OSCILLATIONS →

Periodic Motion ⇒ Any motion that repeats itself over and over again at regular interval of time is called periodic motion

Example ⇒ (i) Motion of any planet around the sun
(ii) Motion of the hands of clock is periodic.

Oscillatory Motion ⇒ If a body moves back and forth repeatedly about its mean position periodically then its motion is said to ~~peri~~ be oscillatory motion.

Example ⇒ The oscillations of mass suspended from a spring.

"Every periodic motion is not oscillatory motion but every oscillatory motion is periodic motion provided that no energy loss take place during oscillatory motion."

Vibration ⇒ When oscillation is very fast and frequency is very high then it is known as vibration.

Periodic Function ⇒ Any function that repeats itself at regular interval of its argument is called a periodic function.

$f(\theta+T) = f(\theta)$

In the above case The function repeats itself after an interval of T

Example Of Periodic function

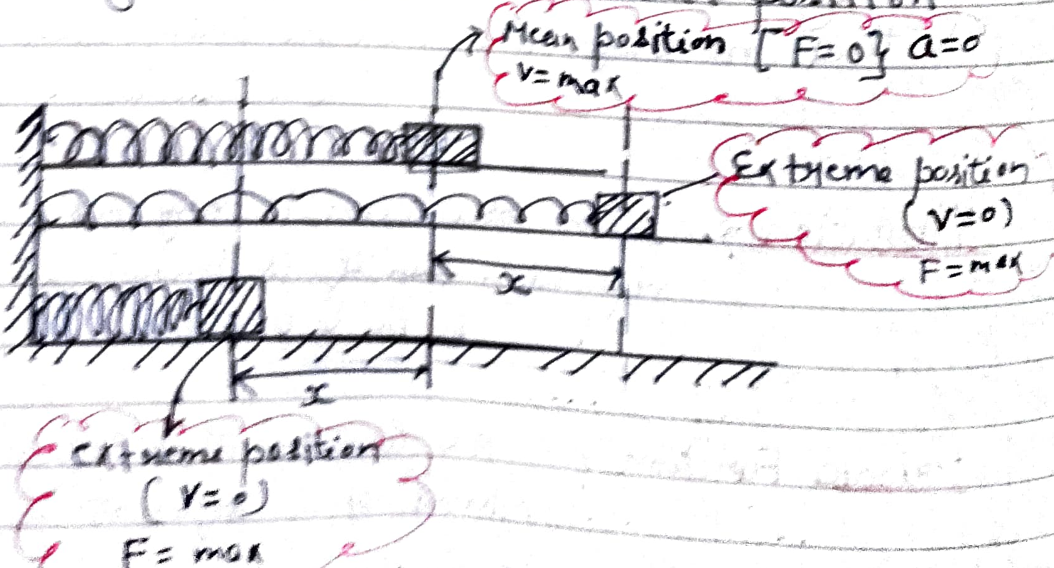
$\sin(\theta+2\pi) = \sin\theta$
$\cos(\theta+2\pi) = \cos\theta$

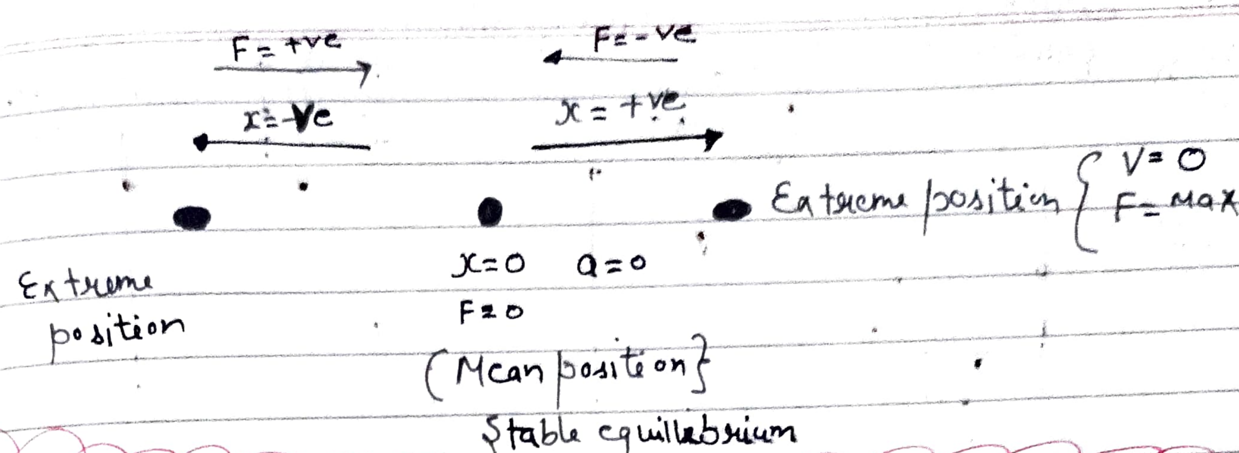
{In this we can see that value of sine and cos repeats with angle of 2π}

Harmonic Motion ⇒ Any oscillatory motion that can be represented in terms of functions sine or cos is called Harmonic motion.

Oscillatory Motion Equation. In terms Of Displacement

In oscillatory motion a force acts on the body which is always directed towards mean position.





☆ In oscillatory motion the force always acts opposite to displacement
☆ In oscillatory motion the displacement is always measured from mean position.

The restoring force in oscillatory motion is given by

$$F = -Kx^n$$

where $n = 1, 3, 5, 7 \ldots$ odd No.

from above equation we can see that

if
$x = -ve \Rightarrow F = +ve$
$x = +ve \Rightarrow F = -ve$

Simple Harmonic Motion [S.H.M]

Simplest oscillatory motion is called simple harmonic motion.

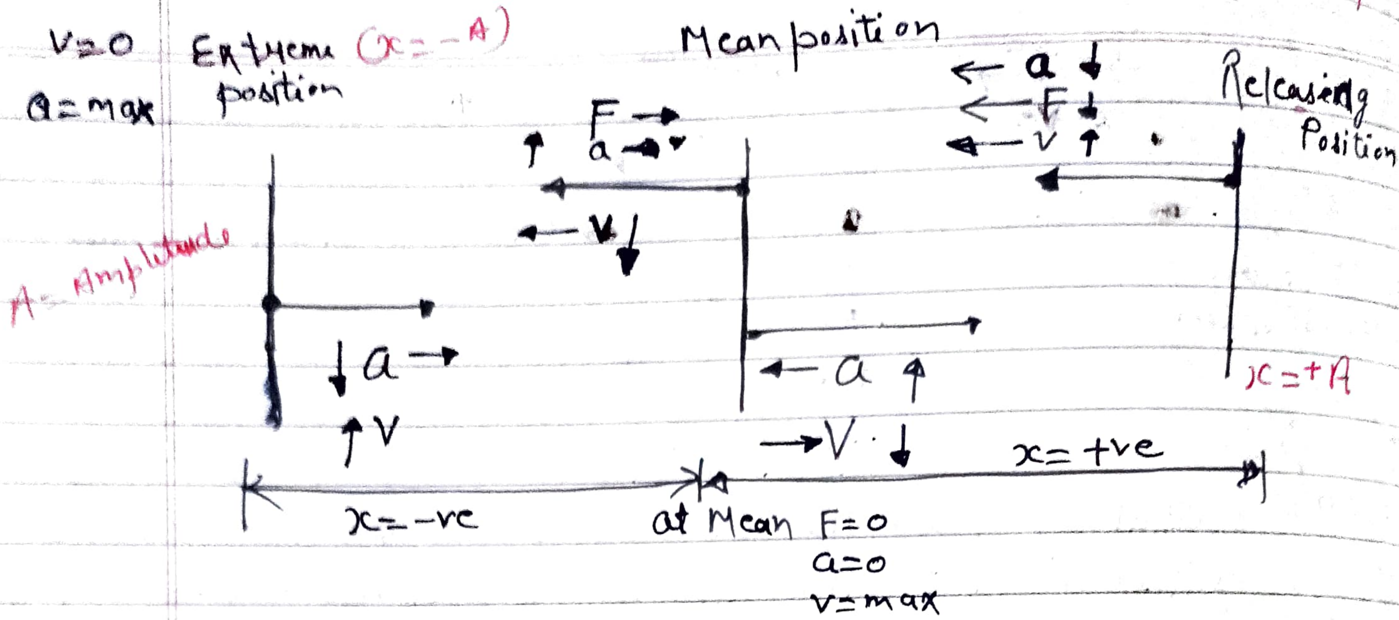
In S.H.M

$$F = -Kx^n$$

$$n = 1$$

$$F = -Kx$$

☆ In S.H.M the body always move in straight path.
☆ Motion of pendulum is said to be S.H.M only if its amplitude is small.



From above diagram we can see that acceleration is always directed towards mean position

$$F = -kx$$

$$ma = -kx$$

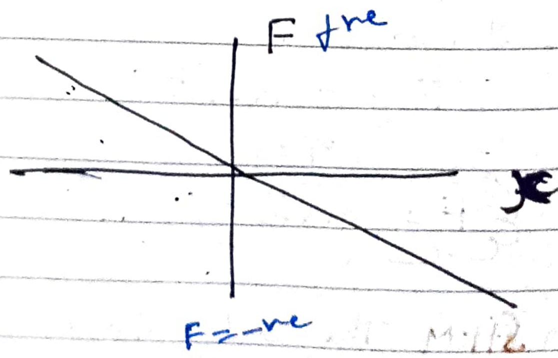
$$a = \frac{-kx}{m} \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\boxed{a = -\omega^2 x}$$

Graphical Relation b/w

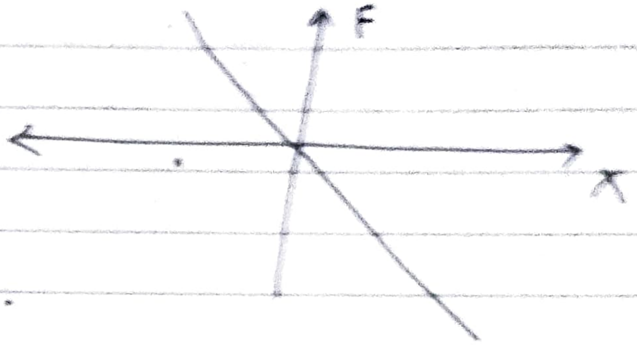
(1) F v/s x

$$F = -kx$$



(ii) a v/s x

$a = -\omega^2 x$



(iii) Velocity versus displacement

$a = -\omega^2 x$

$\left\{ a = \frac{dv}{dt} \right.$

$\frac{dv}{dt} = -\omega^2 x$

$\frac{dv}{\cancel{dt}} \times \frac{dx}{dx} = -\omega^2 x$

{ Multiplying and dividing side by dx

$\frac{v\,dv}{dx} = -\omega^2 x$

$\left\{ \frac{dx}{dt} = v \right.$

$v\,dv = -\omega^2 x\,dx$

integrating both side

$$\int_0^v v\,dv = -\omega^2 \int_{x=+A}^{x} x\,dx$$

{ we know that at $x = A$, $v = 0$ but at some value of displacement x velocity will be r

$$\left[\frac{v^2}{2}\right]_0^v = -\omega^2 \left[\frac{x^2}{2}\right]_A^x$$

$$\frac{v^2}{\cancel{2}} = \frac{-\omega^2}{\cancel{2}}\left[x^2 - A^2\right]$$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$\boxed{V = \pm \omega \sqrt{A^2 - x^2}}$$

from this equation we can see that there are two value of velocity for same displacement

for Graph purpose

$$V = \pm \omega \sqrt{A^2 - x^2}$$

$$V^2 = \omega^2 (A^2 - x^2)$$

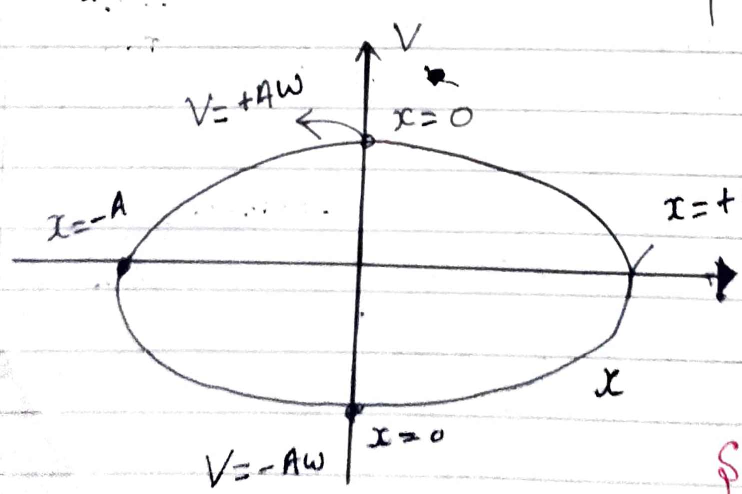
$$\frac{V^2}{\omega^2} + x^2 = A^2$$

$$\boxed{\frac{V^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1}$$

{ ~~Multi~~ Dividing whole equation by A^2

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

Since this equation relate to equation of ellipse



Summary

$F = -Kx$

$a = -\omega^2 x$

$V = \pm \omega \sqrt{A^2 - x^2}$

Displacement-time relationship in Simple Harmonic Motion

From velocity displacement relationship

$$V = \pm \omega \sqrt{A^2 - x^2}$$

$$\frac{dx}{dt} = \pm \omega \sqrt{A^2 - x^2} \qquad \left\{ V = \frac{dx}{dt} \right.$$

$$\frac{dx}{\sqrt{A^2 - x^2}} = \pm \omega dt \quad \text{———} \; \star$$

integrating both side

$$\int_0^x \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \pm \omega dt$$

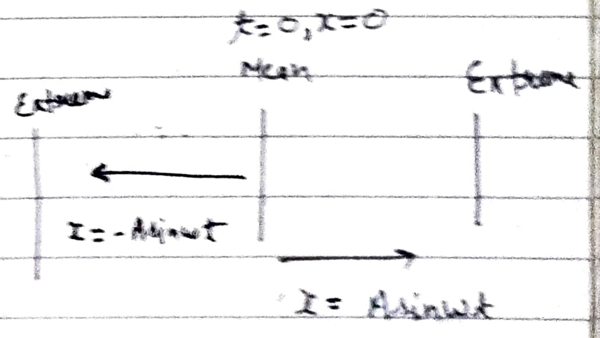
We are assuming that when $t = 0$, $x = 0$ that is particle is at mean position

time $= t$

distance from mean $= x$

$$\left[\sin^{-1} \frac{x}{A} \right]_0^x = \pm \omega t$$

$$\left[\sin^{-1} \frac{x}{A} - \sin^{-1} \frac{0}{A} \right] = \pm \omega t$$



$$\sin^{-1} \frac{x}{A} = \pm \omega t$$

$$\frac{x}{A} = \sin(\pm \omega t)$$

$$x = A\sin(\pm \omega t)$$

$$\boxed{\begin{array}{l} x = A\sin\omega t \\ x = -A\sin\omega t \end{array}}$$

These equation are only valid ~~the~~ when the particle is initially on mean position.

Case-II ⇒ In this case we are assuming that initially the particle is extreme

i.e. when time = 0

then $x = +A$

time = t

displ = x

to get the equation for the above condition

$$\int_A^x \frac{dx}{\sqrt{A^2 - x^2}} = \pm \omega \int_0^t dt$$

integrating equation

$$\left[\sin^{-1}\frac{x}{A}\right]_A^x = \pm \omega t$$

$$\sin^{-1}\frac{x}{A} - \sin^{-1}\frac{A}{A} = \pm \omega t$$

$$\sin^{-1}\frac{x}{A} - 90^\circ = \pm \omega t$$

$$\sin^{-1}\frac{x}{A} = 90 \pm \omega t$$

$$\frac{x}{A} = \sin(90 \pm \omega t)$$

$$x = A\sin(90 \pm \omega t)$$

$$x = A\sin(90 + \omega t) \qquad x = A\sin(90 - \omega t)$$

$$x = A\cos\omega t \qquad x = A\cos\omega t$$

Above equation is valid only when the particle is at extreme (+A) position

S	A
T	C

Case-III ⇒ In this case if we are assuming that the particle is at negative extreme initially. when $t = 0$ when time $= t$
$x = -A$ disp $= x$

$$\int_{-A}^{x} \frac{dx}{\sqrt{A^2 - x^2}} = \int_{0}^{t} \pm \omega\, dt$$

solving it we get

$$x = -A \cos \omega t$$

This equation is only valid when the particle is at negative extreme.

General $x-t$ equation for S.H.M

$$x = A \sin(\omega t + \phi)$$

OR

$$x = A \cos(\omega t + \phi)$$

ϕ → Initial phase

General (v-t), (a-t) equation for S.H.M

$$x = A \sin(\omega t + \phi)$$

$$\frac{dx}{dt} = A\omega \cos(\omega t + \phi)$$

$$V = A\omega \cos(\omega t + \phi)$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}\left[A\omega\cos(\omega t+\phi)\right]$$

$$a = -A\omega^2\sin(\omega t+\phi)$$

$$a = -\omega^2 x \qquad x = A\sin(\omega t+\phi)$$

Differential equation For S.H.M

$$a = -\omega^2 x$$

$$a = \frac{dv}{dt} \quad \text{since} \quad v = \frac{dx}{dt}$$

$$\text{then} \quad a = \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Graph b/w $x-t$, $v-t$ and $a-t$

$$x = A\sin\omega t$$

$$v = A\omega\cos\omega t$$

$$v = A\omega\sin\left(\frac{\pi}{2}+\omega t\right)$$

$$a = -A\omega^2\sin\omega t$$

In these equation we have assumed that the particle is at mean and initial phase is zero

In S.H.M ~~velocity~~ acceleration and displacement are out of phase by π

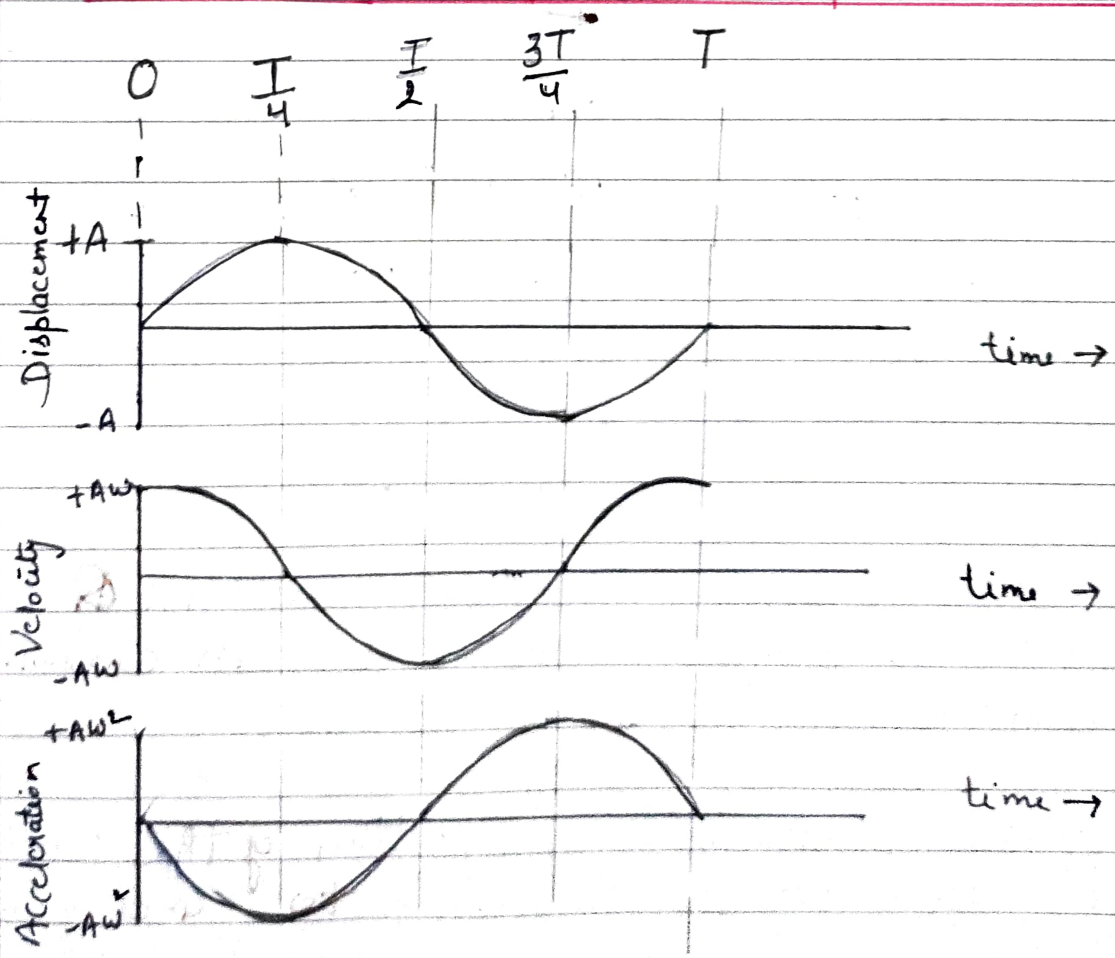
Now putting $\omega = \frac{2\pi}{T}$ in all the equation

$$x = A\sin\left(\frac{2\pi}{T}t\right)$$

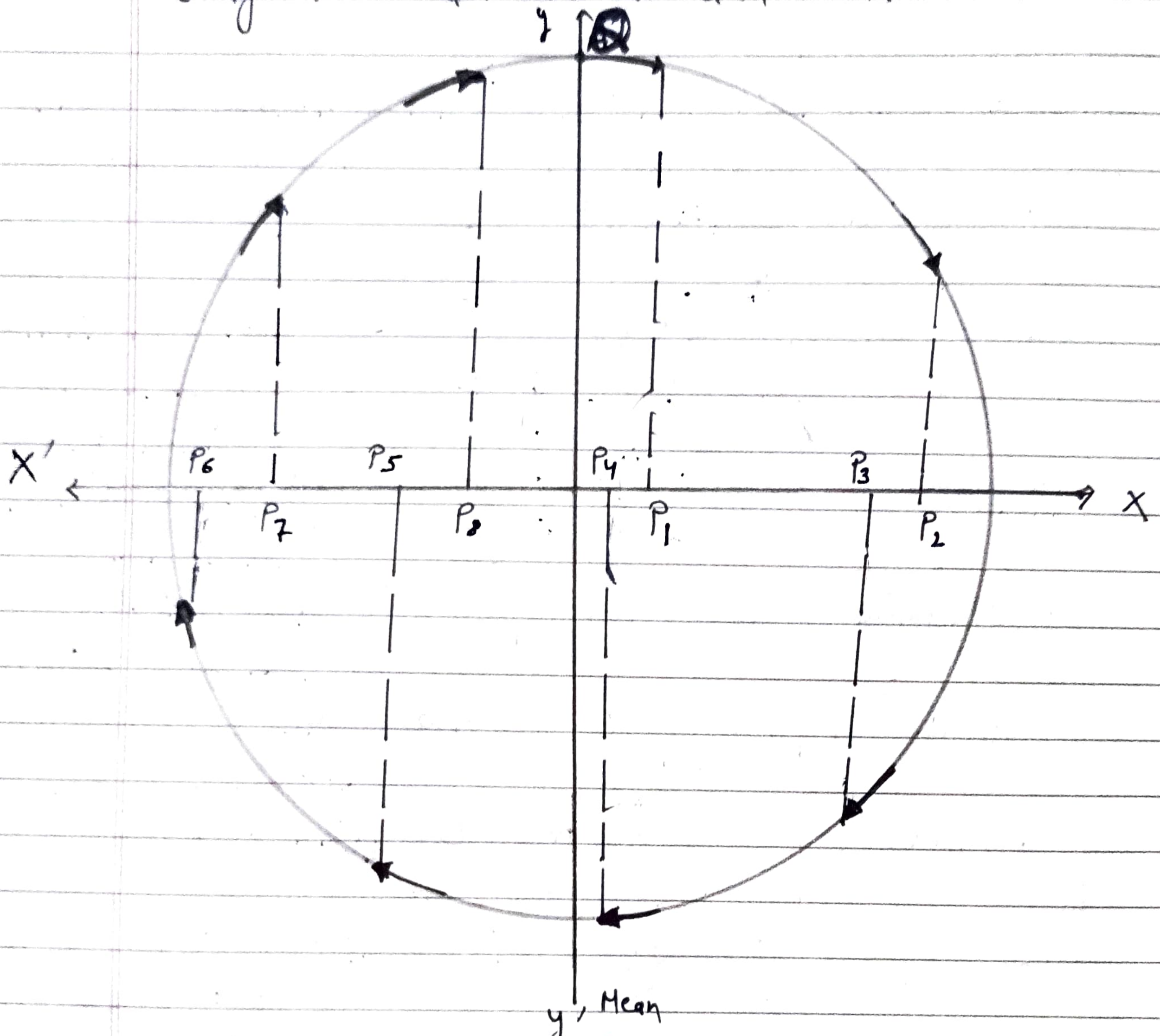
$$v = A\omega\sin\left(\frac{\pi}{2} + \frac{2\pi}{T}t\right)$$

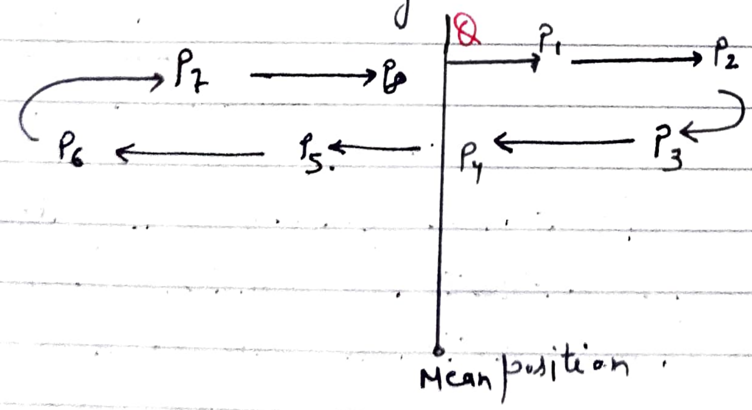
$$a = -A\omega^2\sin\left(\frac{2\pi}{T}t\right)$$

Time	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T
Displacement	0	$+A$	0	$-A$	0
Velocity	$A\omega$	0	$-A\omega$	0	$+A\omega$
Acceleration	0	$-\omega^2 A$	0	$\omega^2 A$	0



Uniform Circular Motion and S.H.M

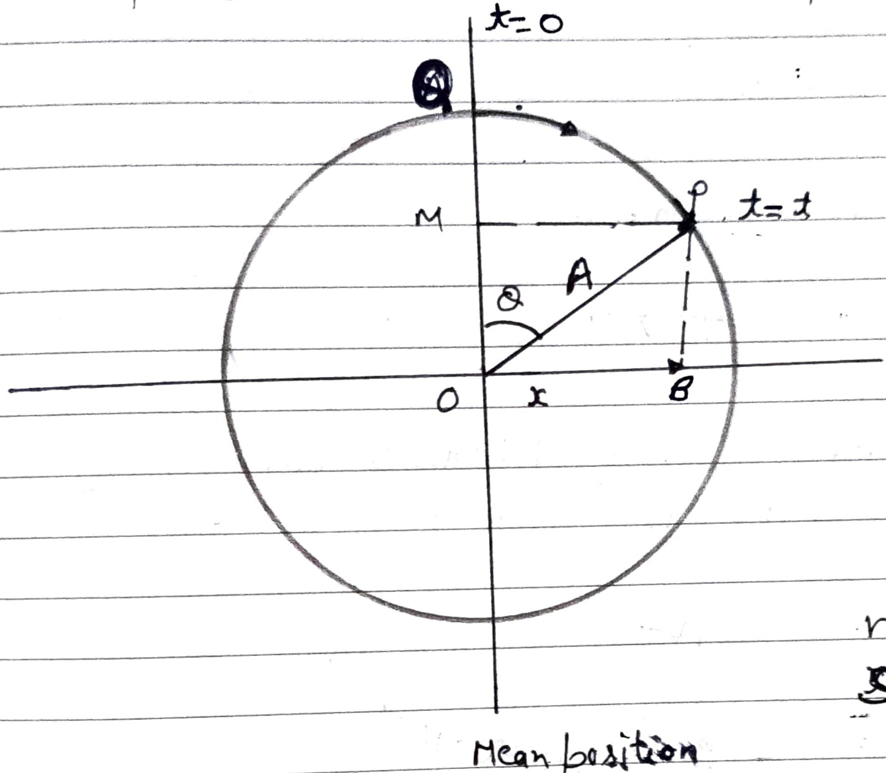




Let us consider a body initially at point Q and starts moving with uniform velocity in clockwise direction.

During its motion the projection of the particle on diameter XX' is moving to and fro motion.

That means during the uniform circular motion of a particle its projection on the diameter performs S.H.M.



From the diagram we can see that when time was equal to t then displacement of the particle from mean position was x.

Now from △OMP

$\sin\theta = \frac{MP}{OP}$

$MP = OP\sin\theta$

$OP = A$

$MP = OB = x$

$$\boxed{x = A\sin\theta}$$

So from this equation it is proved that the projection of particle doing uniform circular motion is doing S.H.M.

Kinetic Energy In S.H.M

At any instant the velocity of particle doing S.H.M is given by

$$V = \pm\omega\sqrt{A^2 - x^2}$$

squaring both side

$$V^2 = \omega^2(A^2 - x^2)$$

Now kinetic energy of particle having mass m doing S.H.M is given by

$$K.E = \frac{1}{2}mv^2$$

$$\boxed{K.E = \frac{1}{2}m\omega^2(A^2 - x^2)}$$

Case I   when: $x = 0$

$$K.E = K.E_{max} = \frac{1}{2}m\omega^2A^2$$

Case - II   when $x = \pm A$

$$K.E = K.E_{min} = 0$$

Potential Energy In S.H.M ⇒

When a particle is displaced from its mean position and Doing S.H.M then the restoring force is given by

$$F = -Kx$$

If we further displace the particle by dx then work done against restoring force is given by

$$dw = \vec{F}\cdot\vec{dx}$$

$F \xleftarrow{180^\circ} \xrightarrow{} dx$

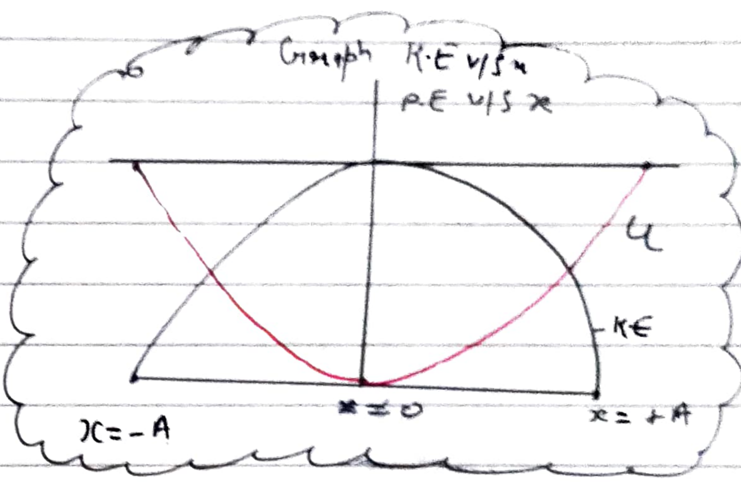
$$dw = F\,dx\cos 180^\circ$$
$$dw = -F\,dx$$
$$dw = -(-Kx)\,dx$$
$$dw = Kx\,dx$$

Total work done in moving the particle from mean position to displacement x

$$\int_0^w dw = K\int_0^x x\,dx$$

$$w = K\left[\frac{x^2}{2}\right]_0^x$$

$$w = \frac{1}{2}Kx^2$$



This work done against restoring force is stored in the form of potential energy

$$w = u$$

$$u = \frac{1}{2}Kx^2$$

$$\omega^2 = \frac{K}{m} \Rightarrow K = m\omega^2$$

$$\boxed{u = \frac{1}{2}m\omega^2x^2}$$

Case-1 When $x = 0$

$u = 0$

Case-2 $x = \pm A$

$u = \frac{1}{2}m\omega^2A^2$

Oscillations Due to Spring

(i) Horizontal Oscillations Of a body on spring

Consider a massless spring lying on a horizontal frictionless surface. Its one end is attached to a rigid support and other end is attached to a body of mass m.

When the mass is displaced from its mean position by x then body starts doing S.H.M. and the restoring force is given by.

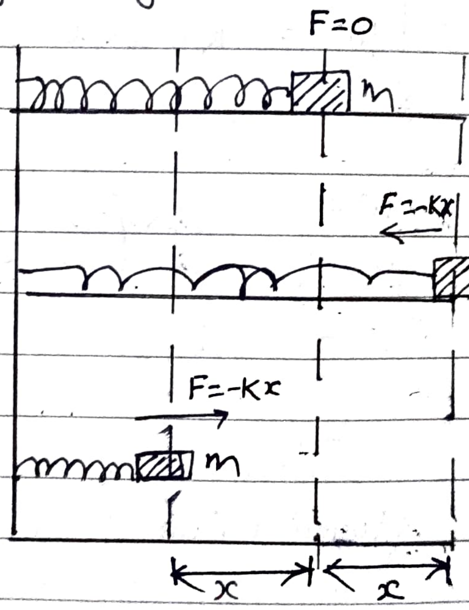
$$F = -Kx$$

$$ma = -Kx$$

$$a = -\frac{K}{m}x$$

$$a = -\omega^2 x \qquad \omega = \sqrt{\frac{K}{m}}$$

$$\boxed{a \propto -x}$$



Since acceleration of the body comes out to be directly proportional to negative of displacement so it confirms S.H.M.

Time period

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{K}{m}}} \Rightarrow T = 2\pi\sqrt{\frac{m}{K}}$$

Frequency

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

(II) ~~Vertical Oscillations of a body on a Spring~~ ⇒

If a spring is ~~...~~ suspended from a rigid support vertically and a body of mass m is attached to its lower end, the spring gets stretched to a distance d due to weight

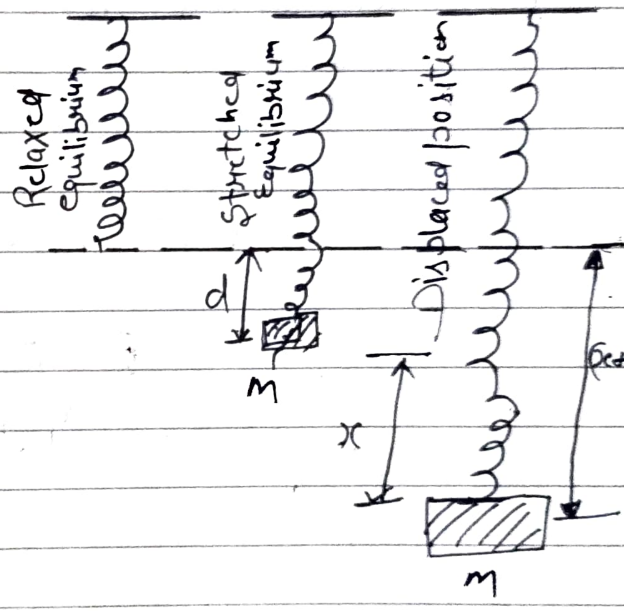
Because of elasticity of the spring a restoring force begin to acts in upward direction

$F_{res} = -Kd$

In equillibrium condition

$mg + (-Kd) = 0$

$mg = Kd$



Now suppose the spring is pulled by displacement x ~~then~~ from its equillibrium position and then released it begin to oscillate up and down. Net upward restoring force is given by

$F = mg - K(d+x)$

$F = mg - Kd - Kx$

Since $mg = Kd$ so

$F = -Kx$

$ma = -Kx$

$a = \frac{-K}{m}x$

$a = -\omega^2 x$ $\left\{ \omega = \sqrt{\frac{K}{m}} \right.$

$\boxed{a \propto -x}$

Since acceleration is directly proportional to negative of disp so body is doing S.H.M

Time period	Frequency
$T = \frac{2\pi}{\omega}$	$\nu = \frac{1}{T}$
$T = 2\pi\sqrt{\frac{m}{K}}$	$\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

Oscillation Of loaded Spring Combination

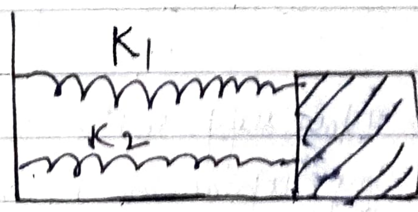
(i) Spring Connected in parallel →

Let two springs having spring constant K_1 and K_2 are connected in parallel.

Let y be the extension produced in each spring. Then restoring force produced in the two spring is

$$F_1 = -K_1 y \quad \text{and} \quad F_2 = -K_2 y$$

Total restoring force



$$F = F_1 + F_2$$

$$F = -y(K_1 + K_2) \quad \text{--- (1)}$$

Let K_p be the force constant for parallel combination then

$$F = -K_p y \quad \text{--- (2)}$$

from (1) and (2)

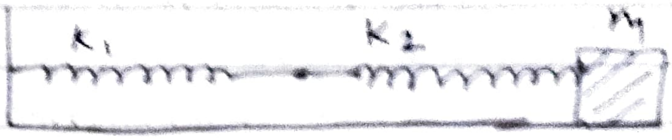
$$K_p = K_1 + K_2$$

Frequency of vibration of parallel combination

$$V_p = \frac{1}{2\pi}\sqrt{\frac{K_p}{m}} = \frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$$

$$\boxed{V_p = \frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}}$$

II Spring Connected in Series



Let x_1 and x_2 be the extension produced in spring having spring constant K_1 and K_2

The restoring force acting in the two spring is given by

$$F = -K_1 x_1 = -K_2 x_2$$ {Restoring force is same in both spring

$$x_1 = \frac{-F}{K_1} \qquad x_2 = -\frac{F}{K_2}$$

Total extension $x = x_1 + x_2$

$$x = \frac{-F}{K_1} + \left(\frac{-F}{K_2}\right)$$

$$x = -F\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$F = \left(\frac{-K_1 K_2}{K_1 + K_2}\right) x \quad \text{—— (1)}$$

Let K_s be the spring constant in series combination

$$F = -K_s x \quad \text{—— (2)}$$

from (1) and (2)

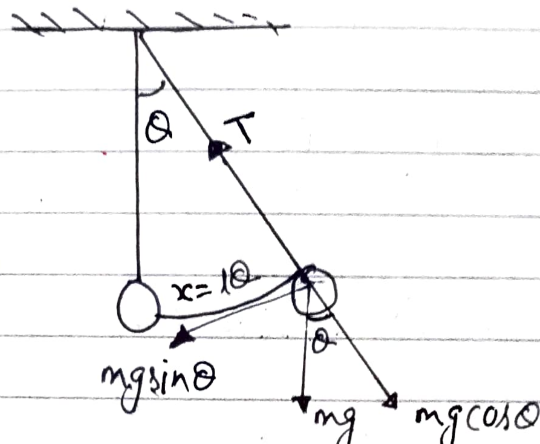
$$K_s = \frac{K_1 K_2}{K_1 + K_2}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K_s}{m}} = \nu = \frac{1}{2\pi}\sqrt{\frac{K_1 K_2}{(K_1+K_2)m}}$$

Simple Pendulum

An ideal simple pendulum consist of a point-mass suspended by a flexible, inelastic and weightless string from a rigid support of infinite mass.

Derivation for time period of Pendulum ⇒



Let pendulum be displaced by angular displacement θ from mean position. Then restoring force acting on the bob of pendulum is

$F = -mg\sin\theta$

for S.H.M the angular displacement is very small so
$\sin\theta \sim \theta$

$F = -mg\theta$

$F = -mg\frac{x}{l}$ $\left\{ \theta = \frac{x}{l} \right.$

$ma = -mg\frac{x}{l}$

$a = -\frac{g}{l}x$

$a = -\omega^2 x$ $\left\{ \omega = \sqrt{\frac{g}{l}} \right.$

Since acceleration is proportional to negative of disp so pendulum is doing S.H.M

Time period of pendulum

$T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{\sqrt{g/l}}$

$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

Frequency of Pendulum

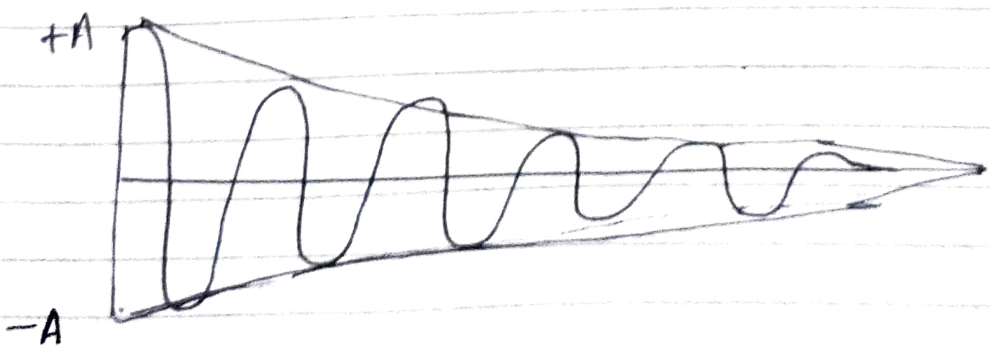
$\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$

Types Of Oscillation

(I) Free Oscillations ⇒ If a body, capable of oscillation, is slightly displaced from its ~~mean~~ position of equilibrium and left to itself, it starts oscillating with a frequency of its own. Such oscillations are called free oscillations.

$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ ν_0 = frequency of free oscillation

(ii) Damped Oscillation ⇒ The oscillations in which the amplitude decreases gradually with passage of time are called damped oscillation.



Differential Equation for damped Oscillators and its solution:

In real oscillator, the damping force is proportional to the velocity of oscillator

$$F_d = -bV$$

b = damping constant

Total restoring force

$$F = -Kx - bV$$

$$m\frac{d^2x}{dt^2} = -Kx - b\frac{dx}{dt}$$

$V = \frac{dx}{dt}$

$$\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + Kx = 0}$$

→ Differential equation for damped oscillations

Solution of above equation is

$$x(t) = ae^{-bt/2m}\cos(\omega' t + \phi)$$

$$\boxed{a' = ae^{-bt/2m}}$$

$$\omega' = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$$

$$\boxed{T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}}}$$

(iii) **Forced Oscillation** ⇒ When a body oscillates under the influence of an external periodic force, not with its own natural frequency but with the frequency of external periodic force, its oscillations are said to be forced oscillations.

Resonant Oscillation ⇒ It is a case of forced oscillation in which frequency of external force matches with natural frequency of oscillation of body.

In this case amplitude of oscillations become very large.

Coupled Oscillation ⇒ A system of two or more oscillators linked together in such a way that there is mutual exchange of energy b/w them is called coupled oscillator