# WAVES

**Wave** ⇒ Any disturbance which can carry energy or momentum or both from one point to another point is called wave.

**Example** ⇒ Sound wave, Light wave, Matter wave.

## CLASSIFICATION OF WAVE

1 **Based On Medium** ⇒

| Mechanical Wave | Electromagnetic Wave |
|---|---|
| Require material medium for its propagation. | It does not require any material medium for its propagation. |
| Example ⇒ Sound Wave. | Example ⇒ Electromagnetic wave. |

**Property of Material particle**

**Elasticity** ⇒ Due to which it can transfer energy

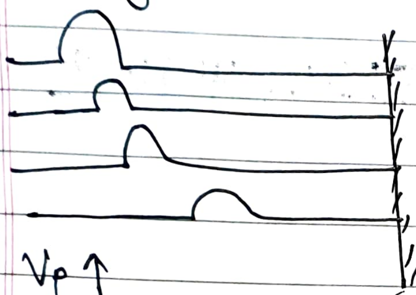
**Inertia** ⇒ Due to which particle return back to its original position.

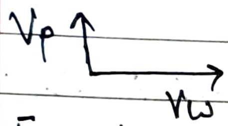
2 **Based On Direction of Motion of Medium particle.**

| Transverse Wave | Longitudinal Wave. |
|---|---|
| In this wave particle of the | In This wave particle of |

medium oscillate perpendicular to the direction of propagation of wave.

During propagation of transverse wave crest and trough are formed.

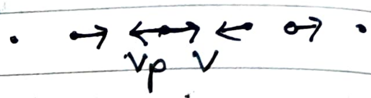





Example ⇒ light wave

the medium oscillate in the direction of propagation of wave.

In this wave propagation Continuous compression and rarefaction will take place.




$V_p \parallel V_w$

**Example** Sound wave

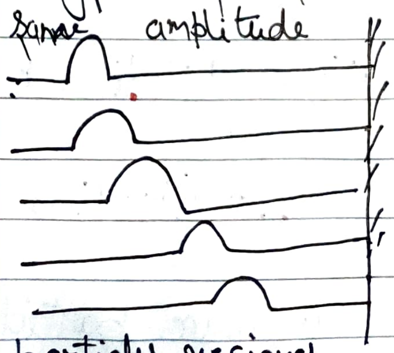
$V_p$ = Velocity of particle
$V_w$ = Velocity of wave.

## 3 Classification Based On Energy

**Progressive Wave**

Energy propagetes from one point to another.

All particles of the medium are oscillating in diffrent phase. with same amplitude

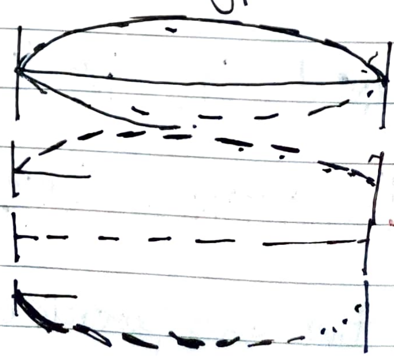


particles recieves energy after some time gap

**Stationary Wave**

Energy do not propagates from one point to other.

All the particle of the medium oscillates in same phase but with different amplitude



# Some Defination Related to wave Motion.

1 **Amplitude (A)** ⇒ It is the maximum displacement of the particle of about the medium about their mean position.

2 **Time Period (T)** ⇒ It is the time taken by the particle of the medium to comple one cycle about their mean position.

3 **Frequency ($\nu$)** ⇒ It is number of wave produced per unit time.

$$\boxed{\nu = \frac{1}{T}}$$

4 **Angular Frequency** ⇒ The rate of change of phase with time is called angular frequency of the wave.

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$

5 **Wavelength ($\lambda$)** ⇒ It is the distance b/w two consecutive crest or trough of the medium.

6 **Wave Number** ⇒ It is defined as the number of wave present per unit length of the medium.

$$\boxed{\bar{\nu} = \frac{1}{\lambda}}$$

7 **Angular Wave Constant** ⇒ It is defined as phase change per unit path difference.

$$\boxed{k = \frac{2\pi}{\lambda}}$$

8 **Wave Velocity** ⇒ The distance covered by a wave per unit time in its direction of propagation is called its wave velocity or phase velocity.

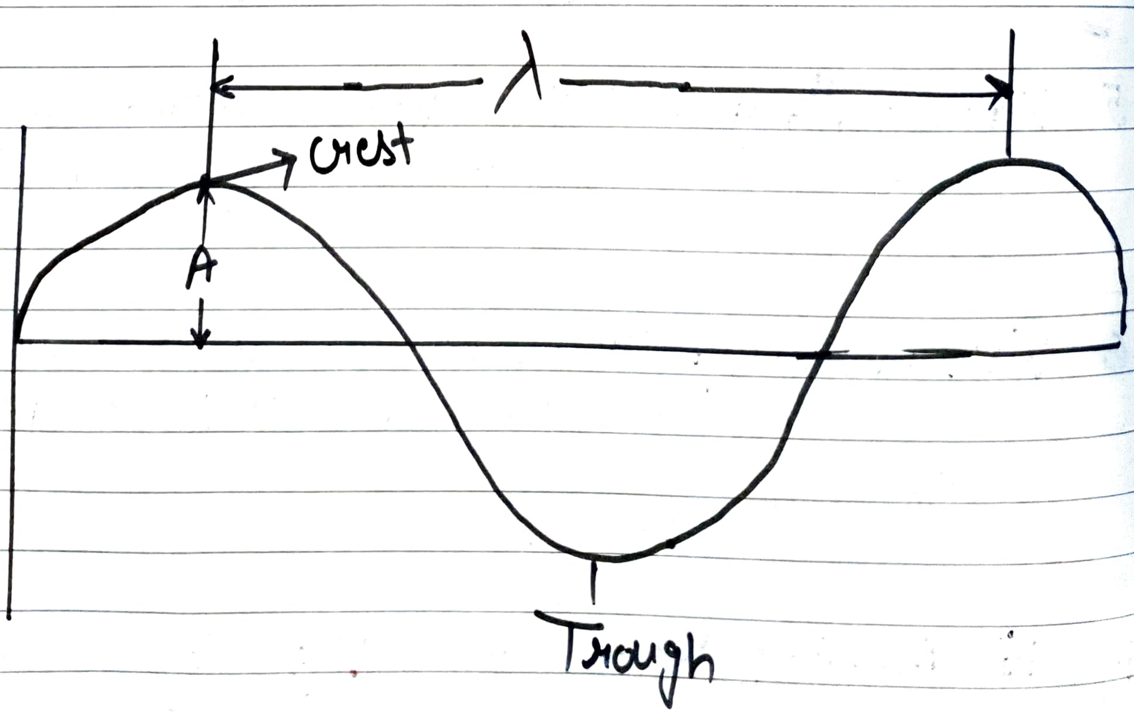
Relation b/w wave velocity, frequency and wave length

$$\text{Wave Velocity} = \frac{\text{Distance}}{\text{time}}$$

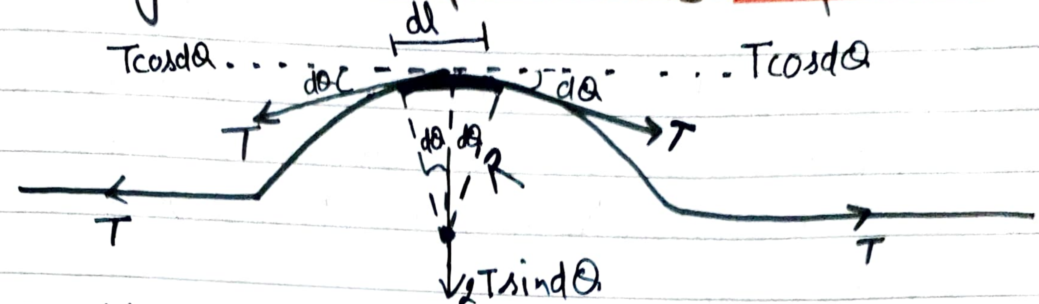
$$v = \frac{\lambda}{T}$$

$$\boxed{\nu = \frac{1}{T}}$$

$$\boxed{v = \nu\lambda}$$



# Speed of Transverse Waves In stretched String




Let us consider a tension T is acting in the string in which transverse wave is propagating. At any moment suppose dl length is at crest. Now assume that dl part of the string is in circular motion of radius R. Let dM be the mass of the string dL. Let m = Mass per unit length.

Now We know That in circular motion

Centripetal force = Centrifugal force

$$2T\sin d\theta = \frac{dMv^2}{R}$$

Since angle is small

$$2T\,d\theta = \frac{dMv^2}{R} \qquad \{dM = m\,dl\}$$

$$2T\,d\theta = \frac{m\,dl\,v^2}{R}$$

$$2\,d\theta = \frac{dl}{R} \qquad \left\{\theta = \frac{l}{r}\right\}$$

$$T\frac{\cancel{dl}}{\cancel{R}} = \frac{m\cancel{dl}\,v^2}{\cancel{R}}$$

$$T = mv^2$$

$$\frac{T}{m} = v^2$$

$$\boxed{v = \sqrt{\frac{T}{M}}}$$

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## Speed Of transverse Wave in solid.

The speed of transverse wave through solid is determined by (i) Elasticity of shape or modulus of rigidity $\eta$ · (II) Mass per unit volume. $\rho$

$$\boxed{V = \sqrt{\frac{\eta}{\rho}}}$$

## Speed Of A longitudinal Wave ⇒

(a) Speed of a longitudinal wave in liquid or gas ⇒

Speed of longitudinal wave through a fluid is determined by two factors:

(i) Bulk ~~elastic~~ modulus of elasticity $(K)$ of the fluid
(ii) Density of the fluid

$$V = C\sqrt{\frac{K}{\rho}}$$

$C = 1$

$$V = \sqrt{\frac{K}{\rho}}$$

(b) Speed of longitudinal wave in solid

$$V = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}}$$

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(c) Speed of longitudinal wave in solid Rod :

$$\boxed{V = \sqrt{\frac{Y}{\rho}}}$$

# SPEED OF SOUND

(i) Newton's Formula for the Speed of Sound in Gas ⇒ According to Newton the propagation of sound in air is an isothermal process.

He argued that small amount of heat is generated ~~com~~ in compressed region is rapidly conducted to the surrounding rarefactions where slight cooling is produced.

Thus the temperature of the gas remains constant.

Then according to Newton speed of sound in air is given by

$$V = \sqrt{\frac{K_{iso}}{\rho}}$$

for an isothermal process

$PV =$ constant

differentiating both side

$$PdV + VdP = 0$$

$$PdV = -VdP$$

$$P = \frac{-VdP}{dV} = -\frac{dP}{\frac{dV}{V}}$$

$P = \frac{dP}{\frac{dV}{V}} = \frac{\text{Volume stress}}{\text{Volume strain}}$

$P = \text{K iso}$

$$\boxed{V = \sqrt{\frac{P}{\rho}}}$$

$P = 1.013 \times 10^5$ Pa

$\rho = 1.293$ Kgm$^{-3}$

$$V = \sqrt{\frac{1.013 \times 10^5}{1.293}} = 280 ms^{-}$$

$$\boxed{V = 280 ms^{-1}}$$

(ii) Laplace's Correction ⇒ Laplace pointed out that sound travels through air under adiabatic condition not isothermal condition as suggested by Newton. This is because of following reason →

(a) ~~A sound~~ As sound travels, through a gas temprature rises in the region of compression and falls in the region of rarefactions.

(b) Gas is poor conductor

(c) The compression and rarefaction forms so rapidly that heat generated in the compression region does not get time to pass into ~~comp~~ regions of rarefaction.

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so According to Laplace the speed of sound in air is given by

$$V = \sqrt{\frac{K_{adia}}{\rho}} \quad — (1)$$

$K$ = kappa

adia ⇒ adiabatic

$$PV^{\gamma} = K$$ { for adiabatic process

Differentiating both side

$$P\gamma \cancel{V}^{\gamma-1} + V^{\gamma}dP = 0$$

$$P(\gamma \cancel{V}^{\gamma-1})dV = -V^{\gamma}dP$$

$$\gamma P = \frac{-V^{\gamma}dP}{V^{\gamma-1}dV}$$

$$\gamma P = -\frac{dP}{\frac{dV}{V}} = K_{adia}$$

$$\left\{ \frac{V^{\gamma-1}}{V^{\gamma}} = V^{-1} \right.$$

$$\boxed{\gamma P = K_{adia}} \quad — (2)$$

from (1) and (2)

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

$$\gamma_{air} = \frac{7}{5}$$

$$P = 1.013 \times 10^{5}\ Pa$$

$$\rho = 1.293\ Kgm^{-3}$$

so

$$V = 331.3\ ms^{-1}$$

# Factors Affecting Speed Of Sound in Gas

(i) **Effect Of pressure** ⇒ The speed of sound in gas in independent of pressure.

(ii) **Effect of Density** ⇒

$$V_1 = \sqrt{\frac{\gamma P}{\rho_1}} \qquad V_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}} \Rightarrow \boxed{V \propto \sqrt{\frac{1}{\rho}}}$$

so velocity decreases with increase in density

(iii) **Effect Of humidity** ⇒ Speed of sound is more in moist air than dry air.

Because density of moist air is less than the density of dry air.

(iv) **Effect of Temprature** ⇒

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{—— (1)}$$

$$PV = RT$$

$$P = \frac{RT}{V}$$

$$P = \frac{RT\rho}{m} \qquad \left\{ V = \frac{m}{\rho} \right\}$$

$$\boxed{\frac{P}{\rho} = \frac{RT}{m}} \quad \text{—— (2)}$$

from equation (1) and (2)

$$V = \sqrt{\frac{\gamma RT}{m}}$$

$$\boxed{V \propto \sqrt{T}}$$

So speed of sound increases with increase in temp.

Effect Of Wind ⇒ If the wind is blowing in the direction found then wind speed will ~~d~~ increase. otherwise it will decrease if wind is blowing against the found.

Amplitude And Frequency ⇒ Generally speed of sound is independent to the frequency and amplitude of sound.

The displacement of particle can only be specified if we know the time and distance

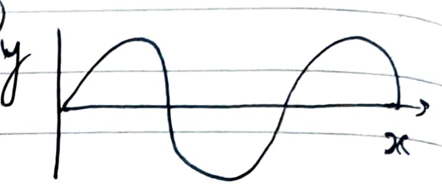
## Equation Of Wave ⇒

Any function will represent equation of wave if it satisfy the following equation

$$\frac{\partial^2 y}{\partial x^2} = K \frac{\partial^2 y}{\partial t^2} \quad — (1)$$

$K \neq 0$

K = square of wave velocity




$$\boxed{y = f(ax + bt)} \quad — (2)$$

y = Displacement of particle
x = Distance

if equation (2) satisfy equation (1) then equation (2) is a equation of wave.

So the wave velocity will be given by

$$\text{Wave Velocity} = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \frac{b}{a}$$

$$\boxed{V = \frac{b}{a}}$$

Condition for wave motion: y should be defined for all values of x and t

if: V = +ve then wave is travelling in negative x axis

if V = −ve then wave is travelling in positive x axis.

## Equation of plane progressive Wave ⇒

Any wave equation That can be represented in the form of function of sine or cos is ~~equ~~ called equation of plane progressive wave.

Q→ Check wether the following is a equation of wave

(i) $y = A\sin\omega t$ (ii) $y = A\sin(\omega t - kx)$ ✓

× (iii) $y = \log(x+2t)$ × not defined for all values

✓ $y = \frac{2}{(3x+2t)^2+4}$ ✓

# General Equation of Plane progressive harmonic Wave

$e^{\infty} = \infty$

$e^{0} = 1$

**Progressive Wave ⇒** A wave that travels from one point to another point of the medium is called progressive wave.

It can be either transverse or longitudinal.

**Plane Progressive Harmonic Wave ⇒** If during the propagation of a wave through a medium, the particle of the medium vibrate simple harmonically about their mean position then the wave is said to be plane progressive ~~wave~~ harmonic wave.

## Equation of Plane progressive wave

$$\boxed{y = A\sin(kx \pm \omega t + \phi)}$$

$+\omega t$ = wave is travelling in negative $x$

$-\omega t$ = wave is travelling in positive $X$

$$W = \frac{2\pi}{T}$$

$$\lambda = \frac{2\pi}{K} \Rightarrow K = \frac{2\pi}{\lambda}$$

**Wave Number =** $K$ = No wave per unit length is known as wave number.

Now Wave Equation can also be written as

$$y = A\sin\left(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{T}t + \phi\right)$$

$$y = A\sin\left\{\frac{2\pi}{\lambda}\left(x \pm \frac{\lambda}{T}t + \phi\right)\right\}$$

$$y = A\sin\left\{\frac{2\pi}{\lambda}\left(x \pm \nu\lambda t + \phi\right)\right\} \quad \left\{\frac{1}{T} = \nu\right.$$

$$\boxed{y = A\sin\left\{\frac{2\pi}{\lambda}\left(x \pm vt + \phi\right)\right\}}$$

[Speed = frequency × wavelength]
$v = \nu\lambda$

Q→(i) If $y = \frac{10}{5 + (x-2t)^2}$ Find (i) wave velocity (ii) Amplitude

Sol

$$v = \frac{\text{coefficient of } t}{\text{coefficient of } x} = \frac{-2}{1} = 2 \text{ m/s}$$

$$A = y_{max} = \frac{10}{5 + \underbrace{(x-2t)}_{\text{Min}=0}} = \frac{10}{5} = 2 \text{ m}$$

(ii) $y = \frac{1}{1+x^2}$ at $t = 0$ sec $\qquad y = \frac{1}{1+(x-1)^2}$ at $t = 2$ sec

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**Phase of a wave ⇒** The phase of harmonic wave is a quantity that gives complete information of wave at any time and at any position.

$$y(x,t) = A\sin(\underbrace{\omega t - kx + \phi_0})$$

$$\text{Phase} \leftarrow \phi = \omega t - kx + \phi_0 \rightarrow \text{Initial phase.}$$

**Phase Change With time ⇒**

$$\phi = \omega t - kx + \phi_0$$

$$\frac{d\phi}{dt} = \omega$$

$$\boxed{d\phi = \omega dt} \qquad \omega = \frac{2\pi}{T}$$

**Phase change with position**

$$\phi = \omega t - kx + \phi_0$$

$$\frac{d\phi}{dx} = -k$$

$$d\phi = -k\,dx \qquad k = \frac{2\pi}{\lambda}$$

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## Particle Velocity ⇒

It is the velocity with which the particle ~~ofsil~~ of the medium vibrate about their mean positions.

The particle velocity can be be determined by differentiating displacement equation with time.

if $y = A\sin(\omega t - kx + \phi)$

$$V = \frac{dy}{dt} = A\omega\cos(\omega t - kx + \phi)$$

$$\boxed{V = A\omega\cos(\omega t - kx + \phi)}$$

Since particles of medium are doing S.H.M so every formula of S.H.M is valid

$$V = \omega A\sin\left[(\omega t - kx + \phi) + \frac{\pi}{2}\right]$$

★★ Particle velocity is ahead of displacement by $\frac{\pi}{2}$

## Acceleration of particle ⇒

$$V = A\omega\cos(\omega t - kx + \phi)$$

$$\frac{dv}{dt} = a = -\omega^2 A\sin(\omega t - kx + \phi)$$

$$\boxed{a = -A\omega^2\sin(\omega t - kx + \phi)}$$

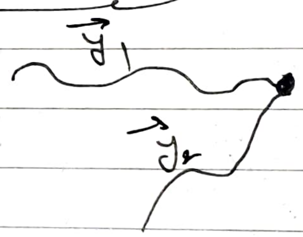
$$a = A\omega^2\sin(\omega t - kx + \phi + \pi)$$

★★ Acceleration is ahead of displacement by $\pi$

# Principle Of Superposition Of Wave ⇒

The principle of superposition of wave state that when a number of wave travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the ~~algebraic~~ Vector sum of the displacement due to individual wave..

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \cdots \vec{y}_n$$




## Superposition Of Wave leads to the following effect.

I. Interference ⇒ When two wave of same frequency moving with same speed in the same direction in a medium superimpose on each other then they give rise to ~~superposi~~ interference.

II. When two wave of same frequency travelling in opposite direction with same speed when they superimpose they produce stationary wave.

III. Beats ⇒ When two wave of slightly different frequency moving with same speed in the same direction in a medium superimpose on each other give rise to Beats.

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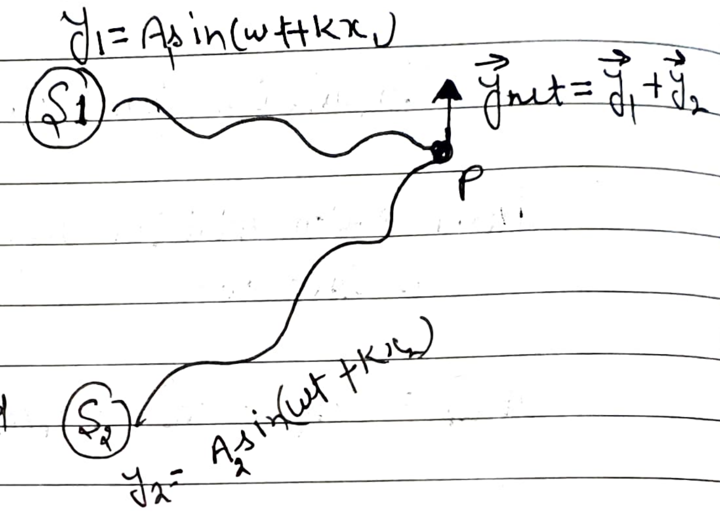
## Comparison b/w Interference, stationary and Beat

| | Interference | Stationary wave | Beat |
|---|---|---|---|
| Speed | Same | Same | Same |
| Direction | Same | Opposite | Same |
| Frequency | Same | Same | Slightly differ |
| Wavelength | Same | Same | Same |
| Amplitude | | | |

# Interference




Let consider two source $S_1$ and $S_2$ are emitting wave with same frequency.

When they reach at point P the displacement produced by them is given by

$$y_1 = A_1 \sin(\omega t + kx_1)$$

$$y_2 = A_2 \sin(\omega t + kx_2)$$

Due to these two wave the ~~disturbance~~ net amplitude of oscillation y the particle will be given by

$$\vec{y}_{net} = \vec{y}_1 + \vec{y}_2$$

$$\vec{y}_{net} = A_1 \sin(\omega t + kx_1) + A_2 \sin(\omega t + kx_2)$$

$$\boxed{y_{net} = A \sin(\omega t + \phi)}$$

Equation of Resultant S.H.M of particle at point P displacement of particle at time t ← $y = A \sin(\omega t + kx)$

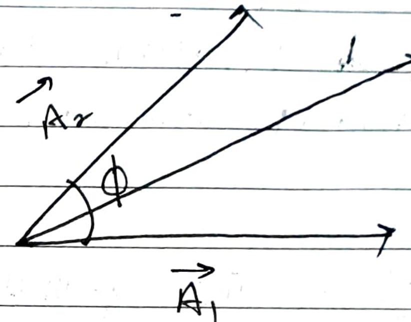
Amplitude ($A$) — Phase angle ($\omega t + kx$)



## I. Amplitude of Resultant S.H.M of particle at Point P

Since the displacement produced by two wave is a vector quantity then they can be represented by two vector.

Let the phase difference b/w the two amplitude when they reach at point P is $\phi$



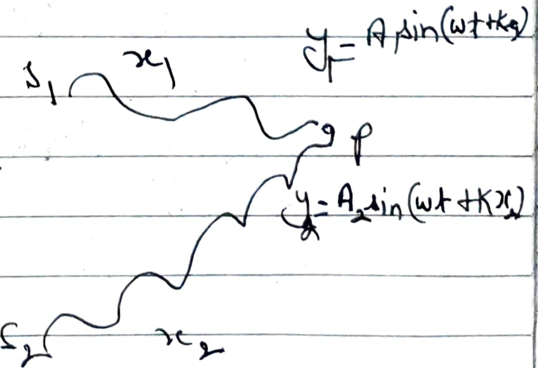

Resultant of Amplitude of oscillation of particle at point P is given by

$$\boxed{A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

from above we can see that the resultant Amplitude depends upon the phase angle difference b/w two wave.

Phase difference at Point P

[Assume $kx_2 > kx_1$]




$$\phi = (\omega t + kx_2) - (\omega t + kx_1)$$

$$\phi = kx_2 - kx_1$$

$$\phi = k(x_2 - x_1)$$

$$\boxed{\phi = \frac{2\pi}{\lambda}\Delta x}$$

From this we can conclude that phase difference will be different for different position hence resultant amplitude will be different at different location

## Important Point

(1) If during interference the wave coming out from two wave have same frequency then amplitude of particle at every point will be fixed and does not change with time. Because phase angle is constant.

That is why we prefer coherent source for visible Interference pattern because in coherent source frequency, wavelength is same for both the source.

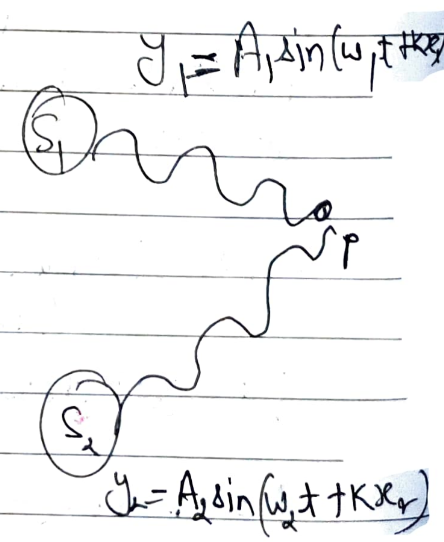
(2) If the two sources are not coherent then

Phase difference will be given by

$$\phi = (\omega_2 t + kx_2) - (\omega_1 t + kx_1)$$

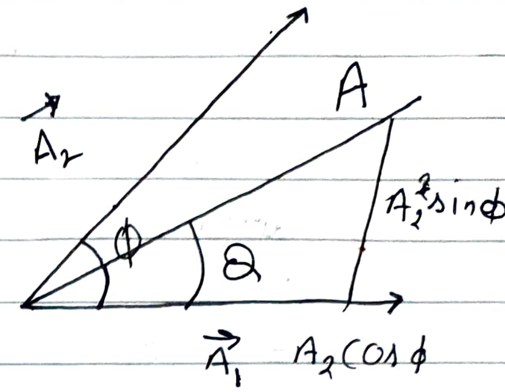
$$\phi = (\omega_2 - \omega_1)t + k(x_2 - x_1)$$

From above we can see that the phase difference is function of time and position. So phase difference at point will vary with time and there will be variation in amplitude



## II) Calculation for Q

$$\tan Q = \frac{P}{B} = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$$




$$\tan Q = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$$

## Intensity Of Resultant Wave

$$I = 2\pi^2 A^2 f^2 \rho V$$

where A = amplitude
f = frequency
V = velocity of wave
ρ = density of medium

$\boxed{I \propto A^2}$ because $2\pi^2 f \rho V$ are constant

$\boxed{I = KA^2}$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$KA^2 = KA_1^2 + KA_2^2 + 2\sqrt{A_1^2K_1}\sqrt{KA_2^2}\cos\phi$$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi}$$

# Type of Interference

| Constructive | Destructive |
|---|---|
| $I = \max$ | $I = \min$ |
| $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$ | $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$ |
| for $I = I_{max}$ | for $I = \min$ |
| $\cos\phi = 1$ | $\cos\phi = -1$ |
| $\cos\phi = 0, 2\pi, 4\pi, 6\pi$ | $\cos\phi = \pi, 3\pi, 5\pi$ |
| Phase difference $\boxed{\phi = 2n\pi}$ | $\phi = (2n+1)\pi$ |
| $\phi = \frac{2\pi}{\lambda} dx$ | $\phi = \frac{2\pi}{\lambda} dx$ |
| $dx = \frac{\lambda\phi}{2\pi}$ | $dx = \frac{\lambda\phi}{2\pi}$ |
| $dx = \frac{\lambda}{2\pi} 2n\pi$ | $dx = \frac{\lambda}{2\pi}(2n+1)\pi$ |
| Path difference $\boxed{dx = n\lambda}$ | $dx = (2n+1)\frac{\lambda}{2}$ |

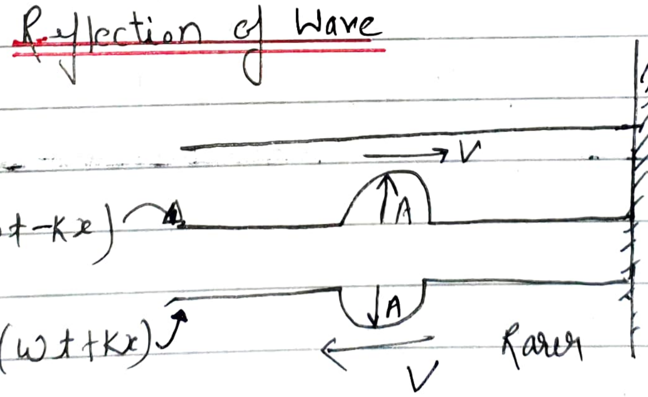
# Reflection and Transmission of Wave on String

| Properties | Reflection | Transmission |
|---|---|---|
| frequency ($\nu$) | Same | Same |
| Velocity (V) | Same | Change |
| Wavelength ($\lambda$) | Same | different |
| Amplitude | Same → fixed End, ~~Same~~ → free End; E=0 | Change |
| Phase | $\phi = 0$ → Denser → Rare; $\phi = \pi$ ~~Same~~ Rarer → Denser | $\phi = 0$ ~~Same~~ |

## Reflection of Wave

$y = A\sin(\omega t - kx)$

$y = -A\sin(\omega t + kx)$




Fixed End [Denser

Energy loss = 0

Transmission = 0

$y = A\sin(\omega t + kx + \pi)$

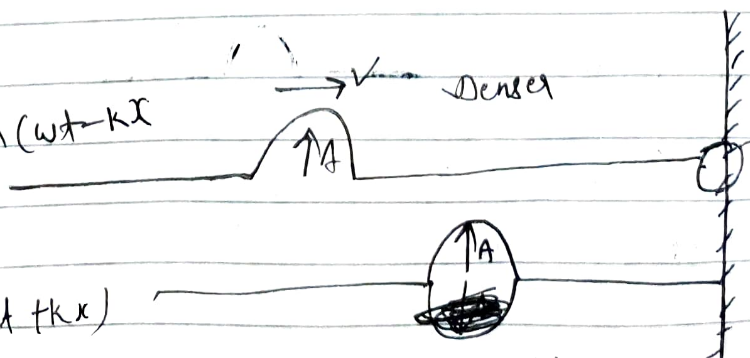
Rarer

free End

Energy loss = 0

$y_i = A\sin(\omega t - kx$

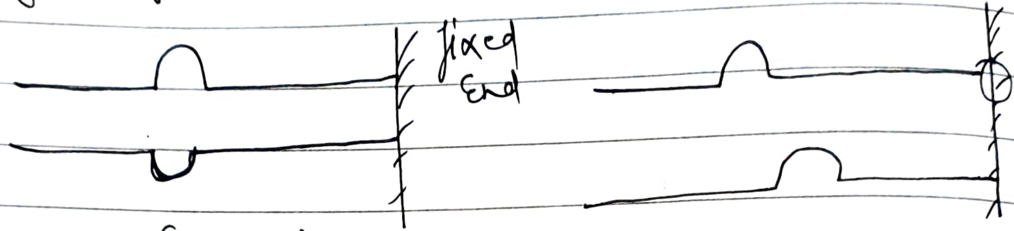
$y_R = A\sin(\omega t + kx)$




No phase change takes place.

Q→1 $y_i = 2\sin(4x - 8t)$ is reflected at $x = 0$ from (i) fixed end (ii) free end. Write the equation for reflected wave?

Ans:




$y_i = 2\sin(4x - 8t)$

$y_R = -2\sin(4x + 8t)$

OR

$y_R = 2\sin(4x + 8t + \pi)$

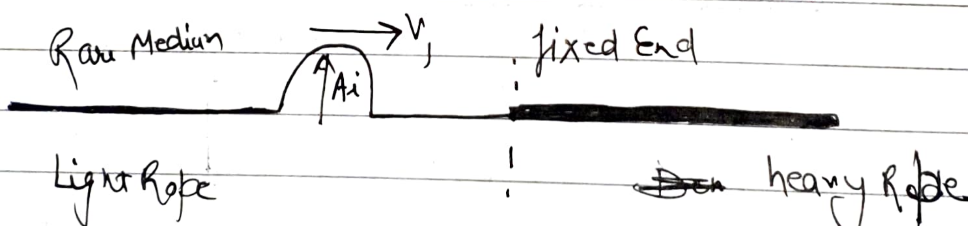
$y_i = 2\sin(4x - 8t)$

$y_R = 2\sin(4x + 8t)$

~~Q→2~~

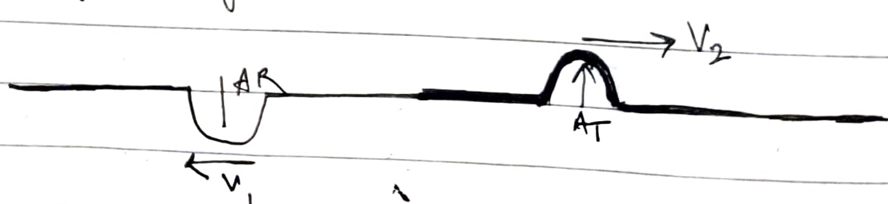
## Combination Of Rope

Case-I
when wave is travelling from rare to denser



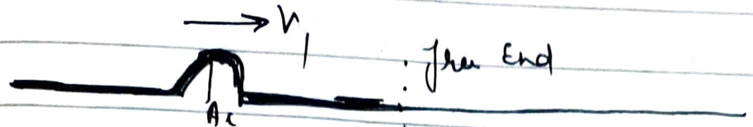

$y_i = A_i \sin(\omega t - kx)$

After Reflection and Transmission

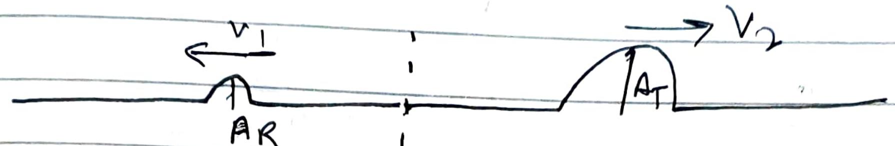



$y_R = -A_R \sin(\omega t + kx)$ $y_T = A_T \sin(\omega t - K_2 x)$

Case-II When wave is travelling from denser to rarer




$y_i = A_i \sin(\omega t - kx)$ Rarer




$y_R = A_R \sin(\omega t + kx)$ $y_T = A_T \sin(\omega t - K_2 x)$

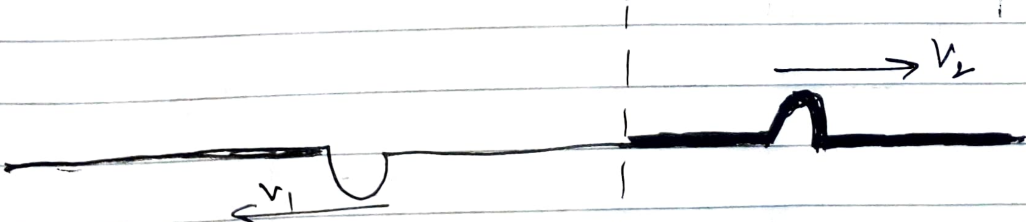
where

$$A_R = \left(\frac{V_2 - V_1}{V_1 + V_2}\right) A_i \qquad A_T = \left(\frac{2V_2}{V_1 + V_2}\right) A_i$$

Q1 If $y_i = 3\sin(2x - 4t)$ is allowed to passed through a rope combination from rarer to denser such that $\mu_2 = 4\mu_1$. Find $y_R$ and $y_T$


$y_i = 3\sin(2x - 4t)$

Sol




$y_R = A_R \sin(\cancel{k_1 x + \omega t} + \pi)$ $y_T = A_T \sin(k_2 x - 4t)$

$y_R = A_R \sin(2x + 4t + \pi)$

$\mu_2 = 4\mu_i$

$A_i = 3$

$\omega = 4$

$k = 2$

$$v = \sqrt{\frac{T}{\mu}}$$

$$v_1 = \frac{4}{2} = 2$$

$$\frac{V_2}{v_1} = \frac{\sqrt{\frac{T}{\mu_2}}}{\sqrt{\mu_1}}$$

$v_1 = 2$

$$\frac{V_2}{V_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{\cancel{\mu_1}}{4\cancel{\mu_1}}}$$

$$\frac{V_2}{V_1} = \frac{1}{2}$$

$$\boxed{V_2 = \frac{V_1}{2}} \Rightarrow V_2 = \frac{\cancel{2}}{\cancel{2}}$$

$V_2 = 1$

$$A_R = \left(\frac{V_2 - V_1}{V_2 + V_1}\right) A_i$$

$$A_T = \left(\frac{2V_2}{V_1 + V_2}\right) A_i$$

$$A_R = \left(\frac{1-2}{3}\right) \times 3$$

$$\boxed{A_R = -1}$$

$$A_T = \frac{2}{\cancel{3}} \times \cancel{3}$$

$$\boxed{A_T = 2}$$

$$\boxed{y_R = -1 \sin(2x - 4t)}$$

$v_1 = f\lambda_1$

$v_2 = f\lambda_2$

$$\frac{2}{1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$\boxed{\lambda_2 = \frac{\lambda_1}{2}}$$

$$k_1 = \frac{2\pi}{\lambda_1}$$

$$k_2 = \frac{2\pi}{\lambda_2}$$

$$\frac{k_1}{k_2} = \frac{\lambda_2}{\lambda_1}$$

$$\boxed{2k_1 = k_2}$$

$k_2 = 2k_1$

$$\boxed{y_T = 2\sin(4x - 4t)}$$

# Stationary Wave

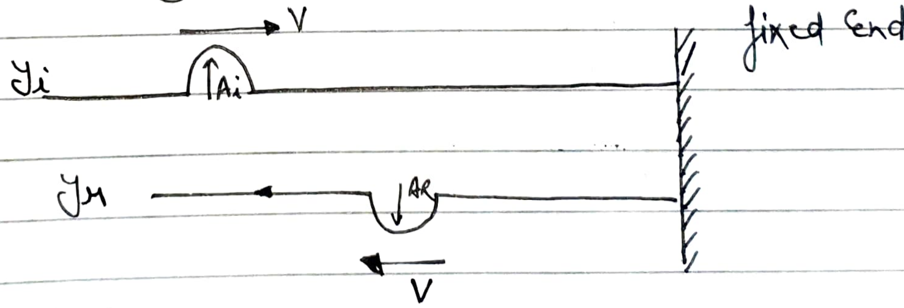
When two identical wave of same amplitude and frequency are travelling in opposite direction with same speed along the same path superimpose each other, the resultant wave ~~dog~~ does not travel in the either direction is called stationary wave or standing wave.

## Standing Wave

In String | In Organan pipe

## Standing Wave In a String

Let us consider a string attached to a wall when shaken generate a wave




Equation of incident wave

$$y_i = A\sin(\omega t - kx)$$

Equation of Reflected Wave

$$y_R = -A\sin(\omega t + kx)$$

Now net displacement of a particle of the rope will be given by

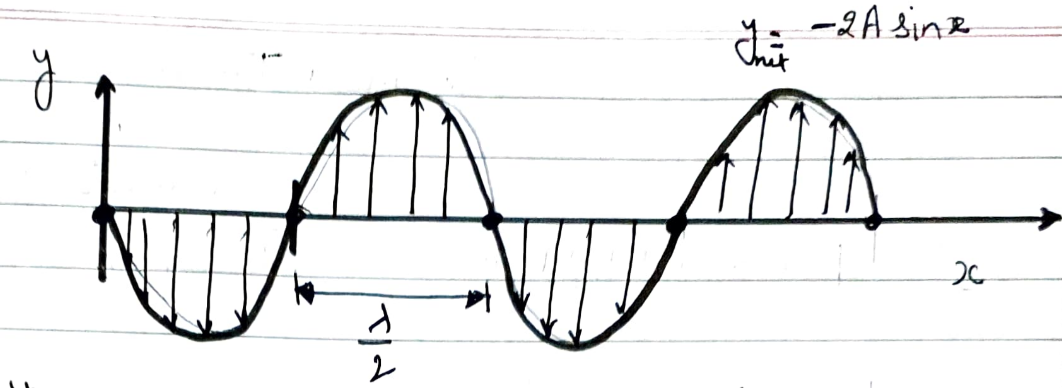
$$\vec{y}_{net} = \vec{y}_1 + \vec{y}_2$$ (According to principle of superposition)

## Formation of stationary wave by Graphical Method.

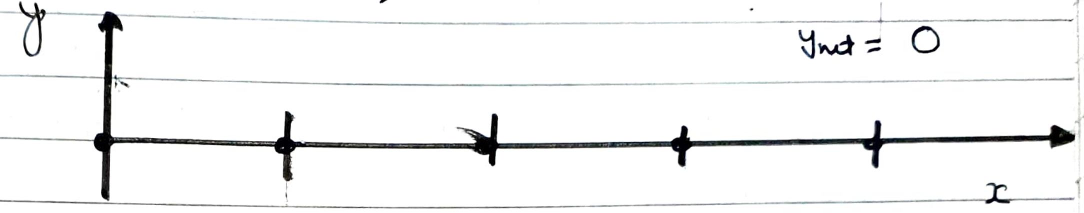
| time | $y_i = A\sin(\omega t - kx)$<br>$y_i = A\sin(\frac{2\pi}{T}t - kx)$ | $y_r = -A\sin(\omega t + kx)$<br>$y_r = -A\sin(\frac{2\pi}{T}t + kx)$ | $y_{net}$ = Standing wave<br>$y_{net} = y_1 + y_2$ |
|---|---|---|---|
| $0$ | $-A\sin kx$ | $-A\sin kx$ | $-2A\sin kx$ |
| $\frac{T}{4}$ | $A\sin(\frac{2\pi}{\cancel{T}} \times \frac{\cancel{T}}{4} - kx)$<br>$A\sin(2\pi - kx)$<br>$A\cos kx$ | $-A\sin(\frac{2\pi}{T} \times \frac{T}{4} + kx)$<br>$-A\sin(2\pi + kx)$<br>$-A\cos kx$ | $0$ |
| $\frac{T}{2}$ | $A\sin(\frac{2\pi}{\cancel{T}} \times \frac{\cancel{T}}{\cancel{2}} - kx)$<br>$A\sin kx$ | $-A\sin(\frac{2\pi}{T}\frac{T}{2} + kx)$<br>$A\sin kx$ | $2$<br>$2A\sin kx$ |
| $\frac{3T}{4}$ | $A\sin(\frac{2\pi}{\cancel{T}} \times \frac{3\cancel{T}}{\cancel{4}_2} - kx)$<br>$A\sin(\frac{3\pi}{2} - kx)$<br>$-A\cos kx$ | $-A\sin(\frac{2\pi}{T} \times \frac{3T}{4} + k)$<br>$-A\sin(\frac{3\pi}{2} + kx)$<br>$A\cos kx$ | $0$ |
| $T$ | $A\sin(\frac{2\pi}{\cancel{T}} \times \cancel{T} - kx)$<br>$A\sin(2\pi - kx)$<br>$-A\sin kx$ | $-A\sin(\frac{2\pi}{T} \times T + kx)$<br>$-A\sin(2\pi + kx)$<br>$-A\sin kx$ | $-2A\sin kx$ |

$y_{net}$ is the equation of standing wave

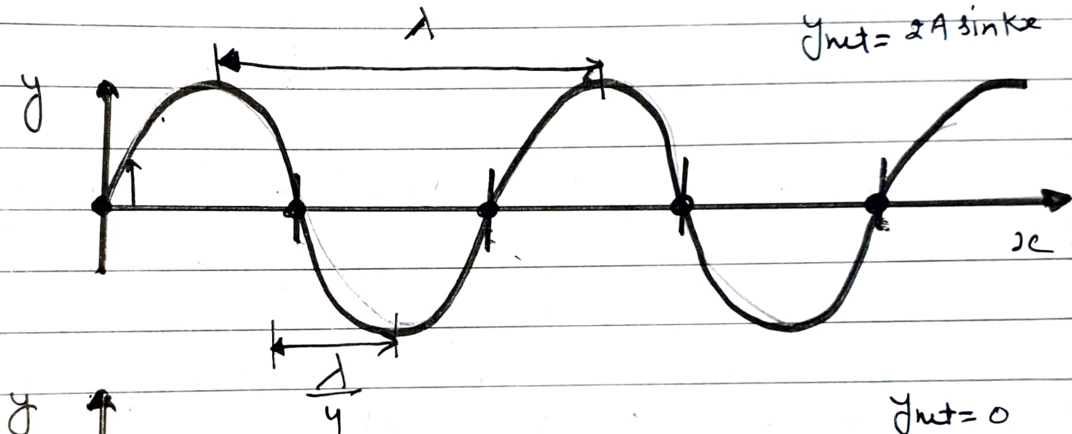
$t = 0$



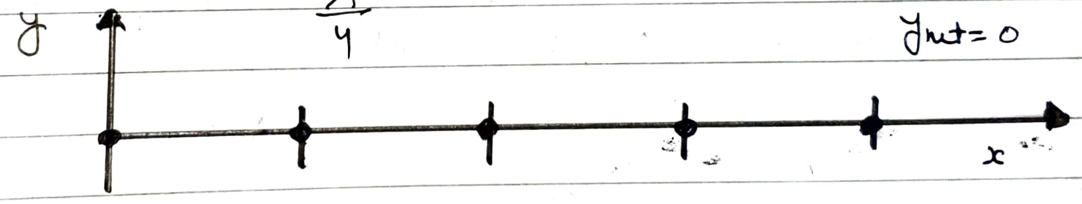

$t = \frac{T}{4}$



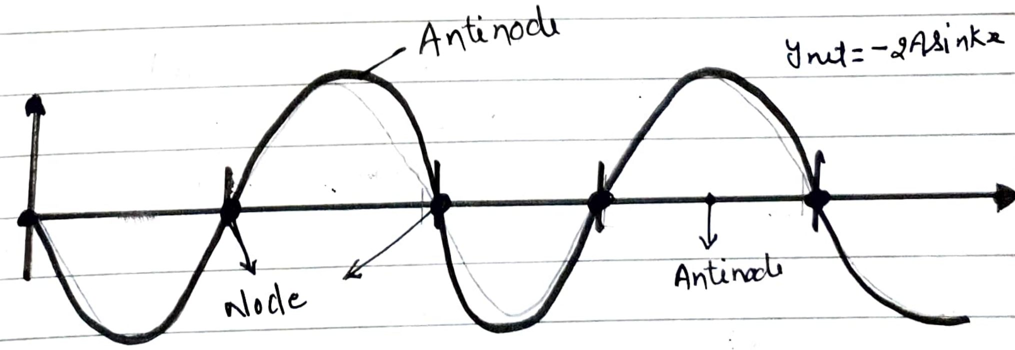

$t = \frac{T}{2}$




$t = \frac{3T}{4}$




$t = T$




Above graph is basically position of all the particle at any instant

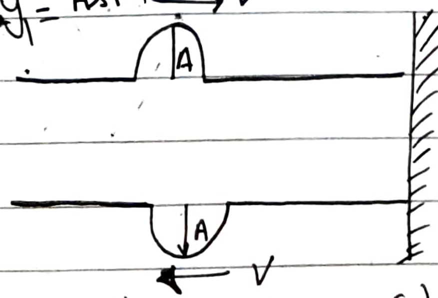
No energy is transferred in stationary wave

* Each particle of the rope is doing S.H.M
* Each particle of rope are in same phase having different value of Amplitude
* The gap b/w two consecutive node and antinode is $\frac{\lambda}{2}$ and two consecute node and antinode is $\frac{\lambda}{4}$

# Analytical Treatment Of Standing Wave

$y = A\sin(\omega t - kx)$ $\xrightarrow{V}$

## ~~(1)~~ Fixed End Reflection



$y_i = A\sin(\omega t - kx)$

$y_R = -A\sin(\omega t + kx)$

$y_R = -A\sin(\omega t + kx)$

$y_{net} = A\sin(\omega t - kx) - A\sin(\omega t + kx)$

$y_{net} = A[\sin(\omega t - kx) - \sin(\omega t + kx)]$

$y_{net} = 2A\cos\omega t \sin(-kx)$

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\boxed{y_{net} = -2A\sin kx \cos\omega t}$$

→ Equation of standing wave

$y_{net} = \underbrace{-2A\sin kx}_{\text{Amplitude}} \underbrace{\cos\omega t}_{\text{Phase}}$

In standing wave amplitude of particle is dependent upon its position ($x$) $2A\sin kx$

**Position Of Node** ⇒ Node is a point where displacement of particle is zero i.e $y_{net} = 0$

$y_{net} = 0$

Since amplitude of particle is independent of time

Then

$2A \sin kx = 0$

$kx = 0, 2\pi, 3\pi, 4\pi, 5\pi \cdots n\pi$

$\frac{2\pi}{\lambda} x = 0, \pi, 2\pi, 3\pi, 4\pi$

$k = \frac{2\pi}{\lambda}$

| Position of first Node. | Position of second Node | Position of third Node |
| --- | --- | --- |
| $\frac{2\pi}{\lambda} \times x = 0$ | $\frac{2\pi}{\lambda} \times x = \pi$ | $\frac{2\pi}{\lambda} \times x = 2\pi$ |
| $x = 0$ | $x = \frac{\lambda}{2}$ | $x = \lambda$ |

**Position of Antinode** $\Rightarrow$ Antinode is that point where the amplitude of the particle is maximum.

$y_{net} = -2A \sin kx \cos \omega t$

Since disp of particle is independent of time so

$y_{net} \propto 2A \sin kx$

for $y_{net} = \max = 1$

$\sin kx = 1$

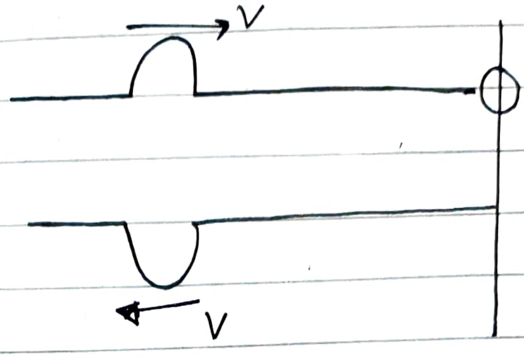
$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

| Position of first Antinode | Position of second Antinode |
| --- | --- |
| $kx = \frac{\pi}{2}$ | $kx = \frac{3\pi}{2}$ |
| $\frac{2\pi}{\lambda} x = \frac{\pi}{2}$ | $\frac{2\pi}{\lambda} x = \frac{3\pi}{2}$ |
| $x = \frac{\lambda}{4}$ | $x = \frac{3\lambda}{4}$ |

## 2/ Stationary wave produced by reflection from free End.

$y_i = A\sin(\omega t - kx)$

$y_R = A\sin(\omega t + kx)$




$y_{net} = y_i + y_R$

$y_{net} = A\sin(\omega t - kx) + A\sin(\omega t + kx)$

$y_{net} = 2A\sin\omega t\cos(-kx)$

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$y_{net} = 2A\cos kx\sin\omega t$ Equation of standing wave.

### Position of node

$y_{net} = 0$

$2A\cos kx = 0$

$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

Position of first Node

$kx = \frac{\pi}{2}$

$k = \frac{2\pi}{\lambda}$

$\frac{2\pi}{\lambda}x = \frac{\pi}{2}$

$x = \frac{\lambda}{4}$

### Position Of Antinode

$y_{net} = \pm 1$

$2A\cos kx = \pm 1$

$kx = 0, \pi, 2\pi, 3\pi$

Position of first Antinode

$kx = 0$

$\frac{2\pi}{\lambda}x = 0$

$x = 0$

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Q→ Find position of Node and Antinode for the given Equation of stationary wave

$$y = 2A\sin\left(kx + \frac{\pi}{3}\right)\cos\left(\omega t + \frac{\pi}{2}\right)$$

Ans:- Position of Node

$y_{net} = 0$

$2A\sin\left(kx + \frac{\pi}{3}\right) = 0$

$kx + \frac{\pi}{3} = 0$

$\frac{2\pi}{\lambda}x = -\frac{\pi}{3}$

$\boxed{x = -\frac{\lambda}{6}}$

Position of Antinode

$y_{net} = \pm 1$

$2A\sin\left(kx + \frac{\pi}{3}\right) = \pm 1$

$kx + \frac{\pi}{3} = \pm 1$

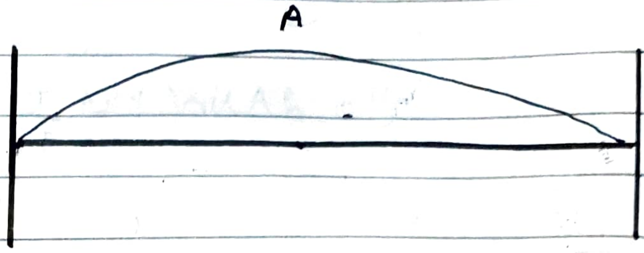
$\frac{2\pi}{\lambda}x = +1 - \frac{\pi}{3}$

$x = \left(\frac{3-\pi}{6}\right)\frac{\lambda}{\pi}$

| Travelling Wave | Standing Wave |
|---|---|
| 1 Energy is transferred | Energy is not transferred |
| 2 $y = A\sin(\omega t - kx)$<br>Max disp of each particle is same | $y = A\sin kx\cos(\omega t)$<br>$A_R = A\sin kx$<br>so each particle is having different amplitude for different particle |
| 3 All the particle are in different phase $(\omega t - kx)$ | All the particle are in same phase $\omega t$ |

# Stationary Wave In a string fixed at Both End.

## First Mode / Fundamental Mode / tone




$$f = \frac{v}{\lambda}$$

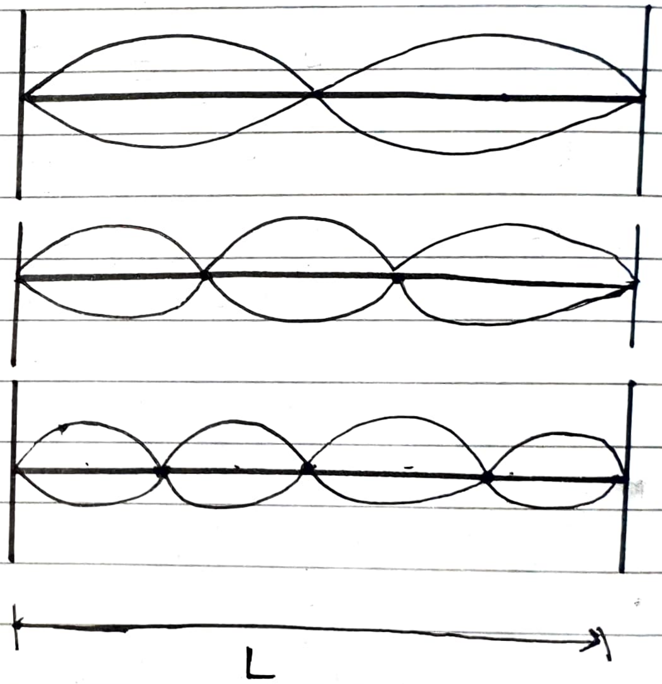
$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\text{tension}}{\text{mass per unit length}}}$$

$$f_\lambda = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$$

$$\frac{\lambda}{2} = L$$

$$\lambda = 2L$$

$$\boxed{f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}}$$




## Second harmonic | First Overtone

$$2\frac{\lambda}{2} = L$$

$$f_2 = 2\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right) - f$$

$$\boxed{f_2 = 2f}$$

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## Third harmonic | Second overtone

$$L = \frac{3\lambda}{2}$$

$$f_3 = \frac{3 \times 1}{2L}\sqrt{\frac{T}{\mu}}$$

$$\lambda = \frac{2L}{3}$$

$$f_3 = 3f$$

$n^{th}$ harmonic

$$\boxed{\frac{n\lambda}{2} = L}$$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

$$\boxed{f_n = nf}$$

so

$$f_1 : f_2 : f_3 = 1 : 2 : 3$$

Q→ In normal mode of vibration of string tied at both ends, the difference in the frequency of ~~2nd~~ fifth harmonic and second overtone is 54 hz. Calculate fundamental frequency.

Ans:

$$f_5 - f_2 = 54$$

$$5f - 3f = 54$$

$$2f = 54$$

$$f = 27$$

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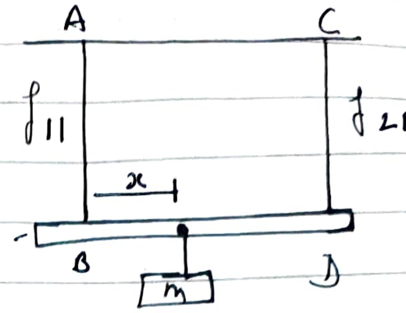
Q→ AB, CD = Massless ~~elastic~~ identical string

BD = Massless rod



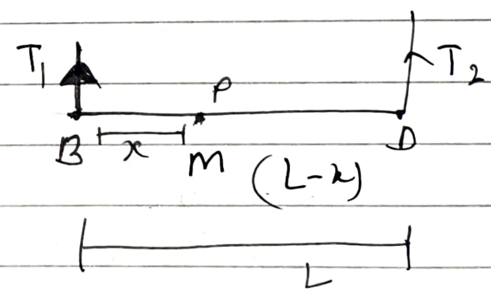

Fundamental frequency of Left wire is double of fundamental frequency of right wire. Find $x$

Sol

$$f_{11} = \frac{1}{2L}\sqrt{\frac{T_1}{\mu}} \qquad f_{21} = \frac{1}{2L}\sqrt{\frac{T_2}{\mu}}$$

$$\frac{f_{11}}{f_{21}} = \sqrt{\frac{T_1}{T_2}}$$

$$2 = \sqrt{\frac{T_1}{T_2}}$$




$$\boxed{T_1 = 4T_2}$$

taking moment along P

$$-T_1 x + T_2(L-x) = 0$$

$$-T_1 x + T_2 L - T_2 x = 0$$

$$\cancel{+} x[T_1 + T_2] = \cancel{+} T_2 L$$

$$x[4T_2 + T_2] = T_2 L$$

$$5x\cancel{T_2} = \cancel{T_2} L$$

$$\boxed{x = \frac{L}{5}}$$

# Stationary Waves In Organ pipe OR Air Column

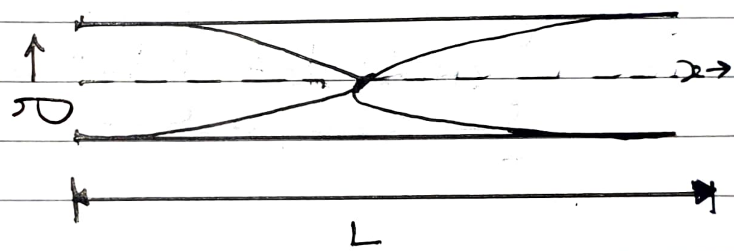
**Organ pipe** ⇒ It is the simplest musical instrument in which sound is produced by setting an air column into vibration.

## Normal Modes Of Vibration Of An Open Organ pipe

### First Mode Of Vibration

$L = 2 \cdot \frac{\lambda_1}{4}$

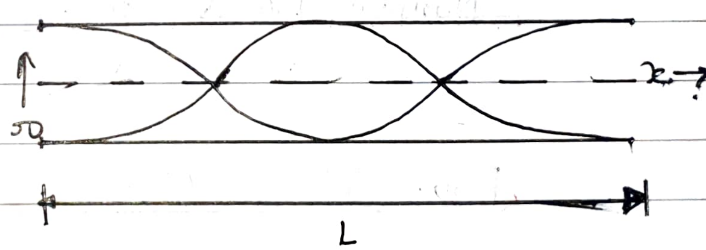
$\boxed{\lambda_1 = 2L}$



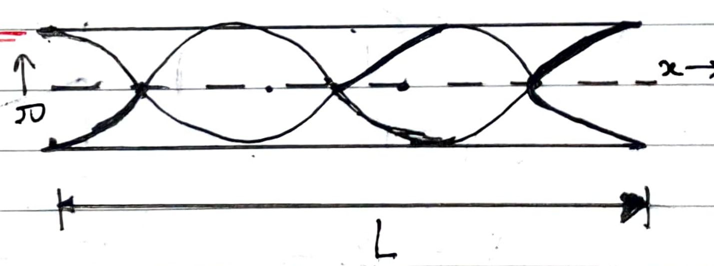
### Frequency of vibration

$\nu_1 = \frac{V}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}}$

$\nu_1 = \nu$



### Second Mode of vibration



$L = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$

$L = \lambda_2$

$\nu_2 = \frac{1}{L}\sqrt{\frac{\gamma P}{\rho}}$

$\nu_2 = 2\nu$

## Third Mode of Vibration

$$L = \lambda_3 + \frac{\lambda_3}{4} + \frac{\lambda_3}{4}$$

$$L = \frac{3\lambda_3}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

$$\nu_3 = \frac{V}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

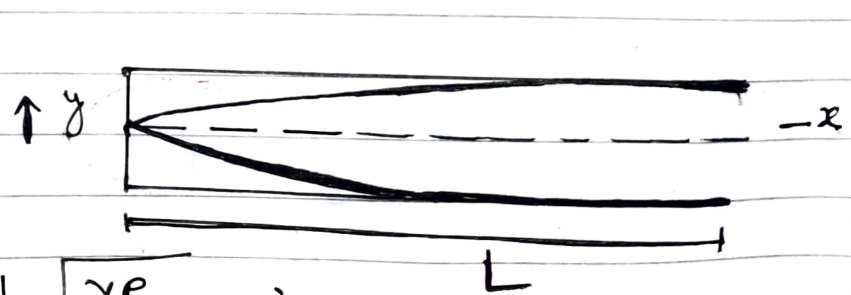
$$\nu_3 = 3\nu$$

$$\nu_1 : \nu_2 : \nu_3 = 1:2:3:$$

## Normal Modes of a Closed Organ Pipe

## First Mode of Vibration

$$L = \frac{\lambda_1}{4}$$



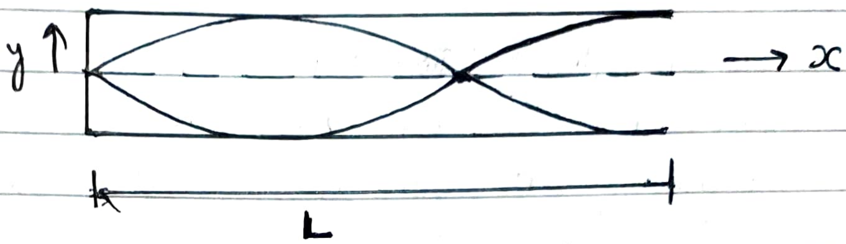

$$\lambda_1 = 4L$$

$$\nu_1 = \frac{V}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu$$

$$\nu_1 = \nu$$

## Second Mode of Vibration

$$L = \frac{3\lambda_2}{4}$$



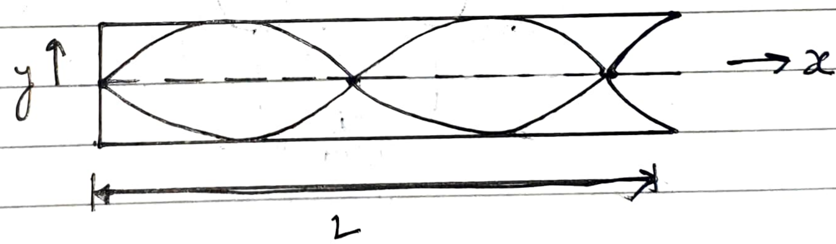

$$\lambda_2 = \frac{4L}{3}$$

$$\nu_2 = \frac{V}{\lambda_2} = \frac{3}{4L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

$$\boxed{\nu_2 = 3\nu}$$

## Third Mode of Vibration

$$L = \frac{5\lambda_3}{4}$$




$$\lambda_3 = \frac{4L}{5}$$

$$\nu_3 = \frac{V}{\lambda_3} = \frac{5}{4L}\sqrt{\frac{\gamma P}{\rho}} = 5\nu$$

$$\boxed{\nu_3 = 5\nu}$$

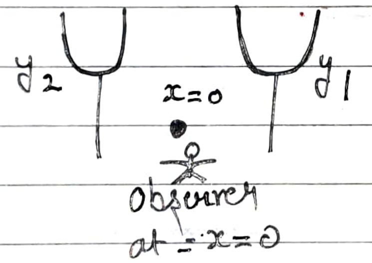
$$\boxed{\nu_1 : \nu_2 : \nu_3 = 1 : 3 : 5}$$

# Beats

Superposition of two sound waves having small difference in frequency travelling in same direction forms beats. When travelling with same speed and in same medium

Let us consider two waves having frequency $\nu_1$ and $\nu_2$ respectively in same medium with same speed

$$y_1 = A\sin(\omega_1 t - k_1 x)$$
$$y_2 = A\sin(\omega_2 t - k_2 x)$$




so equation will change

$$y_1 = A\sin(\omega_1 t)$$
$$y_2 = A\sin(\omega_2 t)$$

Now the displacement of particle will be given by

$$\vec{y} = \vec{y_1} + \vec{y_2}$$

$$y = A\sin(\omega_1 t) + A\sin\omega_2 t$$

$$y = A\left[\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t\right] \qquad \left\{ \begin{array}{l} \omega_1 = 2\pi\nu_1 \\ \omega_2 = 2\pi\nu_2 \end{array} \right.$$

$$y = 2A \sin 2\pi \left(\frac{\nu_1+\nu_2}{2}\right)t \cdot \cos 2\pi \left(\frac{\nu_1-\nu_2}{2}\right)t$$

$$y = \underbrace{2A \cos 2\pi \left(\frac{\nu_1-\nu_2}{2}\right)t}_{A_R} \sin 2\pi \left(\frac{\nu_1+\nu_2}{2}\right)t$$

$$y = A_R \sin 2\pi \left(\frac{\nu_1+\nu_2}{2}\right)t$$

Maximum loud sound
when $A_R$ = Max

For $A_R$ = max

$$\cancel{2A} \cos 2\pi \left(\frac{\nu_1-\nu_2}{\cancel{2}}\right)t = \pm 1$$

$$2\pi \left(\frac{\nu_1-\nu_2}{2}\right)t = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$(\nu_1-\nu_2)\,t = 0$$

$$t = 0$$

$$\pi\left(\nu_1-\nu_2\right)t_2 = \pi$$

$$t_2 = \frac{1}{\nu_1-\nu_2}$$

$$\pi(\nu_1-\nu_2)t_3 = 2\pi$$

$$t_3 = \frac{2}{\nu_1-\nu_2}$$

Difference b/w two loud sound

$$t_2 - t_1 = \frac{1}{\nu_1-\nu_2} - 0$$

$$t_2 - t_1 = \frac{1}{\nu_1-\nu_2}$$

So time gap b/w two loud sound is

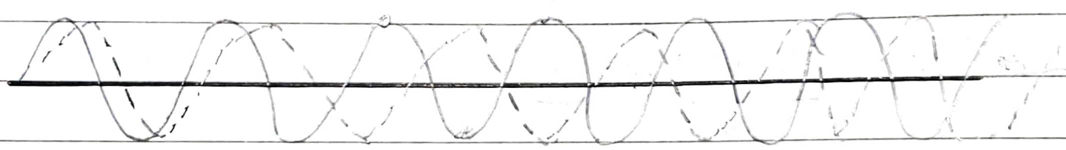
$$T = t_2 - t_1 = \frac{1}{\nu_1-\nu_2}$$

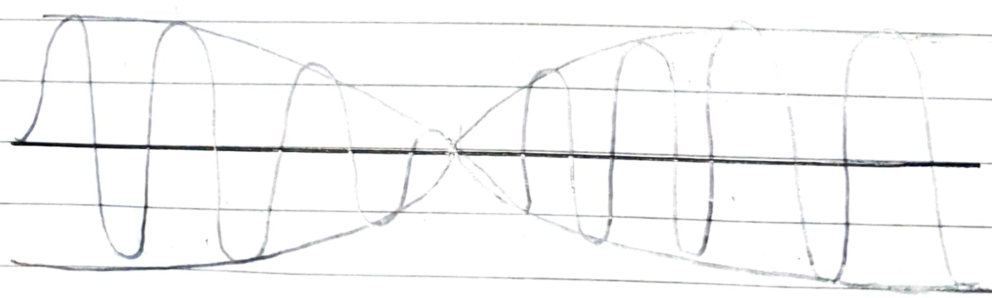
so beat frequency is

$$\nu_{Beat} \frac{1}{T} = \nu_2 - \nu_1$$

$$\boxed{\nu_{Beat} = \nu_2 - \nu_2}$$

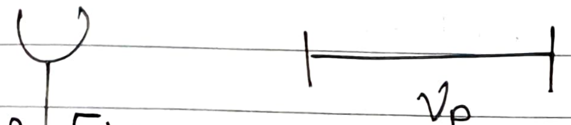
Similiarly beat frequency can also be calculated from minima.





Q→ A tuning fork makes 4 beats/second with a piano. The beat frequency decreases to 2 beats per second when tension in the piano string is increased. The initial frequency of piano was? Given frequency of fork = 512 Hz

Sol



$\nu_F = 512$ $\qquad\qquad\qquad$ $\nu_p$

$$\nu_{Beat} = |\nu_F - \nu_p|$$

$$4 = |512 - \nu_p|$$

so $\nu_p = 516$ OR $508$

$$\nu_{Beat} = |\nu_F - \nu_p'|$$

$$2 = |512 - \nu_p|$$

$\nu_p = 514$ OR $510$

since tension in the string was increased so frequency of piano will also increase since $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$

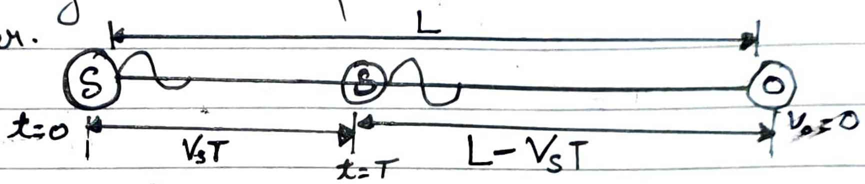
so if we take the initial frequency as 516 then after increasing the tension the beat frequency ~~who~~ would have increased.

But if initial frequency of piano was 508 then after increasing the tension beat frequency would decrease.

# Doppler Effect

The apparent change in the frequency of sound when the source, the observer and the medium are in relative motion is called doppler effect.

**Case-I** Apparent frequency when the source moves towards the stationary observer.




Let us consider a source (S) moving with speed $V_s$ towards an observer O who is at rest with respect to medium

When $\nu$ is the frequency of vibration of the source, Then it sends out sound wave with speed V at a regular interval of $T = \frac{1}{\nu}$

A $t=0$ suppose the source is at a distance L from the observer and emits a compression pulse. It reaches the observer at time.

$$t_1 = \frac{L}{V}$$

Time taken by the second compression to reach observer

$$t_2 = T + \frac{L - V_sT}{V}$$

Time interval b/w two successive compression pulses of the wave to reach observer

$$T' = t_2 - t_1 \quad T + \frac{L - V_s T}{V} - \frac{L}{V}$$

$$T' = \left(1 - \frac{V_s}{V}\right) T$$

$$T' = \left(\frac{V - V_s}{V}\right) T$$

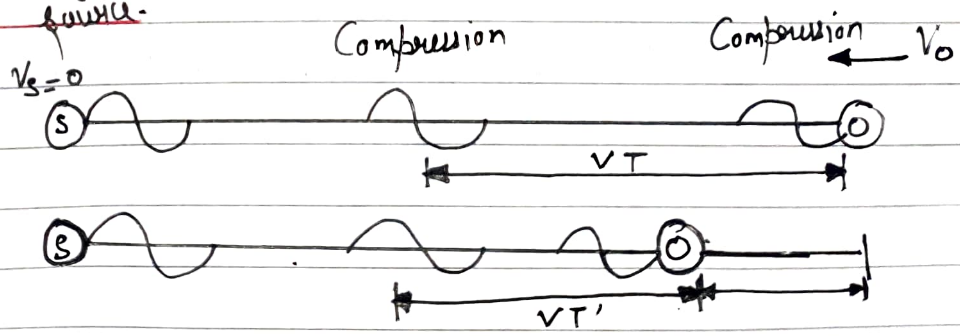
Apparent frequency $\nu' = \frac{1}{T'} = \left(\frac{V}{V - V_s}\right) \frac{1}{T}$

$$\nu' = \left(\frac{V}{V - V_s}\right) \nu$$

Case-II When the source is moving away from the observer

$$\nu' = \left(\frac{V}{V + V_s}\right) \nu$$

Case-III Apparent frequency when the observer moves towards stationary source.




At any instant the separation b/w any two compression pulse is $\lambda = VT$

So when the observer recieves a pulse next pulse is at a distance of VT.

Since the observer is also moving towards the source so the next pulse will reach the observer in T' time

$$T' = \frac{VT}{V + V_o}$$ { $V_o$ = velocity of observer }

Frequency of sound heard by observer is

$$\nu' = \frac{1}{T'} = \frac{V + V_o}{V} \times \frac{1}{T}$$

$$\nu' = \left(\frac{V + V_o}{V}\right)\nu$$

Apparent frequency will increase

(Case-IV) When observer is moving away from the source

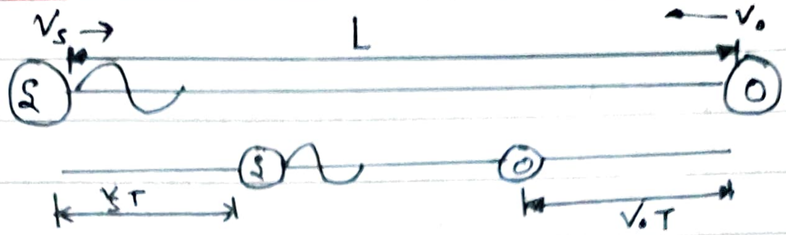
$$\nu' = \left(\frac{V - V_o}{V}\right)\nu$$

Apparent frequency will decrease

Date

Page

## Apparent frequency when both source and observer are in motion.




at $t_0 = 0$ the source (S) and observer O are at a distance L from each other. So when the source emits its first wave source and observer are at L distance from each other. Since observer is also moving so this distance will be covered by wave with $(v+v_o)$ speed

$$t_1 = \frac{L}{v+V_o}$$

at $t = T$ both observer and source have moved towards each other covering distance $V_s T$ by source and $V_o T$ by observer

The new distance b/w source and observer is $L-(V_s+V_o)T$

The second pulse will reach observer

$$t_2 = T + \frac{L-(V_s+V_o)T}{v+V_o}$$

Time interval b/w two successive pulse to reach observer is

$$T' = t_2 - t_1 = \left[T + \frac{L-(V_s+v_o)T}{V+v_o}\right] - \frac{L}{v+v_o}$$

$$T' = \left(\frac{V-V_s}{v+v_o}\right)T$$

## Apparent frequency

$$\nu' = \frac{1}{T'}$$

$$\nu' = \left(\frac{V+V_0}{V-V_s}\right)\frac{1}{T} = \left(\frac{V+V_0}{V-V_s}\right)\nu$$

$$\boxed{\nu' = \left(\frac{V+V_0}{V-V_s}\right)\nu}$$

## Apparent frequency when many factor are contributing at same time

$$\boxed{\nu' = \left[\frac{V \pm V_m \pm V_0}{V \pm V_m \pm V_s}\right]\nu}$$

where $\nu'$ = apparent frequency

$V$ = velocity of wave

$V_m$ = Velocity of Medium
- +ve → in the direction of wave
- −ve → opp to the direction of wave

$V_s$ = Velocity of source
- +ve → Away from observer
- −ve → towards observer

$V_0$ = Velocity of observer
- +ve → Moving towards ~~source~~ source
- −ve → Moving away from source.