases ~~are~~ is independent of pressure.

POISEUILLE'S FORMULA

The volume of liquid flowing out per second through a horizontal capillary tube of length l , radius r and under pressure difference P applied across its ends is given by

$$Q = \frac{V}{t} = \frac{\pi r^4 P}{8\eta l}$$

Stokes' Law

According to Stokes' law the backward viscous force acting on a small spherical body of radius r moving with uniform velocity v through fluid of viscosity η is given by

$$F = 6\pi\eta r v$$

TERMINAL VELOCITY

The maximum constant velocity acquired by a body while falling through a viscous ~~fluid~~ medium is called terminal velocity.

Explanation for terminal velocity \Rightarrow Let us consider a spherical body of radius r falling through a viscous liquid of density σ and coefficient of viscosity η . Let ρ be the density of the body.

~~In equi~~

When the body attains terminal velocity then

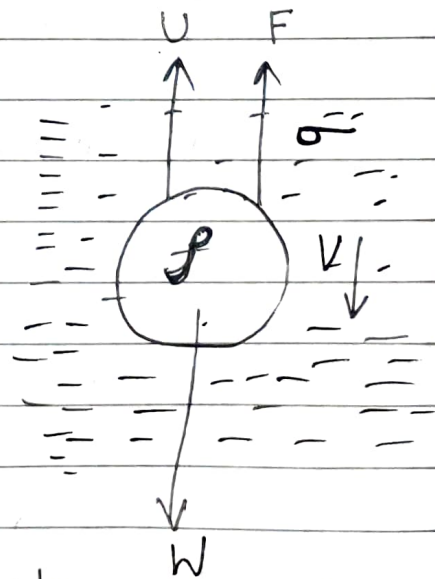
Net upward force acting on the body = Net downward force acting on the body

$$U + F = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

$$3 \cdot 6\pi\eta r v = \frac{4}{3}\pi r^3 g (\rho - \sigma)$$

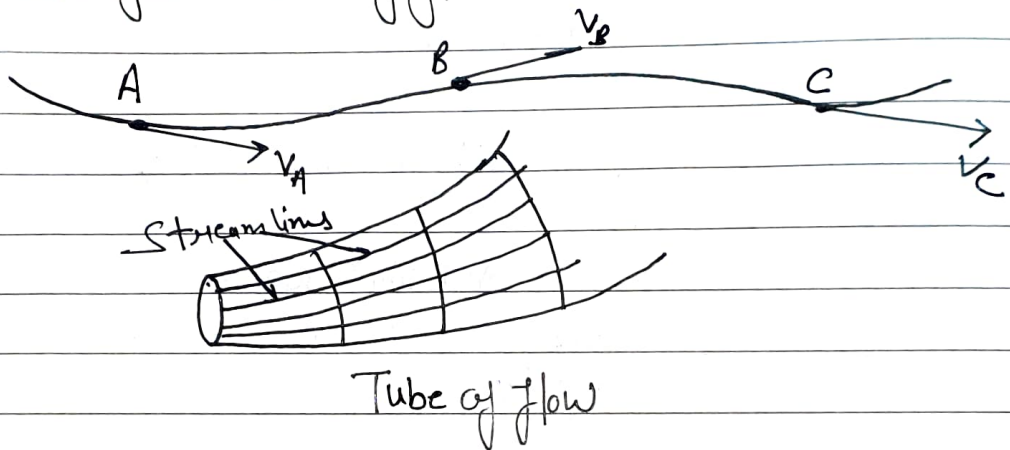
$$v = \frac{2}{9} \frac{r^2 (\rho - \sigma) g}{\eta}$$



where
 W = weight of the body
 U = upthrust force
 F = Drag force.

Streamline Flow / Steady flow

When a liquid flows such that each particle of the liquid passing a given point moves along the same path and has the same velocity as its predecessor, the flow is called streamline flow or steady flow.



Tube of flow \Rightarrow A bundle of streamlines forming a tubular region is called a tube of flow.

Turbulent flow \Rightarrow When the path and the velocity of liquid particle changes continuously, haphazardly then the flow of liquid is called turbulent flow.

LAMINAR FLOW \Rightarrow When liquid flows in such a way that each layer of liquid slide over the other layer. It behave as if different lamina are sliding over one another. Such a flow is called laminar flow.

Velocity Profile \Rightarrow (i) Viscous fluid (ii) Non Viscous fluid



CRITICAL VELOCITY

It is that limiting value of its velocity of liquid upto which the flow is streamlined and above which the ~~lets~~ the flow becomes turbulent.

Factors on which critical velocity depends

- (i) Coefficient of viscosity of fluid (η)
- (ii) Density of fluid (ρ)
- (iii) Diameter of the tube (D)

$$V_c = \frac{\rho \eta}{\rho D}$$

REYNOLD NUMBER

It is dimensionless parameter whose value decides the nature of flow of a fluid through a pipe

$$Re = \frac{\rho V D}{\eta}$$

① If Re is b/w 0 to 2000
the flow is streamlined or
laminar

If Re is above 3000
flow is turbulent

ρ = Density of liquid
 V = Velocity of the liquid
 D = Diameter of pipe
 η = Coefficient of viscosity of fluid

If Re is b/w 2000 and
3000 flow is mixed

Ideal Fluid

An ideal fluid is one which non-viscous, incompressible and its flow is steady and irrotational.

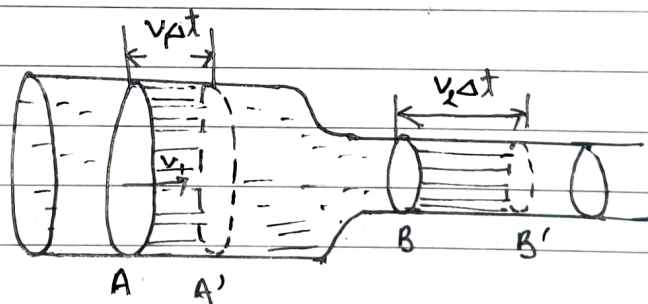
Equation Of Continuity

Let us consider an ideal fluid is flowing b/w A and B section of pipe.

Let

ρ = Density of fluid in pipe

$A_1 = A_2$ = Area of cross-section at point A and B



$v_1 = v_2$ = Velocity of fluid at point A and B

Mass = Volume \times density
Area of cross-section \times length \times density

Mass of fluid entering at point A in Δt time

$$m_1 = A_1 v_1 \Delta t \rho \quad \text{--- (1)}$$

Mass of fluid that flows through section B in time Δt

$$m_2 = A_2 v_2 \Delta t \rho \quad \text{--- (2)}$$

By conservation of mass

$$m_1 = m_2$$

$$a_1 v_1 \rho = a_2 v_2 \rho$$

$$a_1 v_1 = a_2 v_2$$

$$a v = \text{Constant}$$

Equation of Continuity is derived based upon law of conservation of mass.

ENERGY OF FLUID

	<u>Kinetic Energy</u>	<u>Potential Energy</u>	<u>Pressure Energy</u>
	$\frac{1}{2} m v^2$	$m g h$	$P V$
ENERGY per Unit Volume	$\Rightarrow \frac{1}{2} \frac{m v^2}{V}$	$\frac{m g h}{V}$	$\frac{P V}{V}$
	$= \frac{1}{2} \rho v^2$	$\rho g h$	P

BERNOULLI'S PRINCIPLE

As Bernoulli's principle states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an ideal fluid flowing in a pipe remains constant at every point of the fluid.

Mathematically Bernoulli's Principle

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

Proof Of Bernoulli's Principle \Rightarrow

Consider a fluid flowing through a pipe of varying cross-sectional area.

Let

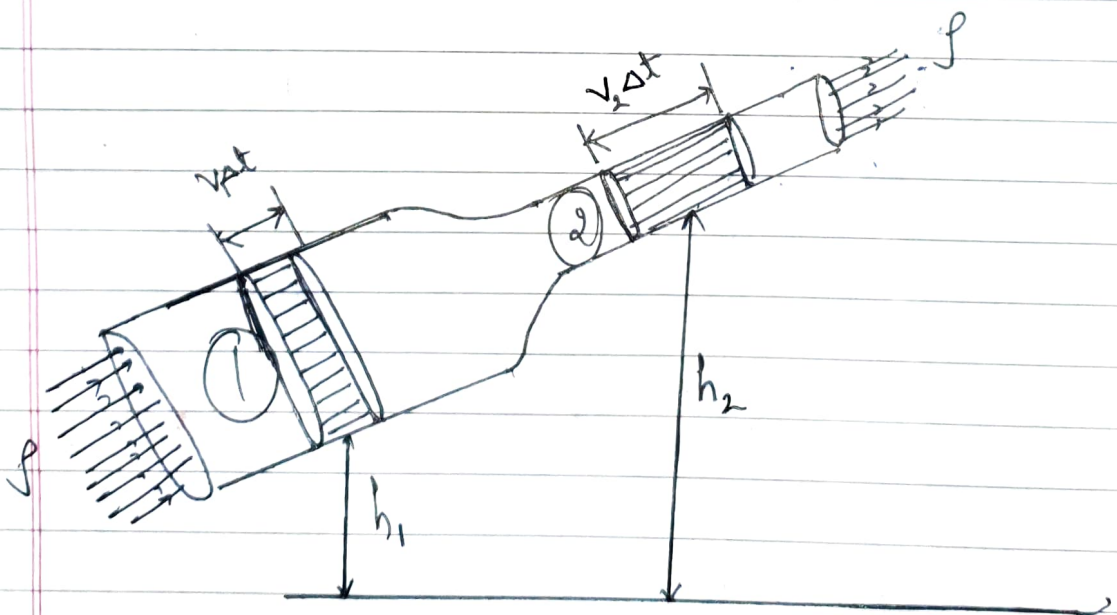
$A_1, A_2 \Rightarrow$ Area of cross-sections at point ① and ②

$v_1, v_2 \Rightarrow$ Velocity of fluid at point ① and ②

$P_1, P_2 \Rightarrow$ Pressure at point ① and ②

$h_1, h_2 \Rightarrow$ Height of point ① and ②

$\rho =$ Density of fluid flowing through pipe



Since fluid flowing through the pipe is incompressible then amount of fluid entering the pipe is equal to fluid leaving the pipe in time Δt

$$m_1 = m_2 = m = \rho \underbrace{A_1 v_1 \Delta t}_{\substack{\text{Density} \times \text{area} \times \text{length} \\ \text{Volume}}}$$

Net work done on the fluid = Work done on the fluid at point ① - Work done on the fluid at point ②

$$\Delta W = P_1 a_1 v_1 \Delta t - P_2 a_2 v_2 \Delta t \quad \left\{ \begin{array}{l} \text{using equation of} \\ \text{continuity } a_1 v_1 = a_2 v_2 \end{array} \right.$$

$$\Delta W = P_1 a_1 v_1 \Delta t - P_2 a_1 v_1 \Delta t$$

$$\Delta W = a_1 v_1 \Delta t (P_1 - P_2) \quad \text{--- ①}$$

Change in Kinetic Energy b/w point ① and ②

$$\Delta K.E = K.E \text{ at point ②} - K.E \text{ at point ①}$$

$$\Delta K.E = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\Delta K.E = \frac{1}{2} m (v_2^2 - v_1^2)$$

$$\Delta K.E = \frac{1}{2} \rho a_1 v_1 \Delta t (v_2^2 - v_1^2) \quad \text{--- ②} \quad \left\{ \begin{array}{l} m = \rho a_1 v_1 \Delta t \end{array} \right.$$

Change in Potential Energy b/w point ① and ②

$$\Delta P.E = mgh_2 - mgh_1$$

$$\Delta P.E = mg(h_2 - h_1)$$

$$\Delta P.E = \rho a_1 v_1 \Delta t g (h_2 - h_1) \quad \text{--- ③}$$

By law of conservation of Energy

Net work done on the fluid = $\Delta K.E + \Delta P.E$.

$$a_1 v_1 \Delta t (P_1 - P_2) = \frac{1}{2} \rho a_1 v_1 \Delta t (v_2^2 - v_1^2) + \rho a_1 v_1 \Delta t g (h_2 - h_1)$$

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{Constant}$$

Bernoulli's theorem is derived by using law of Conservation of energy

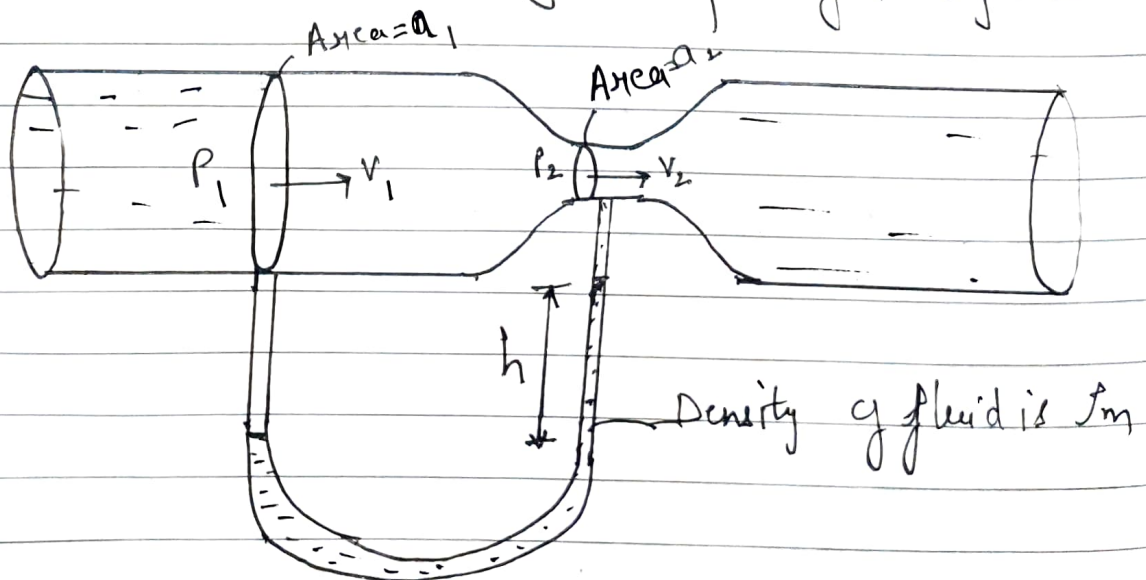
VENTURIMETER

It is a device used to measure the flow of liquid through a pipe.

Principle \Rightarrow It is based on the principle of Bernoulli.

Construction \Rightarrow It consists of a horizontal tube having a wider opening of cross-sectional area A_1 and a narrower neck of cross-sectional area A_2 .

(ii) These two regions of the horizontal tube are connected to a manometer containing a liquid of density ρ_m .



Volume of the liquid flowing out per second

$$Q = a_1 v_1 \quad \text{---} \quad \star$$

Applying Bernoulli's theorem,

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$h_1 \sim h_2$$

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$P_1 - P_2 = \frac{1}{2} \rho v_1^2 \left(\frac{v_2^2}{v_1^2} - 1 \right)$$

from equation of continuity

$$a_1 v_1 = a_2 v_2 \Rightarrow \frac{a_1}{a_2} = \frac{v_2}{v_1}$$

$$\frac{a_1^2}{a_2^2} = \frac{v_2^2}{v_1^2}$$

if h is the height difference in two arms of manometer then

$$P_1 - P_2 = \rho_m g h$$

$$\rho_m g h = \frac{1}{2} \rho v_1^2 \left(\frac{a_1^2}{a_2^2} - 1 \right)$$

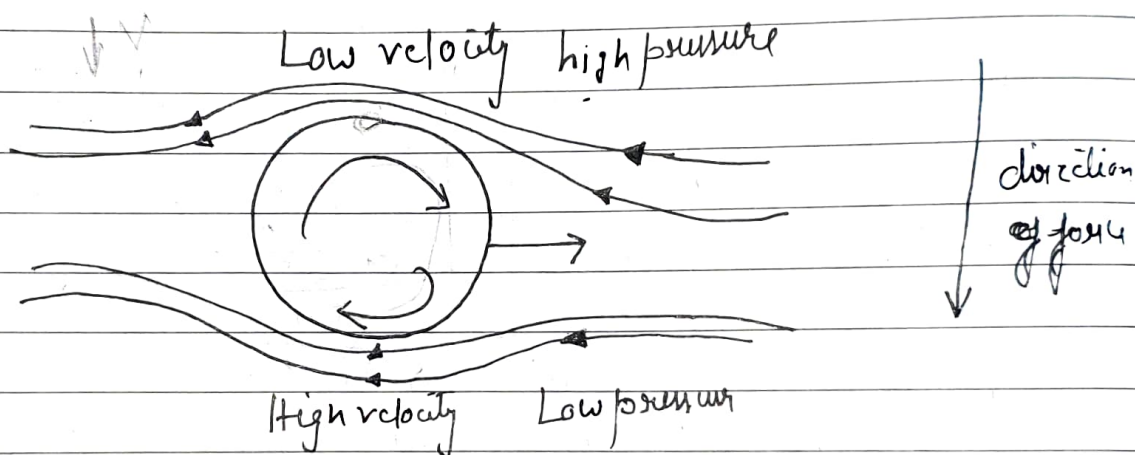
$$v_1 = \sqrt{\frac{2 h \rho_m g a_2^2}{\rho (a_1^2 - a_2^2)}} = a_2 \sqrt{\frac{2 h g \rho_m}{\rho (a_1^2 - a_2^2)}}$$

from \star eqn equation

$$Q = a_1 v_1 = a_1 a_2 \sqrt{\frac{2 h g \rho_m}{\rho (a_1^2 - a_2^2)}}$$

DYNAMIC LIFT \Rightarrow It is the force that acts on body such as aeroplane wing, hydrofoil or spinning ball, by virtue of its motion through a fluid.

Dynamic Lift is caused due to pressure difference b/w two points.



Application of Bernoulli's theorem

- ~~(i)~~ Atomizer or sprayer
- ~~(ii)~~ Explanation of heart attack using Bernoulli's theorem
- ~~(iii)~~ Lift of an aircraft wing.
- ~~(iv)~~ Blowing off the roofs during wind storm.

SURFACE TENSION

COHESIVE FORCE \Rightarrow It is the force of attraction b/w the molecule of substance of same type.

Ex \Rightarrow Force of attraction b/w the molecule of Iron.

ADHESIVE FORCE \Rightarrow It is the force of attraction b/w the molecule of two different substance.

Example \Rightarrow Water wets clothes because of Adhesive force.

Molecular Range \Rightarrow It is the maximum distance upto which a molecule can exert some appreciable force of action on other molecule.

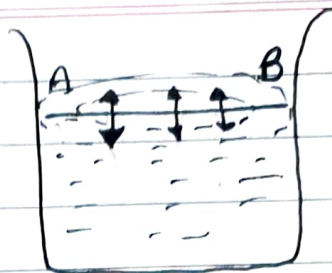
Surface Film \Rightarrow A thin film of liquid near its surface having thickness equal to the molecular range for that liquid is called surface film.

Surface tension \Rightarrow It is the property by virtue of which the free surface of a liquid at rest behaves like an elastic stretched membrane tending to contract so as to occupy minimum surface area.

Surface tension is a force per unit length acting in the plane of the interface between the plane of the liquid and any other surface.

$$\gamma = \frac{F}{l}$$

SI unit of surface tension is N m^{-1}



Dimension = $[M T^{-2}]$

Surface Energy

The extra energy possessed by the molecules of the surface film of unit area compared to the molecules in the interior is called surface energy.

OR

It is equal to the work done in increasing the area of the film by unit amount.

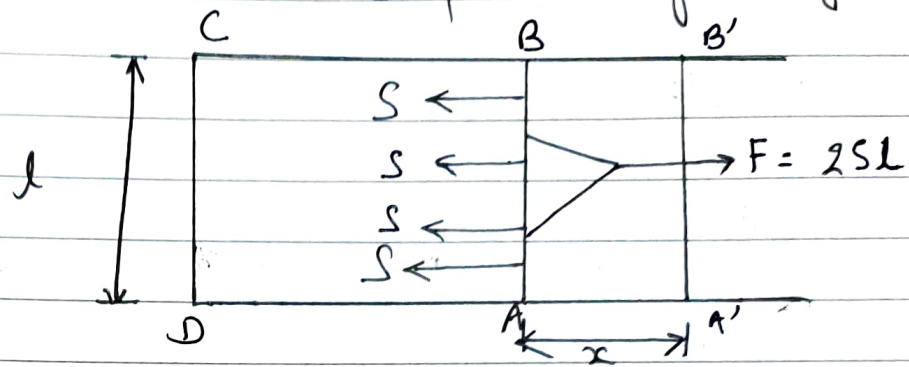
$$\text{Surface Energy} = \frac{\text{Work done}}{\text{Increase in area}}$$

Relationship b/w Surface tension and Surface Energy

Consider a rectangular frame ABCD in which the wire AB is movable. Dip the frame in a soap bubble. A thin film is formed which pulls the wire AB inward due to surface tension with a force

$$F = 2Sl$$

{ The factor 2 is taken because soap has two free surfaces }



Suppose AB is moved out through distance x to the position $A'B'$ then

Work done = Force \times displacement

$$W = 2Slx$$

Increase in area = $2lx$

According to definition

$$\text{Surface Energy} = \frac{\text{Work done}}{\text{Increase in area}}$$

$$\text{Surface energy} = \frac{2Slx}{2lx}$$

$$\text{Surface energy} = S$$

$$\text{Surface energy} = \text{Surface tension}$$

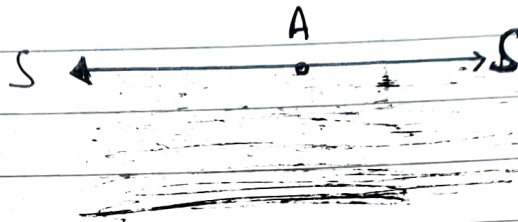
PRESSURE DIFFERENCE

ACROSS a curved liquid surface

~~(i)~~ $P_L = P_V$

vapour side

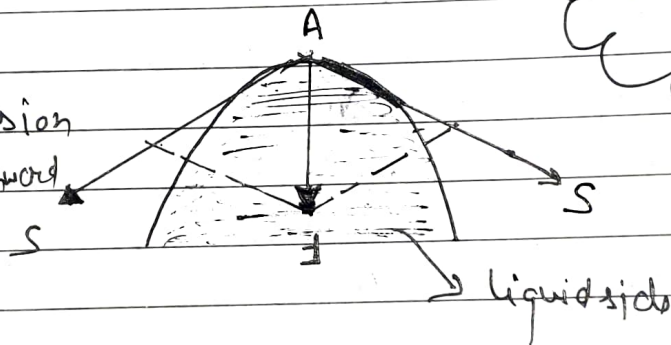
★ Resultant surface tension force is zero



liquid side

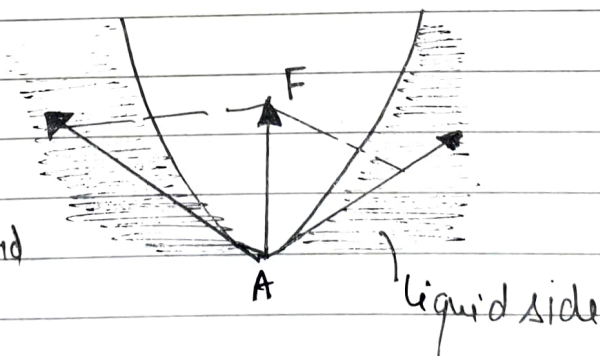
~~(ii)~~ $P_L > P_V$

★ Surface tension act in downward direction

 $P_V =$ Pressure on vapour side

~~(iii)~~ $P_L < P_V$

Resultant surface tension acts in upward direction



Surface tension always acts tangentially to the two surfaces in contact

EXCESS PRESSURE INSIDE A LIQUID DROP

Consider a spherical liquid drop of radius R . Let S be the surface tension of the liquid.

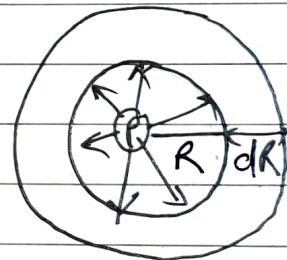
Due to spherical shape, there is an excess pressure P inside the drop over that on outside.

This excess pressure acts normally outwards due to which radius increases from R to $(R + dR)$.

Work done in expanding the liquid bubble

$W = \text{Force} \times \text{displacement}$

$$W = P \times 4\pi R^2 dR \quad \text{--- (1)}$$



Work done = Surface tension \times increase in surface area

$$W = S \times [4\pi(R + dR)^2 - 4\pi R^2]$$

$$W = S \times [4\pi(R^2 + dR^2 + 2RdR) - 4\pi R^2]$$

$$W = S \times [4\pi R^2 + 4\pi dR^2 + 8\pi R dR - 4\pi R^2]$$

$$W = 8\pi S R dR \quad \text{--- (2)}$$

dR^2 is very small and hence neglected.

from (1) and (2)

$$P \times 4\pi R^2 dR = 8\pi S R dR \Rightarrow P = \frac{2S}{R}$$

Excess Pressure in Soap bubble

In case of soap bubble the increase in surface area will be

$$\begin{aligned} \Delta A &= 2(8\pi R dR) \\ \Delta A &= 16\pi R dR \end{aligned} \quad \left[\begin{array}{l} \text{since it is having} \\ \text{two free surfaces} \end{array} \right]$$

$$\text{So work done} = S(16\pi R dR) \quad \text{--- (1)}$$

$$W = P \times 4\pi R^2 dR \quad \text{--- (2)}$$

from (1) and (2)

$$P \times 4\pi R^2 dR = 16\pi S R dR$$

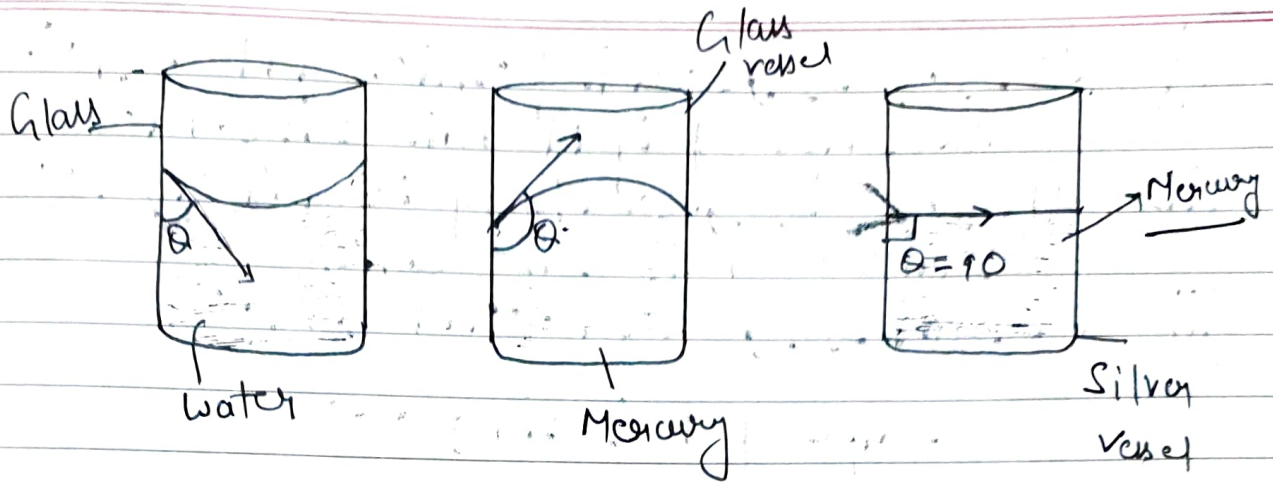
$$P = \frac{4S}{R}$$

ANGLE OF CONTACT

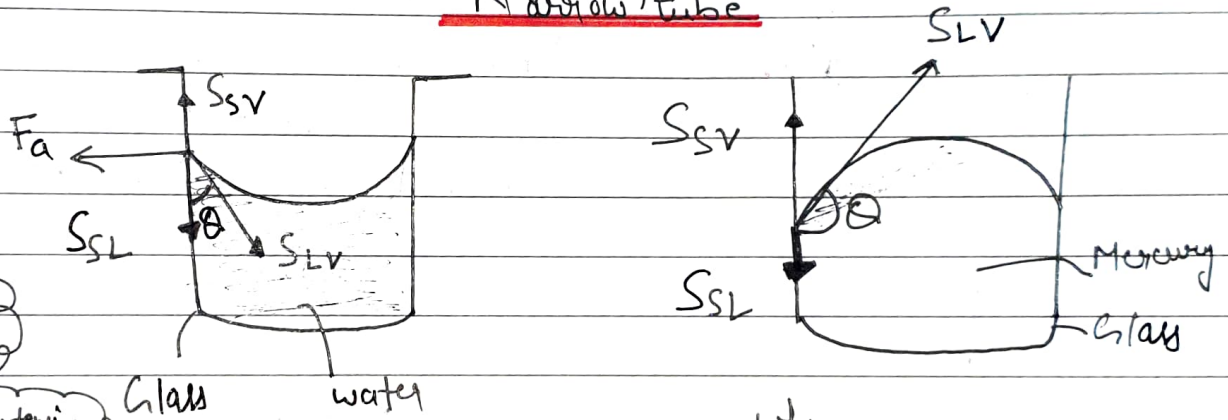
It is defined as the angle θ b/w the tangent to the liquid surface at point of contact and the solid surface inside the liquid.

Factors on which Angle of contact depends

- (i) Nature of solid and liquid in contact
- (ii) Cleanliness of the surface in contact.
- (iii) Temperature of the liquid.



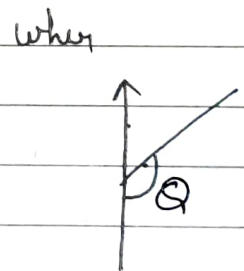
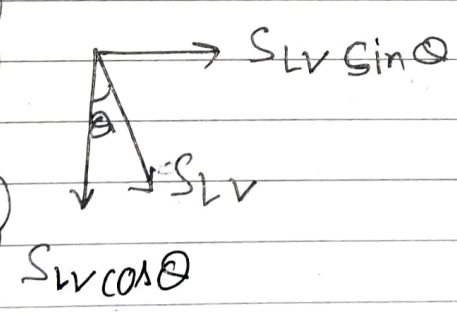
Shape Of Liquid Meniscus In a Narrow tube



$F_a = \text{Adhesive force}$

$S_{SL} = \text{Surface tension b/w solid and liquid}$

$S_{SV} = \text{Surface tension b/w solid and vapour}$



In equilibrium condition Net force in all the direction is zero

$F_a = S_{LV} \sin \theta$

$S_{SV} = S_{SL} + S_{LV} \cos \theta$

$F_a =$

- (i) If $S_{sv} > S_{sl}$, $\cos \theta$ is positive and $\theta < 90^\circ$.
The shape of liquid meniscus is concave.
- (ii) If $S_{sv} < S_{sl}$, $\cos \theta$ is negative and $\theta > 90^\circ$.
shape of meniscus is convex.
- (iii) If $S_{sv} = S_{sl}$, $\cos \theta = 0$ and $\theta = 90^\circ$.

Shape of Meniscus in plane.

☆☆

CAPILLARITY

Capillarity \rightarrow The phenomenon of rise or fall of a liquid in a capillary tube in comparison to the surrounding is called capillarity.

Ascent Formula

Consider a tube (capillary) of radius r is dipped in a liquid of surface tension S and density ρ .

Suppose the liquid wets the sides of the tube then its meniscus will be concave.

The shape of the meniscus of water will be nearly spherical if the capillary tube is of sufficiently narrow bore.

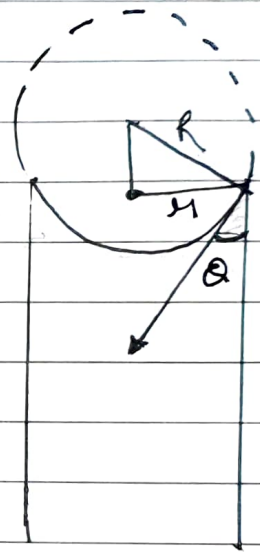
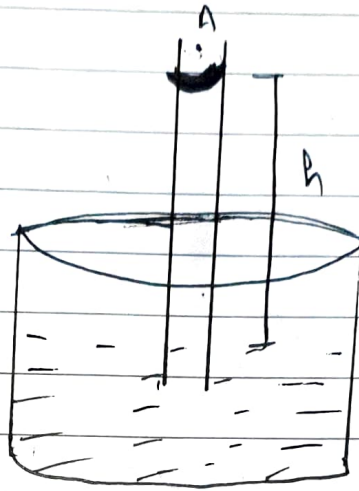
The extra pressure on the concave side at point A

$$P = \frac{2S}{R}$$

$$\frac{\rho}{R} \cos \theta$$

$$R = \frac{\rho}{\cos \theta}$$

$$P = \frac{2S \cos \theta}{\rho} \quad \text{--- (1)}$$



Due to this excess pressure P the liquid rise in the tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure P

$$P = \rho g h \quad \text{--- (2)}$$

from equation (1) and (2)

$$\rho g h = \frac{2S \cos \theta}{\rho}$$

$$h = \frac{2S \cos \theta}{\rho g}$$

if we take into account the volume of the liquid contained meniscus

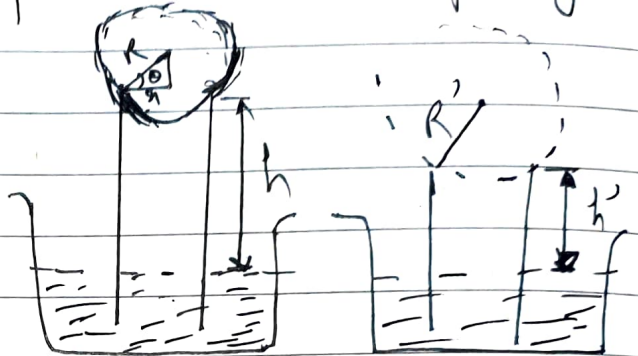
$$h = \frac{2S \cos \theta}{\rho g} - \frac{\rho}{3}$$

Rise of liquid in a tube of Insufficient height

The height to which liquid rises in a capillary tube is given by

$$h = \frac{2S \cos \theta}{r \rho g}$$

$$r = R \cos \theta$$



$$h = \frac{2S \cos \theta}{R \cos \theta \rho g} = \frac{2S}{R \rho g}$$

$$hR = \frac{2S}{\rho g}$$

here $2, S, \rho g$ are constant

since $\frac{2S}{\rho g}$ is constant

\uparrow 0

$$hR = h'R'$$

where R' is the radius of curvature of new meniscus at height h'

Curvature

as $h' < h$ so $R' > R$

$$R' = \frac{hR}{h'}$$

$$R' =$$