UNITS AND MEASUREMENT - 2

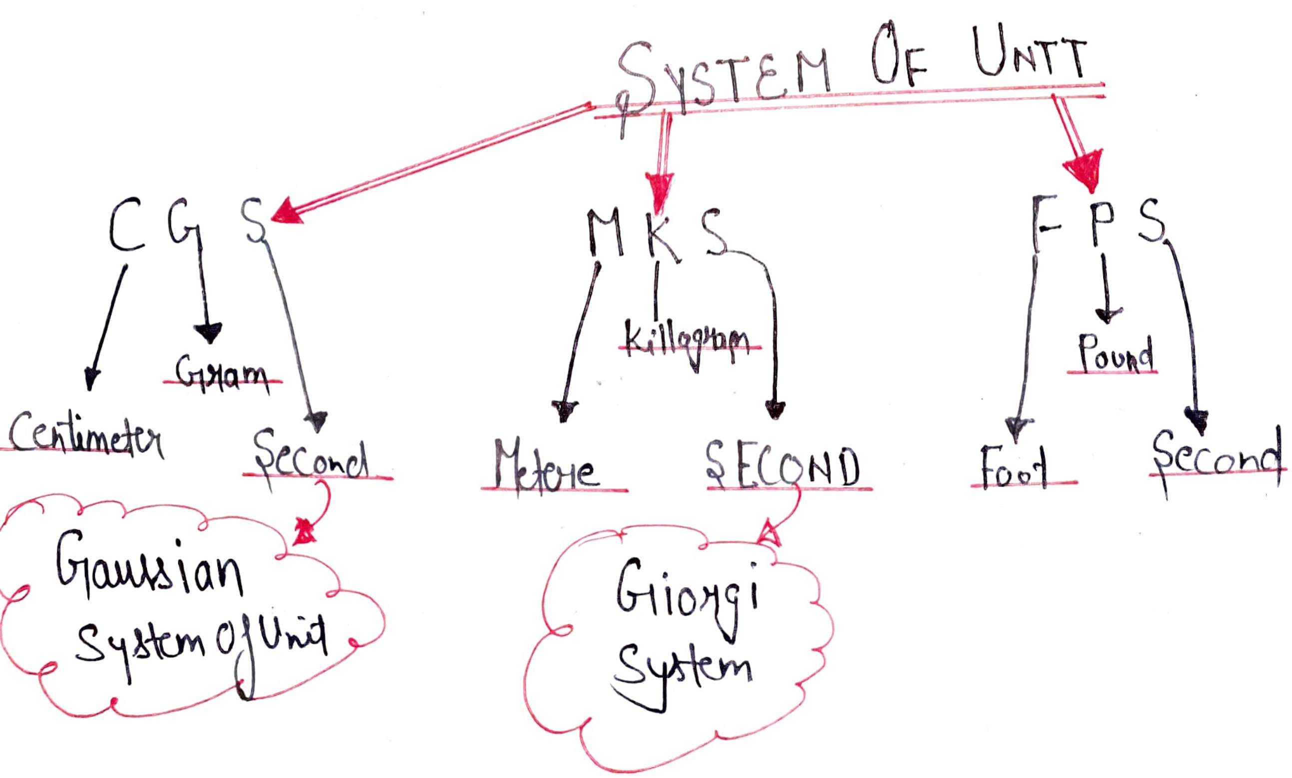
PHYSICAL QUANTITY ⇒ A quantity which can be measured and by which various physical happening can be explained and expressed in form of law is called physical quantity.

Example ⇒ Mass, length, time.

MEASUREMENT ⇒ Measurement of any physical quantity involves comparison with certain internationally accepted refrence standard called unit.

$$\text{Physical Quantity} = \text{Magnitude} \times \text{Unit}$$

$$Q = nu$$



SI System ⇒ It is known as international system of unit and is extended system of units applied to whole physics.

FUNDAMENTAL UNIT ⇒ These are the units which are independ[ent] from other unit and does not require other units to explain them.

There are seven Fundamental unit.

(I) Mass - Kilogram (II) Time - Second (III) Temperature = Kelv[in]
(IV) Current - Ampere (VI) Amount of substance - mole (VII) Distance = met[er] length

DEFINATION OF FUNDAMENTAL UNIT ⇒

Kilogram ⇒ (Unit of Mass) ⇒ Kg ⇒ The Killogram is equal to the mass of international prototype made up of platinum - iridium alloy of cylindrical shape

Meter → It is the length of path travelled by light in vacuum during time interval of ($1/299,792,458$) second.

Second ⇒ Second is the duration of 9,192,631,770 periods of the rad[iation] corresponding to the transition b/w two hyperfine levels of t[he] ground state of cesium-(133) atom.

Kelvin ⇒ It is the fraction $1/273.16$ of thermodynamic temperat[ure] of the triple point of water.

Ampere ⇒ Ampere is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed 1 metre apart in vaccum would produce b/w these conductors a force equal to 2×10^{-7} N per metre of length.

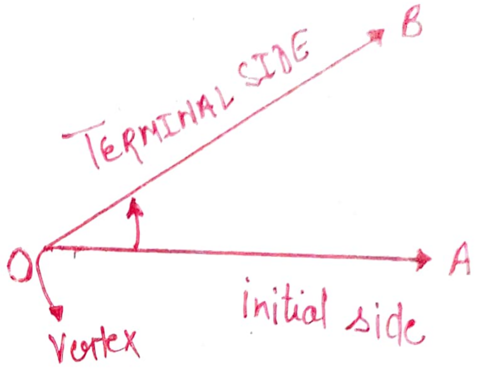
Amount of Substance → The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 Kilogram of Carbon-12

UMINOUS Intensity ⇒ CANDELA → The candela is the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} hz and that has a radiant intensity in that direction of $1/683$ watt per Steradian.

MEASUREMENT OF ANGLE

ANGLE ⇒ Angle is measure of rotation of given ray about its initial point. It is also known as plane angle

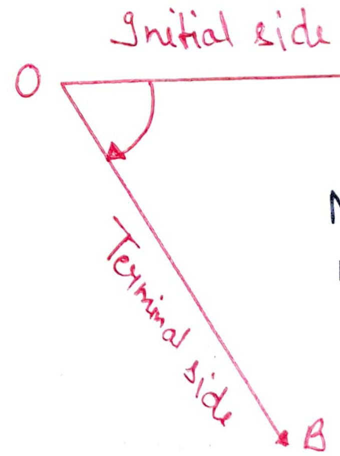
INITIAL Side ⇒ The original ray is called initial side



Positive Angle

Terminal side ⇒ The final position of Ray after rotation is Called terminal side

Positive Angle ⇒ If the ray is rotated in anticlockwise direction Then the angle obtain is taken as positive angle.



Negative Angle

Negative Angle ⇒ If the ray is rotated in Clockwise direction then it is taken as ~~p~~ negative

MEASURE OF Angle ⇒ It is the amount of rotation performed to get the terminal side from initial side.

SYSTEM OF MEASUREMENT OF ANGLE

- (English) Sexagesimal System (degree system)
- (French) Centesimal System or (grade system)
- Circular System (Radian)

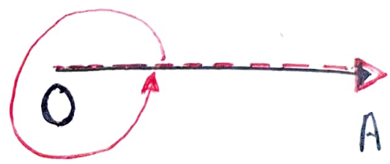
Sexagesimal System.

When the initial ray OA is rotated such that after one rotation it will return back to original position or initial position of OA. Then the angle formed is divided into 360 part

So in one complete rotation
Angle covered will be 360°

Value of one division $= \frac{1}{360}^\circ$

1° is divided into 60 minute



$$1^\circ = 60'$$

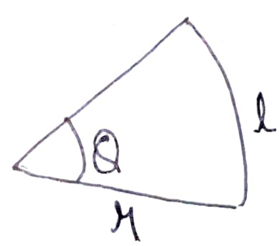
1 minute will be divided into 60 second

$$1' = 60''$$

CIRCULAR SYSTEM [Radian]

Radian ⇒ It is the measure of angle which is given by ration of length of arc to radius of circle.

$$\theta = \frac{l}{r}$$



One radian ⇒ It is the angle substended by an arc having length equal to radius.

In one complete rotation angle will be formed

$$\theta = \frac{2\pi r}{r} \Rightarrow \theta = 2\pi$$

So for one Complete rotation $\theta = 2\pi$ radian

Relationship b/w Degree and Radian.

In Complete rotation $\theta = 360°$

In one Complete rotation = $\theta = 2\pi$ radian

$360° = 2\pi$ radian

$1° = \frac{2\pi}{360}$ radian

$$1° = \frac{\pi}{180}$$

Centesimal System [Grade]

In centesimal system when ray is ~~revoted~~ rotated for Complete circle then angle subtended is ~~36~~ 400^g.

For one Complete revolution = 400^g

$1^g = 100'$ (minute)

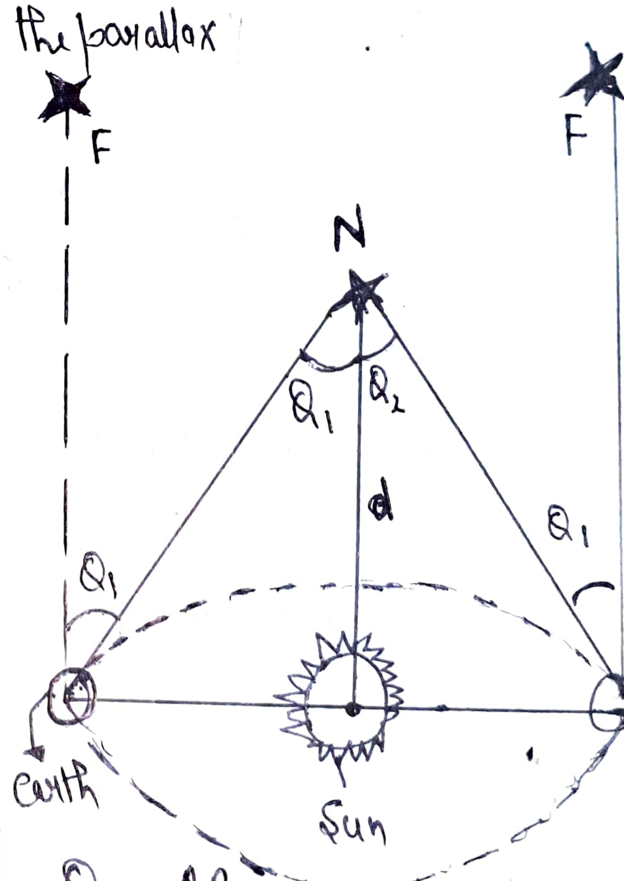
$1' = 100''$ (second)

Measurement Of length By Parallax Method

Parallax ⇒ Parallax is apparent shift in the position of an object with respect to another when we shift our eye sidewise.

Distance of a Near by star by parallax Method

* suppose N is the near by star whose distance is to be found.
* E is a far off star whose direction position is fixed for all position of the earth in orbital motion.
* When the earth is at a point A, the parallax angle b/w distance Star F and Nearby star N is determined
* After six month the the earth is diametrically opposite position B.
* The parallax angle $\angle NBF = Q_2$ is measured
* The parallax angle substended by N on the earth's orbital diameter AB is



$$Q = Q_1 + Q_2$$

$$Q = \frac{\text{Arc}}{\text{Radius}}$$

$$Q = \frac{AB}{AN}$$

$$Q = \frac{AB}{NS}$$

{ Since the star is far away so $AN \sim NS$ }

$$Q = \frac{AB}{d}$$

$$d = \frac{AB}{Q}$$

where Q should be in radian

~~Angles Test And Measurement~~

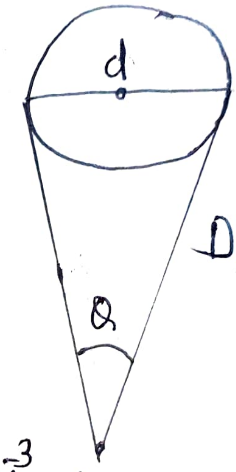
Angle ⇒

NUMERICAL ON PARALLAX

Q→1 The Sun's Angular diameter is measured to be 1920''. The distance D of sun from earth is 1.496×10^{11} m. What is diameter of sun.

Sol

$$\theta = \frac{d}{D}$$



$$d = \theta D$$

$$\theta = 1920'' = \frac{1920}{60 \times 60} \times \frac{\pi}{180} = 9.31 \times 10^{-3}\ \text{rad}$$

$$d = 9.31 \times 10^{-3} \times 1.496 \times 10^{11}$$

$$d = 1.39 \times 10^{9}\ \text{m}$$

SIGNIFICANT FIGURE

In a measured quantity, significant figures are the digits which are absolutely correct plus first uncertain digit.

RULES FOR SIGNIFICANT FIGURE

1) All non zero digits are significant.

Example ⇒ 123.8 ⇒ 4

168.25 ⇒ 5

2) Zeroes appearing b/w two non-zero digits are significant

Example ⇒ 12068 = 5

1.008 = 4

3) Trailing zero after decimal places are significant

Example ⇒ 6.2000 = 5 6.000000 = 7

6.000 = 4

4) Power of 10 are not counted as significant figure.

Example ⇒ 1.28×10^{-6} = 3

108.3×10^{4} = 4

5) If measurement is less than one, then all zeroes occuring to the left of last non-zero digit are not significant

Example ⇒ 0.0026 = 2 0.00260 = 3

0.086 = 2

6) Change in units of measurement of a quantity does not change the number of significant figure

Example ⇒ 7.2cm → 2

0.072 m → 2

0.000072Km → 2

7.2×10^{7} nm → 2

(7) Terminal or trailing zeroes in a number without decimal points are not significant.

Example ⇒ 264000 mm → 3

8200 → 2

(8) Exact Measurement have infinite number of significant figure.

Example ⇒ 40 Eggs ⇒ Infinite

46 Student in a class ⇒ Infinite

(9) All the constant have infinite significant figure

Example ⇒ $\pi = \frac{22}{7}$ ⇒ Infinite

Speed of light in vacuum ⇒ Infinite

G ⇒ Infinite

h ⇒ Infinite

Avogadro Number

Question For PRACTICE

(1) Count total No of significant figure

(1) 4.080 cm

(2) 10.00 cm

(3) 0.079 m

(4) 950

(5) 4.070800

(6) 7.090×10^{5} m

(7) 12 Banana

(8) 7.28×10^{-9} m

ROUNDING OFF A DIGIT

(1) If the number lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However if it is more than 5, then the cut of digit is increased by 1.

Example ⇒ Round off upto three significant digit

(I) 6.269 Ans:- 6.27

(II) 6.263 Ans:- 6.26

(2) If the digit to be dropped is 5 followed digit other than zero, then the preceeding digit is increased by 1

Rounded off to three

Example ⇒ $x = 12.658 \Rightarrow 12.7$

$x = 16.251 \Rightarrow 16.3$

(3) If the digit to be dropped is simply 5 or 5 followed by zero then the preceding digit is left unchanged if it is even OR Increased by 1 if it is odd.

Rounded off to two digit

Example ⇒ $x = 6.25 \Rightarrow 6.2$

$x = 6.25 \Rightarrow 6.2$

$x = 6.35 \Rightarrow 6.4$

$x = 6.350 \Rightarrow 6.4$

Question Based on Rounding of

Q-1 Round off the following Numbers to three significant figure.

(a) 24572 ⇒

(b) 24937 ⇒ 24.9

(c) 36.350 ⇒ 36.4

(d) 42.450×10^9 ⇒

Alzebraic Operation With Significant Figure

(1) ~~Addition And Substraction~~ ⇒ When two measured values added or substracted the Least Number of digits after the decimal in n. Then sum or difference also, contain number of digits after decimal should be n.

Ex

$$\begin{array}{r} 1.26 \\ +\ 1.356 \\ \hline 2.616 \end{array}$$

Ans:- 2.62

$$\begin{array}{r} 1.682 \\ 2.9 \\ +\ 3 \\ \hline 7.582 \end{array}$$

Ans:- 8

After Rounding off to two digit after decimal because the Number that we have added have two digits minimum after decimal

$$\begin{array}{r} 2.69 \\ -\ 1.8 \\ \hline 0.89 \end{array}$$

Ans:- 0.9

MULTIPLICATION
OR
Division

Suppose in the measured value to be multiplied or divided the least number of significant digits be ~~one~~ n. Then in the product or quotient, the number of significant digits should also n

Example ⇒

(1)
$$\begin{array}{r} 2.3 \\ \times 1.2 \\ \hline 46 \\ 23 \\ \hline 2.76 \\ \hline \end{array}$$

Now the answer should be round off to two significant digit because the ~~digits~~ no. of digit which are significant are two

(2)
$$\left.\begin{array}{r} 2.36 \\ \times 8 \end{array}\right\} \longrightarrow$$ Least No. of significant digit is one so answer should also be rounded off to one significant digit.

$$\begin{array}{r} \hline 18.88 \\ \hline \end{array}$$

Ans:- 20

(3)
$$\frac{1.96}{2} = 0.98$$

After Rounding off Answer will be 1

ERROR

Accuracy ⇒ It refers to the closeness of a measurement to the true value of the physical quantity.

Precision ⇒ It refers to the closeness of two or more measurement to each other.

Example ⇒ Suppose you have a ball of mass 10.12 kg. You gave this Ball to two students to measure the mass of the ball

Readings Taken by student A	
1	10.11
2	10.13
3	10.12
4	10.11

Readings taken by A is accurate because it is closer to true value

Reading taken by student B	
1	10
2	10
3	10
4	10

Readings taken by B is precise because they are close to each other

* Accurate reading or measurement can also be precise but precise may or may not be accurate

Error In Measurement

Error in measurement is equal to the difference b/w the true value and measured value.

$$\text{Error} = \text{True Value} - \text{Measured Value}$$

TYPE OF ERROR

(I) Constant Error

(II) Systematic Error →
* Instrumental error
* Personal error
* Error due to external cause.
* Imperfection in experimental technique

(III) Random Error

(IV) Least Count Error

(V) Gross errors OR Mistakes.

Method To minimize Error ⇒ The causes of random error are unknown and we can not remove the completely. But we can minimize the error by taking large no. of measurement at different time and condition. Now if take the average of measurement then it will have minimum percentage of error.

True Value ⇒ The average of all the measurement is known as true value.

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \dots a_n}{n}$$

True Value.

a_1, a_2, a_3 → These are measurement

Absolute Error ⇒ The difference b/w true value of a quantity and the measured value of the quantity is known as absolute error.

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

Δa_1 = Absolute error in measurement 1

Absolute error can be positive or negative

Mean Absolute Error ⇒ Arithmatic mean of the magnitude of absolute errors in all the measurements is called mean absolute error.

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|}{n}$$

* The final result of measurement can be written as

$$a = a_m \pm \Delta a_{mean}$$

* This means that value of a is likely b/w $a_m + \Delta a_{mean}$ and $a_m - \Delta a_{mean}$

Relative Error ⇒ Ratio of mean absolute error to the true value or mean value of the measured quantity.

$$\text{Relative error} = \frac{\Delta a_{mean}}{a_m}$$

Percentage Error = $(\text{Relative error} \times 100) = \frac{\Delta a_{mean}}{a_m} \times 100$

Q→ The diameter of a wire is measured by screw gauge was found as 1.625, 1.626, 1.624, 1.623 and 1.627.
Calculate (I) Mean value of diameter [True value] (II) absolute error (III) Mean absolute error (IV) Relative error (V) Percentage error (VI) Express the results in terms in terms of percentage error

Sol

$$a_m = \frac{1.625 + 1.626 + 1.624 + 1.623 + 1.627}{5} = 1.625$$

$$a_m = 1.625$$

** If in the answer there are more than 3 digits after decimal then we need to round off to three digit

(b) Absolute error

$\Delta a_1 = 1.625 - 1.625 = 0$

$\Delta a_2 = 1.625 - 1.626 = -0.001$

$\Delta a_3 = 1.625 - 1.624 = 0.001$

$\Delta a_4 = 1.625 - 1.623 = 0.002$

$\Delta a_5 = 1.625 - 1.627 = -0.002$

c Mean absolute error

$$\Delta a_m = \frac{0 + 0.001 + (0.001) + (0.001) + (0.002) + (0.002)}{5} = 0.0012$$

$$\boxed{\Delta a_m = 0.0012}$$

(d) Relative error $= \frac{0.0012}{5} = \frac{\Delta a_m}{a_m} = 0.00024$

$$\boxed{\text{Relative error} = 0.00024}$$

e Percentage error $= \frac{\Delta a_m}{a_m} \times 100 = \cancel{\frac{0.0012}{5} \times 100} =$

$$= \frac{0.0012}{5} \times 100 = 0.024\%$$

Diameter $= (6.25 \pm 0.024\%)$

Combination Of Error

Let a and be b are two measurements having Δa and Δb as their absolute error respectively in their measurement.

1. Error in the Combination of ~~two~~ Measurement

$$x = a + b$$

$$x \pm \Delta x = (a \pm \Delta a) + (b \pm \Delta b)$$

$$x \pm \Delta x = (a+b) \pm (\Delta a + \Delta b)$$

$$x \pm \Delta x = x \pm (\Delta a + \Delta b)$$

$$\Delta x = \pm (\Delta a + \Delta b)$$

{ Δx = absolute error in the measurement of x

(2) Errors In difference

$$x = a - b$$

$$(x \pm \Delta x) = (a \pm \Delta a) - (b \pm \Delta b)$$

$$(x \pm \Delta x) = (a-b) \mp \Delta b \pm \Delta a$$

$$(x \pm \Delta x) = x \pm (\Delta a + \Delta b)$$

$$\Delta x = \pm(\Delta a + \Delta b)$$

{ Δx = It is the absolute error when a and b are subtracted

Error will be same in addition and subtraction

(III) Error in Multiplication

$$x = ab$$

$$x \pm \Delta x = (a \pm \Delta a)(b \pm \Delta b)$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = a\left(1 \pm \frac{\Delta a}{a}\right) b\left(1 \pm \frac{\Delta b}{b}\right)$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = x\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

$$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a \Delta b}{ab}$$

Product of two small term will be very small. eg = 0.1 × 0.1 → 0.01

$$\boxed{\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)}$$

$\frac{\Delta a \Delta b}{ab}$ = is a very small term so it will be ignored

(IV) Error in Division

$$x = \frac{a}{b}$$

$$(x \pm \Delta x) = \left(\frac{a \pm \Delta a}{b \pm \Delta b}\right)$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)}$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = x\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$$

Bionomial expansion
$(1+x)^m = 1 + mx$
$(1+x)^{-m} = 1 - mx$
This is used when x is very small

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \mp \frac{\Delta b}{b}\right)$$

$$\cancel{1} \pm \frac{\Delta x}{x} = \cancel{1} \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b} \pm \frac{\Delta a \Delta b}{ab}$$

Since Δa, and Δb are very small so their product is very small

$$\boxed{\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)}$$

(V) Error in Quantity raised to some power

$$x = \frac{a^m b^n}{c^p}$$

taking log on both side

$$\log x = m \log a + n \log b - p \log c$$

Differentiating both side w.r.t K

$$\frac{1}{x}\frac{dx}{dK} = \frac{m}{a}\frac{da}{dK} + \frac{n}{b}\frac{db}{dK} - \frac{p}{c}\frac{dc}{dK}$$

$$\frac{dx}{x} = m\frac{da}{a} + n\frac{db}{b} - p\frac{dc}{c}$$ We can have directly written this

writing above in terms of fractional error

$$\pm\frac{\Delta x}{x} = \pm m\frac{\Delta a}{a} \pm n\frac{\Delta b}{b} \pm p\frac{\Delta c}{c}$$

$$\frac{\Delta x}{x} = \pm\left(m\frac{\Delta a}{a} + n\frac{\Delta b}{b} + p\frac{\Delta c}{c}\right)$$

Q→ If $Z = \frac{A^p B^q}{C^r}$, then find maximum fractional error in Z.

Sol^n = $$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Question Based Upon Combination of error

Q.1 Two length wire are (2.681 ± 0.01) and (2.16 ± 0.002) in meter respectively. Calculate the ~~error in~~
(i) Sum of length of two wire
(ii) Difference of length of two wire, Result should be with error.

Sol (I) $L = l_1 + l_2$ $\quad \Delta L = \pm(\Delta l_1 + \Delta l_2)$

$l_1 = 2.681 \quad l_2 = 2.16$

$\Delta l_1 = 0.001 \quad \Delta l_2 = 0.002$

$L = 2.681 + 2.16$ $\quad \Delta L = \pm(0.001 + 0.0002)$

$L = 4.841$ $\quad \Delta L = \pm 0.0012$

$L = 4.84$

Result should be rounded off to minimum no. of digit after decimal

So Sum of length of wire $= 4.84 \pm 0.0012$

(II) $L = l_1 - l_2 = 2.681 - 2.16$

$L = 0.521$

$L = 0.52$

The result should be rounded off to two ~~decimal~~ digits after decimal

Difference of two wire $= 0.52 \pm 0.0012$

Q→2 The current voltage relation of diode is given by $I = (e^{\frac{1000V}{T}} - 1)$ where the applied voltage V is in volts and temperature T is in kelvin. If a students makes an error measuring ± 0.01V while measuring the current of 5mA at 300K, What will be the error in the value of current in mA

Sol

$$I = e^{\frac{1000V}{T}} - 1$$

$$I + 1 = e^{\frac{1000V}{T}}$$ { Taking log on both side }

$$\log(I+1) = \frac{1000V}{T}$$

$$\frac{1}{(I+1)}\frac{dI}{dV} = \frac{1000}{T}\frac{dV}{dV}$$ { Differentiating both side w.r.t V }

$$dI = \frac{1000(I+I)dV}{T}$$

$$dI = \frac{1000}{300}(5+1)\,0.01 = 0.2\,mA$$

$$dI = 0.2\,mA$$

Q→3 The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary $d = 1.25 \times 10^{-2}$ m rise of water $h = 1.45 \times 10^{-2}$ m using $g = 9.80\,m/s^2$ and the simplified relation $T = \frac{rhg}{2}$. Find the possible error in surface tension

Ans:-

$$T = \frac{rhg}{2} \times 10^3 \frac{N}{m} = dhg \times 10^3 N/m$$

$d = 1.25 \times 10^{-2}$ m

$\Delta d = 0.01$

$h = 1.45 \times 10^{-2}$

$\Delta h = 0.01$

{ error in measured value is equal to least count of the instrument from which the value is being measured ~~which is equal to~~ }

$$\frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h}$$

$$\frac{\Delta T}{T} = \frac{0.01}{1.25 \times 10^{-2}} + \frac{0.01}{1.45 \times 10^{-2}}$$

$$\frac{\Delta T}{T} = 0.015$$

$$\frac{\Delta T}{T} \times 100 = 1.5\%$$

Que → Two resistors of resistances $R_1 = (100 \pm 3)\Omega$ and $R_2 = 200 \pm 4$ are connected (a) In series (b) In parallel. Find the equivalent resistance in parallel and series combination using relation $R_S = R_1 + R_2$ and $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$, $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

Sol (a) $R = R_1 + R_2 = 100 + 200 = 300\,\Omega$

$\Delta R = \Delta R_1 + \Delta R_2 = 3 + 4 = 7$

$R = (300 \pm 7)\,\Omega$

(b) $= \frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{100 \times 200}{(100 + 200)}$

$R' = \frac{20000}{300} = \frac{200}{3}\Omega$

differenting both side

$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

differentiating w.r.t x

$-\frac{2dR'}{R'^2 dx} = -\frac{2}{R_1^2}\frac{dR_1}{dx} - \frac{2dR_2}{R_2^2 dx}$

$\frac{dR'}{R'^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}$

$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$\Delta R' = R'^2\left[\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right]$

$\Delta R' = \left(\frac{200}{3}\right)^2\left[\frac{3}{100^2} + \frac{4}{(200)^2}\right]$

$\Delta R' = 1.8$

$\boxed{R' = \left(\frac{200}{3} \pm 1.8\right)\Omega}$

LEAST COUNT

It is the smallest value that a instrument can measure

Name of Instrument	Value of List Count
Meter Scale	0.1cm = 1mm
Vernier Scale	0.01cm = 0.1mm
Screw Gauge	0.001cm = 0.01mm

- Least count of any instrument help us in determining the maximum possible error that can occur during measurement

Name Of Instrument	Maximum possible error
Meter scale	0.1cm
Vernier scale	0.01cm
Screw Gauge	0.001 cm

Most accurate measurement will be done by instrument having Lowest Least Count

Q→ The dimension of an object is measured and found to be 2.81 cm. Name the instrument that has been used to measure. Also write the maximum possible error in measurement of the dimension.

Sol Name of Instrument = Vernier Scale
Maximum Error = 0.01 Cm

Q→ Out the following measurements which one is more accurate , 2.16 , 2.168 , 2.0000 , 2.1

Ans:- 2.0000 is the most accurate measurement because it has been measured by instrument which is having lowest value of Least Count.

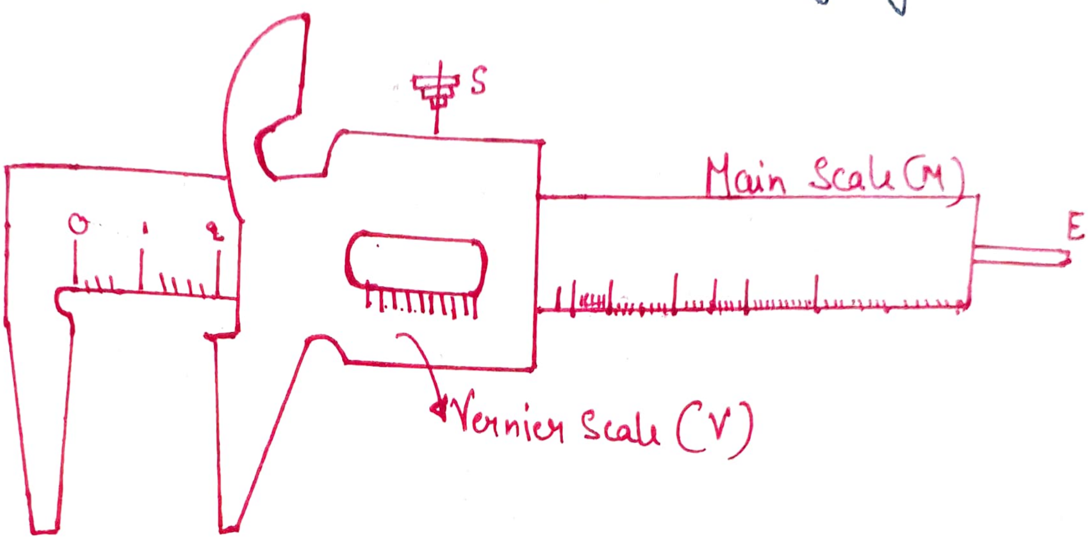
VERNIER CALLIPERS

Vernier calliper has two scale

(i) Main Scale ⇒ It consist of a steel metallic strip M, graduated in cm and mm. The least count of this scale is 0.1cm or 1mm.

(ii) Vernier Scale ⇒ Vernier scale V slides on ~~mett~~ metallic strip M. It can be fixed at any position by screw S.

Vernier scale has 10 division over a length of 9mm.



The division on vernier scale are smaller then on the main scale.

Least Count of vernier scale ⇒

$$n\ V.S.D = (n-1)\ MSD$$

V.S.D = Vernier Scale Division

MSD = Main Scale division

$$1\ V.S.D = \frac{n-1}{n}\ M.SD$$

$$\text{Least Count} = 1\ MSD - 1\ V.SD$$

$$= 1\ MSD - \frac{(n-1)}{n}\ MS.D = \frac{1}{n}\ MSD$$

$$\text{Least Count} = \frac{\text{Value of } 1\ M.SD}{\text{Total No. of division on verniers cale}}$$

In generaneral the value of 1 M.S.D is 0.1cm and total No. of division on vernier scale is 10. So

$$\text{Least Count} = \frac{0.1}{10} = \frac{1\,M.S.D}{\text{Total No. of division on vernier scale}}$$

$$\boxed{\text{Least Count} = 0.01\,cm}$$

Reading a Vernier scale =

$$\boxed{\text{Measured dimension} = N + n \times \text{Least Count}}$$

N = main scale Reading before on the left of zero of the Vernier scale.

n = no. of vernier scale division which just Coincide with main scale.

Screw Gauge

It is based upon the principle of micrometer screw.

It has two scale

① Linear Scale OR pitch Scale ⇒ It has least Count 0.1cm. ie the value of one division is equal to 0.1

② Circular Scale ⇒ It has 100 division.

When circular scale Complete one rotation then this scale moves one division of pitch scale.

$$\text{Least Count of the screw gauge} = \frac{\text{Value of one division on pitch scale}}{\text{Total No. of division on circular scale}} = \frac{0.1}{100}$$

$$= 0.001\,Cm$$

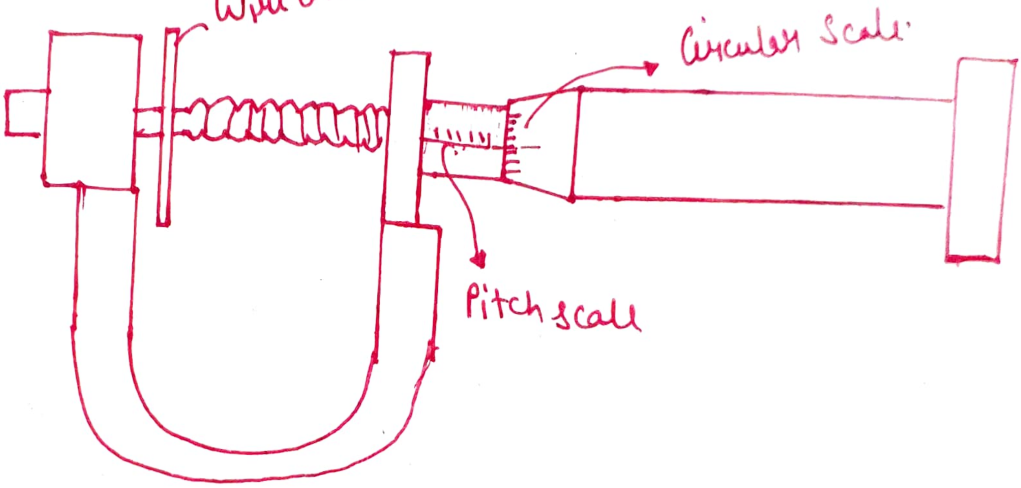
$$\boxed{\text{Least count} = 0.001\,Cm}$$

Measurement by Screw Gauge

Total Reading = N + n × Least Count

where N = No. of complete division on linear scale

n = No. of division of circular scale that coincide with reference line.



→ ~~A~~ In an experiment, the angles are required to be measured using an instrument. 29 division of the main scale exactly coincides with 30 division on the vernier scale. If the smallest division of the main scale ~~coincides~~ is half degree (= 0.5°). Then least count of instrument will be ?

Sol

$$\text{Least Count} = \frac{\text{Value of 1 Division of Main scale}}{\text{Total No. of division on vernier scale.}}$$

$$\text{Least Count} = \frac{0.5}{30} = \frac{5}{300} = \frac{1}{60}^{\circ} = 1 \text{ minute}$$

Least Count = 1 minute

Question Based Upon Instrument

Q→1 A spectrometer gives the following reading when used to measure the angle of prism

Main scale reading = 58.5 degree

Vernier scale reading = 09 division

Given that 1 division on main scale corresponds to 0.5 deg. Total division on the vernier scale is 30 and match with 29 division main scale. What is the angle of the prism from the above data.

Sol

$$\text{Angle of Prism} = \left(\text{Main scale Reading}\right) + \left(\text{Vernier Scale Reading} \times \text{Least Count}\right)$$

$$\text{Least Count} = \frac{\text{Value of One Main scale division}}{\text{Total No. of division on vernier scale}} = \frac{0.5}{30} = \frac{1}{60}$$

$$\text{Angle of prism} = (58.5) + \left(9 \times \frac{1}{60}\right) = 58.65^\circ$$

Angle of prism = 58.65°

DIMENSIONS

Dimension Of a physical Quantity ⇒ The dimensions of a physical quantity are the power to which the fundamental quantities must be raised to represent that quantity completely.

Example Velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{L}{T} = [M^0 L T^{-1}]$

Dimensions of fundamental Quantity

Mass	M
Length	L
Time	T
Temperature	K OR Θ
Electric Current	A
Amount of Substance	mol^{-1}
Luminous Intensity	Cd

Dimensions Of Some Important Constant

1) Gravitational Constant (G) = $F = \frac{G m_1 m_2}{r^2}$

Where m_1, m_2 are mass
F = Force
r = distance

$G = \frac{F r^2}{m_1 m_2} = \frac{(MLT^{-2})(L^2)}{M M}$

$G = [M^{-1} L^3 T^{-2}]$

2) Planck Constant (h) = $E = h\nu$

$h = \frac{E}{\nu} = \frac{\text{Energy}}{\text{frequency}} = \frac{ML^2T^{-2}}{T^{-1}}$

$h = ML^2T^{-1}$

(3) Boltzmann Constant ⇒

Energy $= \frac{1}{2} \times$ Boltzmann constant $\times$ temperature

$\downarrow$

$[ML^2T^{-2}] = K_B \times [\theta]$

$[ML^2T^{-2}\theta^{-1}] = K_B$

(4) Gas Constant = R ⇒

$PV = nRT$

P = Pressure
T = temperature
n = mole
V = Volume

$\frac{PV}{nR} = \,\,$ $R = \frac{PV}{nT}$

$R = \frac{[ML^{-1}T^{-2}][L^3]}{[mol][\theta]}$

$R = [ML^2T^{-2}\theta^{-1}mol^{-1}]$

(5) Coefficient of viscosity = η

$F = \eta A \frac{dv}{dx}$

F = force
A = Area
dv = change in velocity
dx = change in distance

$\eta = \frac{F\,dx}{A\,dv} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]}$

$\eta = [ML^{-1}T^{-1}]$

(6) Specific heat (S) =

$Q = Sm\Delta T$

Q = heat
m = mass
ΔT change in temp

$S = \frac{Q}{m\Delta T} = \frac{ML^2T^{-2}}{M\,\theta}$

$S = [L^2T^{-2}\theta^{-1}]$

(7) Thermal Conductivity (K) = $[MLT^{-3}\theta^{-1}]$

Dimensionless Constant

(1) $\sin\theta$, $\cos\theta$, $\tan\theta$ and other trignometric function are dimensionless.

(2) $\log x = [M^0L^0T^0]$

(3) Any Number like 2, 3 have no dimension = $[M^0L^0T^0]$

(4) $e^x = (2.71)^x =$ Dimensionless = $[M^0L^0T^0]$

(5) $\pi = [M^0L^0T^0]$

Principle Of Homogeneity of Dimension

According to this principle, a physical ~~quantity~~ equation will be dimensionally correct if the dimensions of all the terms occuring on both sides of the equation are same.

Ex = $S = ut + \frac{1}{2}at^2$

$[L] = [LT^{-1}T] + [LT^{-2}T^2]$

$L = L + L$

$L = 2L$

$\boxed{L = L}$

$L + L = L$

$T + T = T$

$L - L = L$

$T - T = T$

$\frac{L}{L} = [M^0L^0T^0]$

$L \times L = L^2$

Principle of homogeneity also mean that we can add only two similar quantity

Distance - Distance = possible

Distance - time = not possible

Application Of Dimensional Analysis

- Checking the correctness of formula
- Conversion of unit
- Derivation of formula

Checking the correctness of formula

For a correct equation the dimension of quantity on both side of R.H.S and L.H.S must be same

ex

$$v^2 - u^2 = 2as$$

$$[LT^{-1}]^2 - [LT^{-1}]^2 = [LT^{-2}]L$$

$$[L^2T^{-2}] = [L^2T^{-2}]$$

Since both side dimension are same so it is correct.

Q→1 Check the correctness of the formula

(I) $\int \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$ where x and a are distance

(II) $T = 2\pi\sqrt{\frac{l}{g}}$ l = length, g = acceleration due to gravity

Sol (I) $\int \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$

$$\frac{L}{\sqrt{L^2-L^2}} = \frac{1}{a}$$

$\left\{ \sin^{-1}\frac{a}{x} \right.$ is dimensionless

$$\frac{L}{L} = \frac{1}{L}$$

$$M^0L^0T^0 \neq M^0L^{-1}T^0$$

So the formula is incorrect

(II)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$[M^0L^0T] = \sqrt{\ldots}$$

$$[M^0L^0T] = \ldots$$

$$[M^0L^0T] = \ldots$$

both side a... equal so c...

CONVERSION OF UNIT

Physical Quantity $= \underbrace{n}_{\text{Multiplication factor}} \underbrace{U}_{\text{unit}}$

When we convert a physical quantity from one unit to other unit dimension of the physical quantity will remain same

$$n_1 U_1 = n_2 U_2$$

$$1\,\text{Km} = 1000\,\text{meter}$$

But Both Km and meter has same dimension.

★ Convert one newton into dyne

Sol

$$n_1 U_1 = n_2 U_2$$

$$1\,N = n_2\,\text{dyne} \quad \text{—— ★}$$

$$n_2 = \frac{n_1 U_1}{U_2}$$

$$n_2 = 1\,\frac{[M_1 L_1 T_1^{-2}]}{[M_2 L_2 T_2^{-2}]}$$

Dimension of force

$$n_2 = \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]^{-2}$$

$M_1 = Kg \quad M_2 = g$

$L_1 = m \quad L_2 = cm$

$T_1 = T_2 = sec$

$$n_2 = \left[\frac{Kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2}$$

$$n_2 = \left[\frac{1000g}{g}\right]\left[\frac{100cm}{cm}\right]\left[\frac{s}{s}\right]^{-2}$$

$$n_2 = 1000 \times 100$$

$$n_2 = 10^5$$

Putting the value of n_2 in ★ equation

$$1\,N = 10^5\,\text{dyne}$$

Q→3 The value of universal gravitational quantity constant is $6.67\times10^{-11} N/m^2$ in SI unit? Find the value of G in cgs?

Q→4 Convert one ~~newton~~ Joule into erg?

Derivation of formula

Q→1 The force acting on a body depends upon mass of the body and acceleration produced in the body. Then derive for the formula of acceleration?

Sol

$$F \propto m^a a^b$$

$$F = K m^a a^b \quad \text{---} (1) \quad \{\text{where K is dimensionless constant}\}$$

Writing dimension on both side.

$$[MLT^{-2}] = [M]^a [LT^{-2}]^b$$

$$MLT^{-2} = M^a L^b T^{-2b}$$

using principle of homogeneity powers of fundamental quantity on both side must be same.

$a=1$ $b=1$ $-2b = -2$

$b=1$

Putting the value of a and b is equation (1)

$$F = K m^1 a^1$$

$$F = Kma$$

if $K=1$ $F = ma$

Q→ (2) Kinetic energy (K.E) of a body ~~having~~ depends upon mass of the body (m) and velocity (v) of the body? Give derivation for energy of the body.

Sol

$K.E \propto m^a v^b$

$K.E = K\, m^a v^b$ — (1) where K = dimensionless Quantity.

Writing dimension on both side

$[ML^2T^{-2}] = [M]^a [LT^{-1}]^b$

$ML^2T^{-2} = M^a L^b T^{-b}$

Using principle of homogenity the power of fundamental quantity on both side must be equal

$a = 1$ $b = 2$ $-2 = -b$

$b = 2$

Putting the value of a and b in equation (1)

$K.E = K m v^2$

if $K = \frac{1}{2}$

$K.E = \frac{1}{2} m v^2$

Q→ (3) The force acting on a body moving in a circular path depends upon mass of the body (m) velocity of the body (v) and radius of circular path (r). Then derive for the formula of force.

Q→4 The energy E of an oscillating body in simple harmonic motion depends upon its mass (m) frequency (ν) and amplitude (a). Using the method of dimensional analysis derive the formula for E. 1

2

Q→5 If force 'F' velocity 'V' and time 'T' are fundamental quantities. Find the dimension of Energy 'E'.

Sol

$E \propto F^a V^b T^c$

$E = K F^a V^b T^a$ { where K is dimensionless quantity }

$[ML^2T^{-2}] = [MLT^{-2}]^a [LT^{-1}]^b [T]^c$ { writing dimension on both side }

$[ML^2T^{-2}] = M^a L^a T^{-2a} L^b T^{-b} T^c$

$ML^2T^{-2} = M^a L^{a+b} T^{-2a-b+c}$

using principle of homogeneity

$1 = a$

$a+b = 2$

$1+b = 2$

$b = 2-1$

$b = 1$

$-2a-b+c = -2$

$-2(1)-1+c = -2$

$c = -2+3$

$c = 1$

$E = KFVT$

Limitation Of dimensional Analysis

1 The method does not give any information about the dimensionless constant K.

2. ~~If~~ It fails when a physical quantity depends upon more than three quantities.

3 It ~~is~~ fails when physical quantity is sum or difference of two or more physical quantity.

4) It fails to derieve relationship which involves trignometric, logarithm and exponential functions.

5) Some times it is difficult to identify the factors on which physical quantity depends.