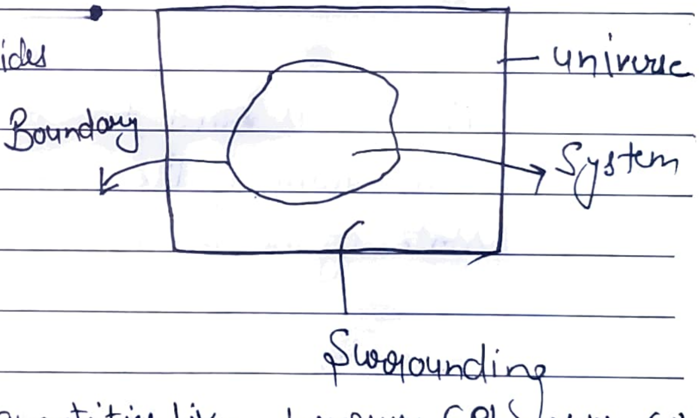
THERMODYNAMICS - 12

It is the branch of science that deals with concept of heat and temprature and their inter conversion of heat and other forms of energy.

Thermodynamic System ⇒ An assembly of a very large number of particle having certain value of pressure, volume and temprature is under in our observation is called thermodynamic system.

Surrounding ⇒ It is that part of ~~atmos~~ universe which is under in our observation is called surrounding.

Boundary ⇒ It is a line that divides system from surrounding.



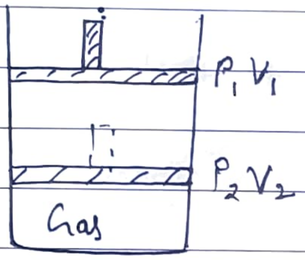
Thermodynamic Variable ⇒ The quantities like pressure (P), volume (V) and temprature (T) which decides the state of a system is called thermodynamic variable.

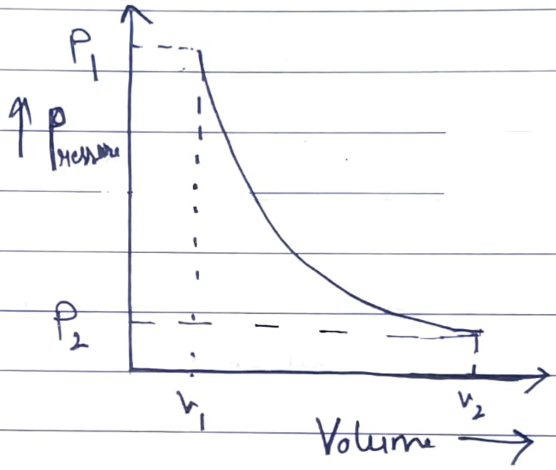
Equation Of state ⇒ The mathematical relationship b/w thermodynamic variable is known as equation of state.

Ex $\boxed{PV = nRT}$

INDICATOR DIAGRAM

A graphical representation of the state of system with the help of any two thermodynamic variable is called indicator diagram.





THERMODYNAMIC EQUILIBRIUM

A system is said to be in the state of thermodynamic equilibrium if the macroscopic variable describing the thermodynamic state of the system do not change with time.

A system in the state of thermodynamic equilibrium satisfy following type of equilibrium simultaneously.

(i) Mechanical Equilibrium ⇒ There is no unbalanced force in the interior or b/w the system and surrounding.

(ii) Thermal Equilibrium ⇒ All parts of the system and surrounding are at same temperature.

(iii) Chemical Equilibrium ⇒ The system does not undergo any

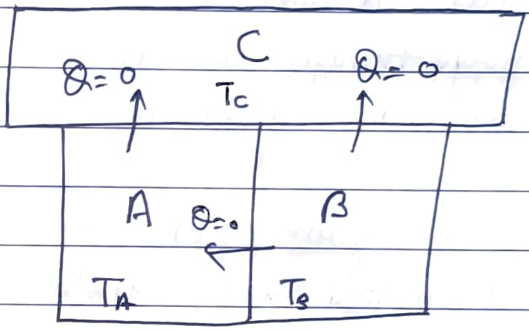
Internal Energy of Real Gas = K·E + PE
" " " Ideal gas = K·E

spontaneous change in its internal structure due to chemical reaction.

ZEROTH LAW OF THERMODYNAMIC

It state that if two system A and B are ~~in~~ separately in Thermal equilibrium with a third system C, then A and B are also in thermal equilibrium with each other.

$T_A = T_B = T_C$



Temperature ⇒ It is one of the seven fundamental quantity which decides the direction of flow of heat.

Heat always flow from high temperature to low temperature.

Internal Energy ⇒ It is sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest.

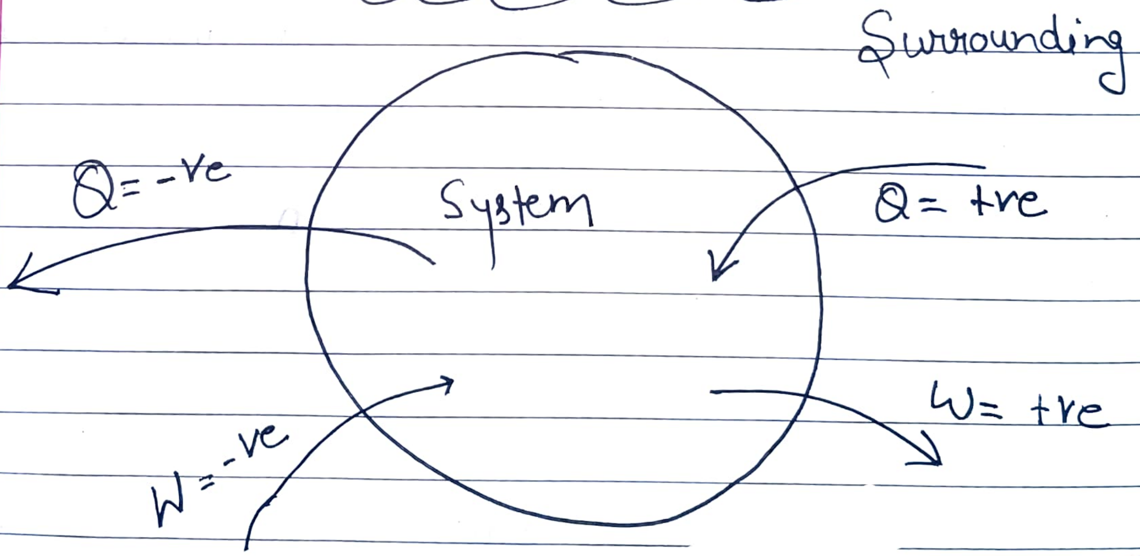
* Internal energy of a system is Thermodynamic state variable.

Pressure has no role in internal energy of Ideal gas where as it play role in case of real gas

- ☆ Internal Energy of an ideal gas is purely in the form of kinetic energy.
- ☆ Internal energy of the system depends upon pressure, volume and temperature.

	Heat	Work
(i)	Heat is a mode of energy transfer due to temperature difference	Work is a mode of energy transfer which does not require temperature difference.
(ii)	Heat is a form of energy transfer that produces random motion	Work is a mode of energy transfer that produces organised motion.

SIGN CONVENTIONS



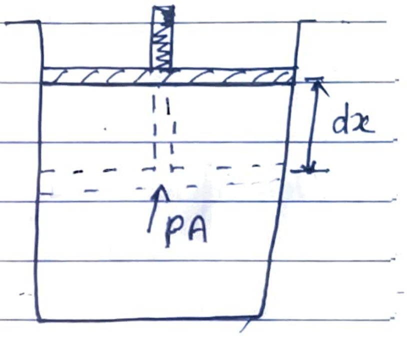
- ☆ Increase in the internal energy of the system is positive.
- ☆ Decrease in the internal energy of system is negative.

Work done During Expansion =>

Let us consider an ideal gas contained in a cylinder of cross-sectional area A provided with frictionless movable piston.

Let P be the pressure of the gas.

Force exerted by the gas on the piston



$F = P \times A$

Suppose due to this pressure the ~~f~~ piston expands by small distance dx

work done in expansion

$dw = F\,dx$

$dw = P\,A\,dx$ $\left\{ A\,dx = dV \right.$

$\boxed{dw = P\,dV}$

total work done

$$W = \int dw = \int_{V_1}^{V_2} P\,dV$$

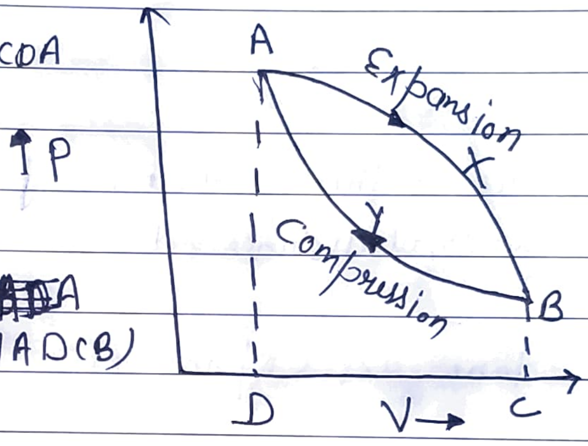
Cyclic Process => Any process in which the system returns to initial state ~~is called~~ ~~Thermodynamic process~~. after undergoing a series of changes is known as a cyclic process.

Non Cyclic Process ⇒ Any process in which the system does not return to its initial state after undergoing a series of changes is known as non cyclic process.

Work Done During Cyclic Process

Work done during expansion ⇒ + area of AXBCDA

Work done during compression ⇒ − area of ~~BYAD~~A (CBYADCB)



Net work done during cyclic process = $W_1 + W_2$

$W = \text{area } AXBCD + (-\text{ ar } BYADCB)$

OR

Area Inclosed by the curve gives the work done in cyclic process.

FIRST LAW OF THERMODYNAMICS

According to first law of Thermodynamics if some heat is supplied to a system which is capable of doing work, Then the quantity of heat absorbed by the system will be equal to the sum of the increase in its internal energy and the external work done by the system on the surroundings.

OR

According to first law of thermodynamics energy can neither be created nor be destroyed but it can be converted from one form into another form.

$$\boxed{\Delta Q = \Delta U + \Delta W}$$

ΔQ = Heat supplied to the system

ΔU = Change in the internal energy of the system.

ΔW = Work done by the system on the surrounding.

$$\Delta W = P\Delta V$$

$$\boxed{\Delta Q = \Delta U + P\Delta V}$$

Specific Heat Of a Gas ⇒ ☆ ☆

The amount of heat required to raise the temperature of one gram of gas through 1°C is called specific heat of the gas.

★ But the specific heat of gas is not fixed. A gas can have any value of specific heat depending on the ~~Condition~~ Conditions under which it is heated.

A gas can have infinite No. of specific heat

Specific heat of Gas can have following value

→ Specific heat of gas is zero if ~~it~~ gas is compressed suddenly.

$$C = \frac{\Delta Q}{m\Delta T} \qquad \left\{ \Delta Q = 0 \right.$$

$$\boxed{C = 0}$$

⇒ Specific heat is infinite when the gas expand suddenly when heat is supplied

The heat supplied is completely used in expansion so there is no rise in temprature

$$\Delta T = 0$$

$$\boxed{C = \frac{\Delta Q}{m \times 0} = \infty}$$

⇒ Specific heat of gas is positive when heated in such way that fall in temp~~tem~~perature due to expansion is less than the rise in temprature due to heat supplied.

$$C = \frac{\Delta Q}{m\Delta T} = \text{a positive Value}$$

⇒ Specific heat of gas is negative if fall in temp due to expansion is more than rise in temp due to heat supplied.

{ Out of solid, liquid and gas, gas has highest internal energy }

Principal Specific heat of Gas

Molar specific heat at constant volume [C_V]

It is defined as the amount of heat required to raise the temp of 1 mole of gas through 1°C at constant volume.

Molar specific heat at constant pressure [C_P]

It is defined as the amount of heat required to raise the temp of 1 mole of gas through 1°c at constant pressure.

Out of C_P and C_V
$C_P > C_V$

☆ C_P is greater than C_V because in C_P heat supplied is used for two purpose i.e for work done and increasing temp {$dQ = du + PdV$}

☆ C_V is less because heat supplied is completely used for increasing internal energy {$du = dQ$}

Relationship b/w C_P and C_V { Mayer's formula }

Consider one mole of gas is heated to raise its temp by dT°c

According to first law of thermodynamics

$$dQ = dU + PdV$$

if the heat is absorbed at Constant volume then

$$dV = 0$$

$$dQ = du + PdV$$

$$dQ = du$$

Differentiating both side w.r.t temp

$$\frac{dQ}{dT} = \frac{du}{dT}$$

$$\boxed{C_V = \frac{du}{dT}} \quad \text{——} ①$$

$$\left\{ \begin{array}{l} dQ = mCdT \\ m = 1 \\ dQ = CdT \\ \frac{dQ}{dT} = C \end{array} \right\}$$

if the heat is absorbed at constant pressure Then

$$dQ = du + PdV$$

Differentiating both side w.r.t T

$$\frac{dQ}{dT} = \frac{du}{dT} + P\frac{dV}{dT}$$

$$\left\{ \frac{du}{dT} = C_V \right.$$

$$C_p = C_V + P\frac{dV}{dT}$$

$$p - C_V = P\frac{dV}{dT} \quad \text{——} ③$$

from ideal gas equation

$$PV = nRT \qquad n = 1$$

$$PV = RT$$

Differentiating both side w.r.t T keeping pressure constant

$$\frac{PdV}{dT} = \frac{RdT}{dT}$$

$$\frac{PdV}{dT} = R \quad \text{—} ④$$

from ③ and ④ equation

$$\boxed{C_p - C_V = R}$$

Thermodynamic Process

A thermodynamic process is said to occur if the thermodynamic variables of a system undergo a change with time.

Various Type Of Thermodynamic Process

(i) Isothermal Process ⇒ It occurs at constant temp. $\Delta T = 0$, $\Delta U = 0$

(ii) Isobaric process ⇒ It occurs at constant Pressure. $\Delta P = 0$

(iii) Isochoric process ⇒ It occurs at constant Volume $\Delta V = 0$

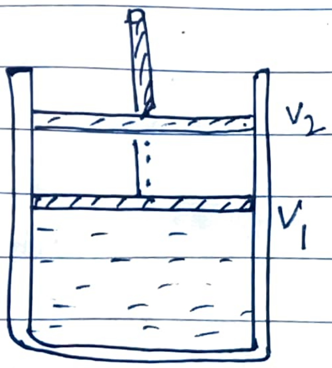
(iv) Adiabatic process ⇒ No exchange of heat b/w system and surrounding $\Delta Q = 0$

(V) **Quasi-Static process** ⇒ It is infinitely slow process such that system remains in thermal and chemical equilibrium with the surroundings.

Work done during an Isothermal expansion.

Consider n mole of an ideal gas is inclosed in cylinder having conducting walls provided with frictionless and movable piston.

work done during expansion is given by



$$dw = PdV$$

total work done in expanding from V_1 to V_2

$$\int_0^W dw_{iso} = \int_{V_1}^{V_2} PdV \quad — ①$$

Greek word ← adiabatic = a + dia + bates

$\downarrow$ Not, $\downarrow$ Through, $\downarrow$ heat

: from ideal gas equation

$$PV = nRT$$

$$P = \frac{nRT}{V} \quad -②$$

Substituting the value of ② in equation ①

$$\int_0^W dW_{iso} = \int_{V_1}^{V_2} \frac{nRT\,dV}{V}$$

$$W_{iso} = nRT \int_{V_1}^{V_2} \frac{1}{V}\,dV$$

$$W_{iso} = nRT\,[\ln V]_{V_1}^{V_2}$$

$$W_{iso} = nRT\,[\ln V_2 - \ln V_1]$$

$$W_{iso} = nRT \ln \frac{V_2}{V_1}$$

$$W_{iso} = 2.303\, nRT \log \frac{V_2}{V_1}$$

$\ln = 2.303 \log_{10}$

ADIABATIC Process

An adiabatic process ~~to~~ is one in which the pressure, volume and temperature of the system changes but there is no exchange of heat b/w the system and the surroundings.

Condition for an adiabatic process to take place.

(i) The walls of the container must be perfectly insulated.

Mass of a body is increased when it is heated

(ii) The process of compression and expansion should be sudden.

example ⇒ Propagation of sound wave in air is an example of adiabatic process

Adiabatic Relation b/w P, V and T

(i) According to First law of thermodynamics

$$dQ = du + dW$$

for one mole of gas

$$du = C_v\,dT \quad \text{and} \quad P\,dV = dw$$

and for adiabatic process $dQ = 0$

so first law of thermodynamics become

$$0 = C_v\,dT + P\,dV \qquad \text{——} ①$$

from ideal gas equation

$$PV = RT \qquad \{n = 1$$

$$P\frac{dV}{\cancel{dT}} + V\frac{dP}{\cancel{dT}} = R\frac{dT}{\cancel{dT}} \qquad \{\text{differentiating both side w.r.t } T$$

$$P\,dV + V\,dP = R\,dT$$

$$dT = \frac{P\,dV + V\,dP}{R} \qquad \text{——} ②$$

from ① and ② equation

$$C_v\left[\frac{P\,dV + V\,dP}{R}\right] + P\,dV = 0$$

{ $PV = nRT$ is valid for all the process }

$$C_v . PdV + C_v VdP + RPdV = 0$$

$$(C_v + R) PdV + C_v VdP = 0$$

$$C_p PdV + C_v VdP = 0$$

{ $C_p - C_v = R$
$C_p = C_v + R$ }

Dividing both side by $C_v PV$

$$\gamma \frac{dV}{V} + \frac{dP}{P} = 0$$

{ $\frac{C_p}{C_v} = \gamma$ }

integerating both side

$$\gamma \int \frac{dV}{V} + \int \frac{dP}{P} = 0$$

$$\gamma \log_e V + \log_e P = C$$

{ $m \log n = \log n^m$ }

$$\log_e V^\gamma + \log_e P = C$$

$$\log_e PV^\gamma = C$$

taking exponential on both side

$$PV^\gamma = e^C$$

$$\boxed{PV^\gamma = K}$$

{ $K = e^C$ }

(iii) Adiabatic Relation b/w P and T

$$PV = RT$$

$$V = \frac{RT}{P} \quad — ①$$

$$PV^\gamma = K \quad — ②$$

from ① and ②

$$P\left(\frac{RT}{P}\right)^{\gamma} = K$$

$$P^{1-\gamma} T^{\gamma} = \frac{K}{R^{\gamma}}$$

$$\therefore \boxed{P^{1-\gamma} T^{\gamma} = \text{constant}}$$

$$P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$$

Adiabatic Relation b/w V and T

$$PV = RT$$

$$P = \frac{RT}{V} \quad \text{—} ①$$

$$PV^{\gamma} = K \quad \text{—} ②$$

from ① and ②

$$\frac{RT}{V} V^{\gamma} = K$$

$$RT\,V^{\gamma-1} = K$$

$$T V^{\gamma-1} = \frac{K}{R}$$

$$\frac{K}{R} = \frac{\text{Constant}}{\text{Constant}} = \text{const}$$

$$T V^{\gamma-1} = \text{Constant}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

Work done During Adiabatic Expansion

$$W_{adia} = \int_{V_1}^{V_2} P\,dV$$

for an adiabatic process

$$PV^{\gamma} = K$$

$$P = \frac{K}{V^{\gamma}} = KV^{-\gamma}$$

$$W_{adia} = \int_{V_1}^{V_2} KV^{-\gamma}\,dV$$

$$W_{adia} = K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$W_{adia} = \frac{K}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} = K$$

$$W_{adia} = \frac{1}{1-\gamma}\left[KV_2^{1-\gamma} - KV_1^{1-\gamma}\right]$$

$$\frac{1}{1-\gamma}\left[P_2V_2^{\gamma}V_2^{1-\gamma} - P_1V_1^{\gamma}V_1^{1-\gamma}\right]$$

$$W_{adia} = \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right]$$

$$\left\{\begin{array}{l} P_1V_1 = nRT_1 \\ P_2V_2 = nRT_2 \end{array}\right\}$$

$$W_{adia} = \frac{1}{1-\gamma}\left[nRT_2 - nRT_1\right]$$

$$W_{adia} = \frac{nR}{1-\gamma}\left[T_2 - T_1\right]$$

$$\boxed{W_{adia} = \frac{nR}{\gamma-1}\left[T_1 - T_2\right]}$$

Application Of First law Of Thermodynamic

(i) To Isothermal process

$dQ = du + PdV$

for isothermal $du = 0$

$\boxed{dQ = PdV}$

(ii) To Isochoric process

$dQ = du + Pdv$

$dv = 0$

$\boxed{dQ = du}$

(iii) To Isobaric process

$dQ = du + PdV$

(iv) For Boiling

$dQ = du + PdV$

$dQ = MLv$

$dV = V_2 - V_1$

$MLv = du + P(V_2 - V_1)$

$du = U_2 - U_1$

$\boxed{MLv = (U_2 - U_1) + P(V_2 - V_1)}$

(v) For Melting process

$dQ = du + PdV$

in Melting $dV = 0$

$dQ = mL_f$ $\quad du = U_2 - U_1$

$\boxed{ML_f = (U_2 - U_1)}$ $\quad \boxed{Pdv = 0}$

(vi) For adiabatic

$dQ = du + PdV$

for adiabatic $dQ = 0$

$\boxed{-du = PdV}$

(Efficiency of heat engine is more in hilly area then plains)

Heat ENGINE

It is a device which converts continuously heat energy into mechanical energy in a cyclic process.

Construction Of heat Engine ⇒ It consist of following part

(i) **Source** (T_1) ⇒ It is a heat reservoir from which if we remove any amount of heat its temperature will remain same. It is always at higher temp.

(ii) **Sink** (T_2) ⇒ It is a heat reservoir at low temp. If we reject any amount of heat in it its temprature will remain same.

(iii) **Working Substance** ⇒ Working substance is any material which performs mechanical work when heat is supplied to it.

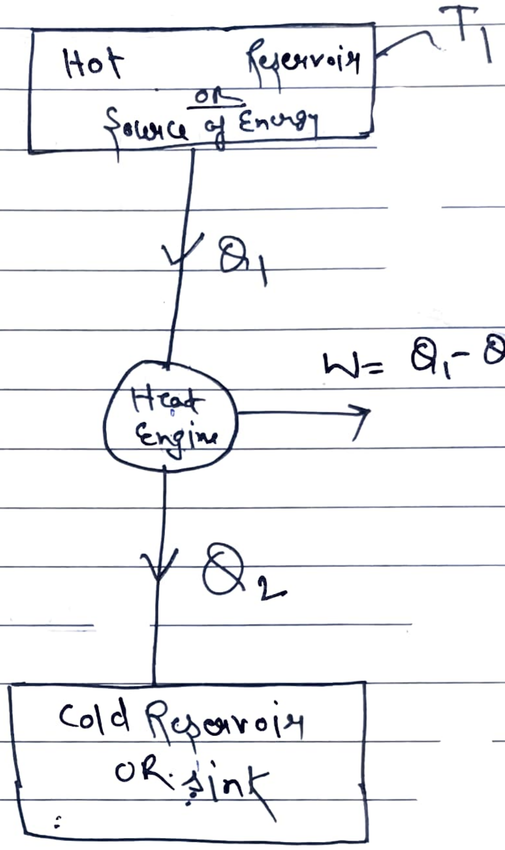
Ex → Mixture of ~~p~~ air and diesel.

Working Of heat Engine ⇒ In every cycle the working substance absorbes heat (Q_1) from the source and converts it into mechanical work (W) and reject heat (Q_2) into the sink

Efficiency Of a heat Engine ⇒ It is defined as the ratio of net work done by the engine to the heat supplied by the ~~engine~~ source.

$\eta_{\text{petrol engine}} = 26\%$ $\eta_{\text{diesel engine}} = 40\%$ $\eta_{\text{steam engine}} = 12 \text{ to } 16\%$

$$\eta = \frac{\text{Work done by Engine}}{\text{Heat absorbed from the source}}$$



$$\eta = \frac{W}{Q_1}$$

$$W = Q_1 - Q_2$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\boxed{\eta = 1 - \frac{Q_2}{Q_1}}$$

Efficiancy of Engine Can not be 1 or 100% because Q_2 can never be zero

Types Of Heat Engine

External Combustion Engine (EC)	Internal Combustion Engine (IC)
Heat Energy needed in this is produced out side the cylinder by burning fuel	Heat energy required by engine is produced inside the cylinder by burning the fuel inside it
ex Steam Engine	Ex → Diesel engine OR petrol Engine

Limitations Of First Law Of Thermodynamics

(i) It does not indicate the direction Of transfer Of heat
(ii) It does not tell any thing about condition under which heat can be converted into mechanical energy
(iii) It does not indicate the extent to which heat energy can be converted into mechanical work continuously.

SECOND LAW OF THERMODYNAMICS

(i) Kelvin-Planck Statement ⇒ It is impossible to construct an engine whose sole aim is to convert heat extracted from a reservoir into work without any other effect.

(ii) Clausius Statement ⇒ According to Clausius it is impossible to extract heat from low temprature body and reject it to a high temprature body without any external work.

Limitation Of Second law Of Thermodynamics.

(i) It can not be proved directly.
(ii) It is only applicable to cyclic process.
(iii) It makes no predictions as to what will happen under certain conditions ~~the~~ ~~to~~ but simply state what will happen under given condition.

Reversible Process ⇒ A process is said to be reversible if it can return back to its initial state without leaving any impact on the surrounding.

Necessary Condition for Reversible process

(i) The process must be quasi-static
(ii) The dissipative forces such as viscosity, friction, inelasticity etc. should be absent.

Irreversible process ⇒ Any process which can not return to its initial state is called irreversible process.

Example ⇒ (i) Rusting Of Iron
(ii) Sudden expansion or contraction of a gas.

Importance Of Reversibility ⇒ Thermodynamics deals with efficiency of heat engine. And according to second law of thermodynamics no engine can have efficiency of 100%.

But the engine based upon reversibility have highest efficiency as compared to engines developed upon irreversibility concept.

CARNOT ENGINE

It is an ideal reversible engine that operates b/w two temprature T_1 (source) and T_2 (sink)

It was conceived by French Engineer Sadi Carnot.

Construction ⇒ Carnot heat engine has following main parts

(i) **Cylinder** ⇒ It is main part of the engine and it has conducting base and insulated walls.

(ii) **Source** ⇒ It is a heat reservoir at higher temprature T_1 from which the engine draws heat. Its temprature remain constant even if we remove any amount of heat from it.

(iii) **Sink** ⇒ It is a heat reservoir at lower temprature T_2. to which if we reject any amount of heat the temprature will remain same.

(iv) **Working Substance** ⇒ The working substance is an ideal gas contained in the cylinder.

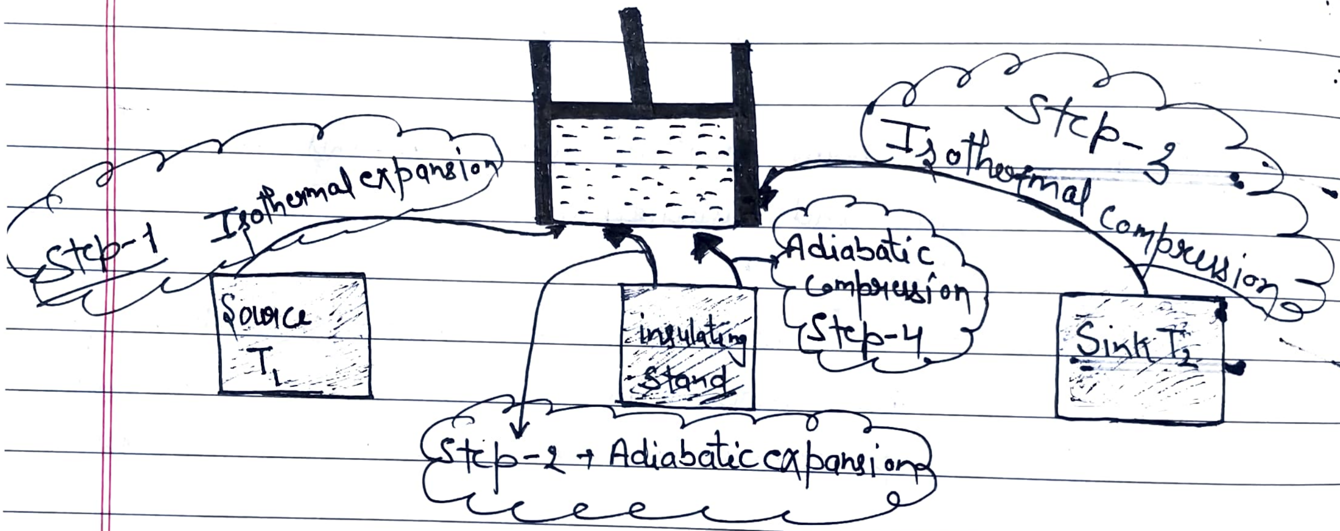
(v) **Insulating Stand** ⇒ It is used to isolate the cylinder base from surrounding

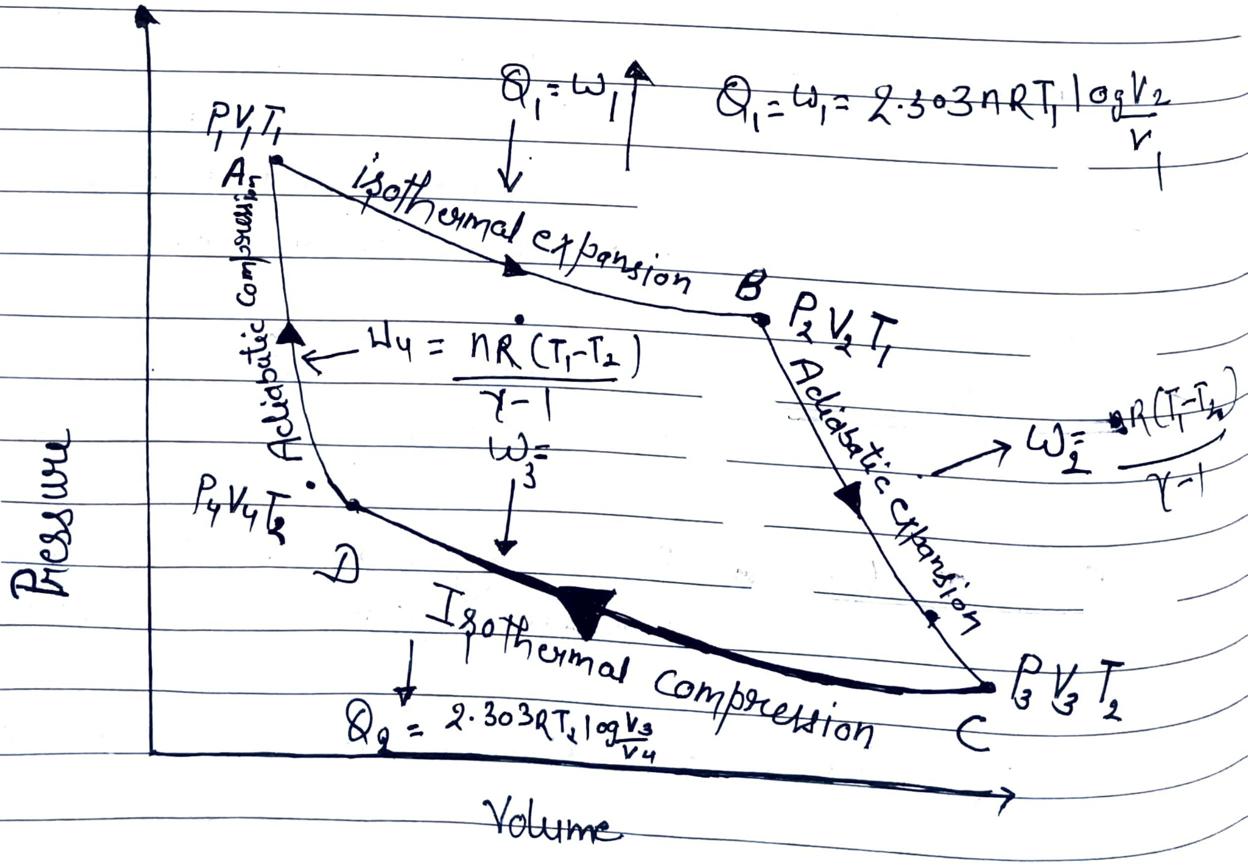
CARNOT Cycle ⇒ The working substance is carried through a reversible cycle of following four steps:-

Step-1 Isothermal Expansion (A – B) ⇒ In this step the source is brought in contact with base of cylinder and heat is supplied to the and piston moves outwords and gas expands without any increase in temprature

Heat absorbed by the cylinder

$W_1 = Q_1 = 2.303\,nRT_1 \log \frac{V_2}{V_1}$





Step-2 Adiabatic Expansion (B-C) ⇒ In this step force is removed and insulating stand is connected to the base of the cylinder and the gas expands using its own internal energy due to which its temprature decreases to T_2.

Work done by gas in expansion

$$W_2 = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

Step-3 Isothermal Compression [C-D] ⇒ Now insulating stand is removed and sink is connected to the base of the cylinder. The piston moves inwards and gas is compressed. The extra heat generated in the cylinder due to compression is rejected to the sink and temprature of the cylinder remains constant.

$$\text{Heat rejected to sink} = W_3 = Q_2 = 2.303\, nRT_2 \log\left(\frac{V_3}{V_4}\right)$$

Step-4 ⇒ Adiabatic Compression [D-A] ⇒ Now in this step Insulating stand is attached to the base of the cylinder and piston moves inwards, due to this gas is compressed and internal energy increases and temprature rises to T_1

$$\text{Work done during this process} = W_4 = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

Net work done by gas = $W_1 + W_2$ {during steps 1 and step-2

Net work done on gas = $W_3 + W_4$ {During step 3 and 4

Net work done by the gas in one complete cycle

$$W_{net} = W_1 + W_2 - (W_3 + W_4)$$

{ Since work done by the system is +ve

{ and on the system is negative

$$W_2 = W_4$$

$$\boxed{W_{net} = W_1 - W_3} = Q_1 - Q_2$$

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta = \frac{1 - 2.30\, nRT_2 \log \frac{V_3}{V_4}}{1 - 2.30\, nRT_1 \log \frac{V_2}{V_1}}$$

☆

Now for adiabatic process in step-2

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad \text{— ①}$$

For adiabatic process in step-4

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{— ②}$$

from equation ① and ②

$$\left(\frac{V_3}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1}$$

$$\frac{V_3}{V_4} = \frac{V_2}{V_1} \quad — ③$$

Putting value of ③ in ☆ equation

$$\eta = 1 - \frac{2.303\, nRT_2 \log\left(\frac{V_2}{V_1}\right)}{2.303\, nRT_1 \log\left(\frac{V_2}{V_1}\right)}$$

$$\boxed{\eta = 1 - \frac{T_2}{T_1}}$$

Important point Regarding Carnot Engine =)

~~(i)~~ Efficiency of Carnot Engine can not be 100% because for that $T_1 = \infty$ $T_2 = 0$ but this temp is not possible

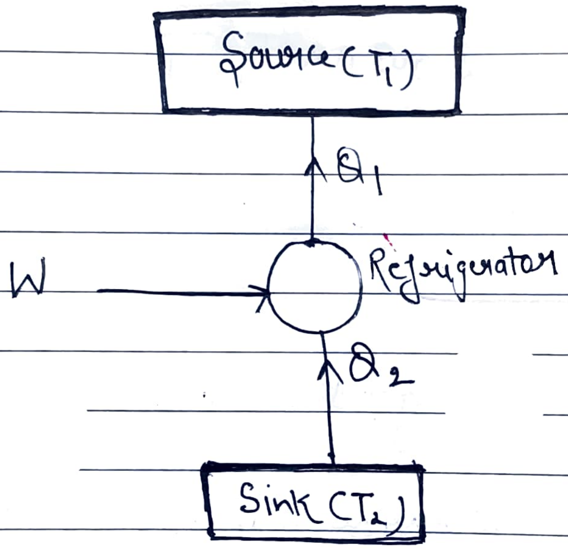
~~(ii)~~ If $T_1 = T_2$ $\eta = 0$ so in order to design a engine source and sink must have different temp.

~~(iii)~~ $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ since $T_2 = 0$ is not possible in the same way $Q_2 = 0$ is also not possible so it shows 100% conversion energy into work is not possible.

REFRIGERATOR

It is a carnot' heat engine working in reverse direction

Working ⇒ In refrigerator the working substance absorbs Q_2 heat from sink (T_2) by doing W amount of work and rejects Q_1 heat to the source at temp T_1



$$Q_1 = W + Q_2$$

$$W = Q_1 - Q_2$$

- ☆ In domestic refrigerator food and ice makes sink and surrounding acts as source.
- ☆ Work is done by electric motor.
- ☆ Freon [CCl_2F_2] is used as working substance.

In Refrigerator the working substance is carried through following cycle of four step.

(i) The gas is allowed to expand suddenly (adiabatically) from high to low pressure. Due to working substance converts into liquid-vapour mixture.

(ii) This cold fluid is allowed to absorb heat Q_2 isothermally and due to which liquid-vapour mixture completely converts into vapour.

(iii) Now the vapour is compressed adiabatically till heats up to the temprature of surroundings.

(iv) Finally the vapour is compressed isothermally in contact with the surrounding and gets converted into fluid.

Coefficient Of performance (β) = It is defined as the ratio heat removed per cycle to the mechanical work required to do it

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\beta = \frac{Q_2}{Q_2\left(\frac{Q}{Q_2} - 1\right)}$$

$$\beta = \frac{1}{\frac{Q_1}{Q_2} - 1} \Rightarrow \beta = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$\boxed{\beta = \frac{T_2}{T_1 - T_2}}$$