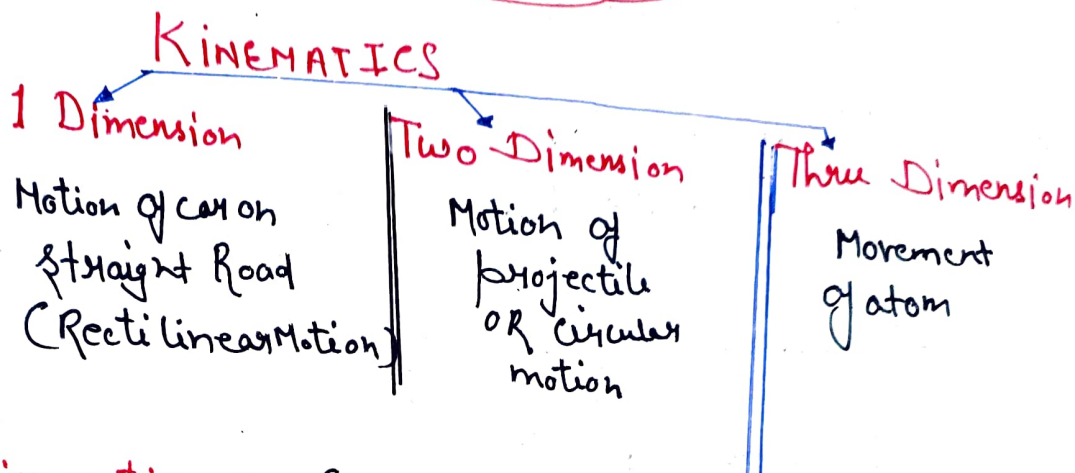
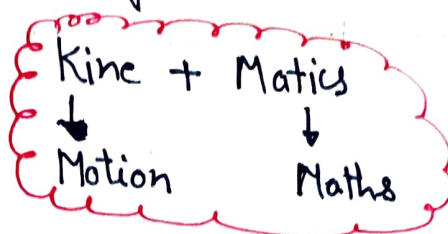


# KINEMATICS

(1)

The branch of Mechanics that deals with the object in motion without considering the cause of motion.



Variable of Kinematics  $\Rightarrow$  (I) Distance (II) Displacement (iii) Speed  
(iv) Velocity (v) Time (vi) Acceleration (vii) Jerk

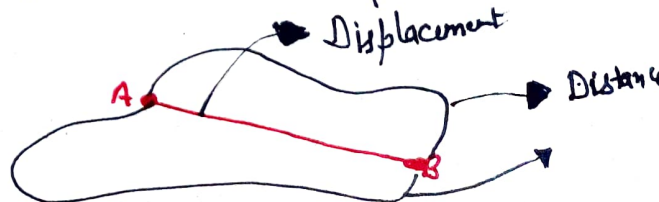
## Distance

- \* It is the actual path length covered by body from initial point to the final point
- \* It is a scalar quantity
- \* It can be zero or positive
- \* Distance of a body can never predict the direction of motion
- \* Its SI unit will be in meter (m)

$\text{Distance} \geq \text{displacement}$

## Displacement

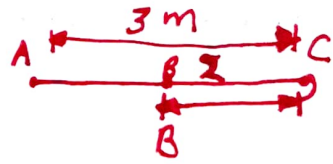
- \* It is the shortest path length from initial to the final point
- \* It is a vector quantity
- \* It can be positive, zero or negative
- \* It can also not predict the direction of motion.
- \* Its SI unit is also meter.



## Question Based Upon Displacement / Distance (2)

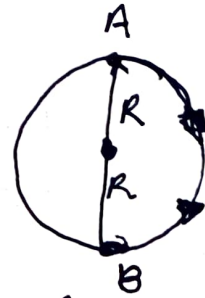
Q → Find the value of distance and displacement when a body is going from point A to B

(I) Body goes from A to C and returns back to B



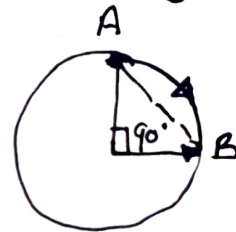
Sol Distance =  $AC + BC = 3 + 2 = 5 \text{ m}$   
 Displacement =  $AB = 1 \text{ m}$

(II) Distance =  $\pi R$   
 Displacement =  $2R$



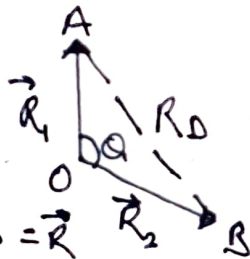
(III) Distance =  $\frac{2\pi R}{4} = \frac{\pi R}{2}$

Displacement =



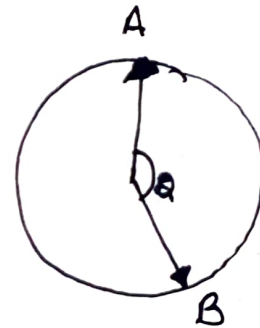
Displacement =  $\sqrt{2} R$

(III) Displacement -



Displacement =  $AB = \vec{R}$

using  $\Delta$  law of vector addition



Distance =  $\frac{\theta}{360} \times 2\pi R$

$1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}$

$$\vec{AO} + \vec{OB} = \vec{AB}$$

$$\vec{AB} = \vec{OB} - \vec{AO} = \vec{R}_2 - \vec{R}_1$$

$$\vec{R}_0 = \vec{R}_2 - \vec{R}_1$$

$$|\vec{R}_0| = \sqrt{R_1^2 + R_2^2 - 2R_1R_2 \cos \theta}$$

$$R_1 = R_2 = R$$

$$R_0 = \sqrt{R^2 + R^2 - 2R^2 \cos \theta}$$

$$R_0 = \sqrt{2R^2 + 2R^2 \cos \theta}$$

$$R_0 = \sqrt{2R^2(1 + \cos \theta)}$$

$$R_0 = \sqrt{2R^2 \cdot 2 \sin^2 \frac{\theta}{2}}$$

$R_0 = 2R \sin \frac{\theta}{2}$

Displacement

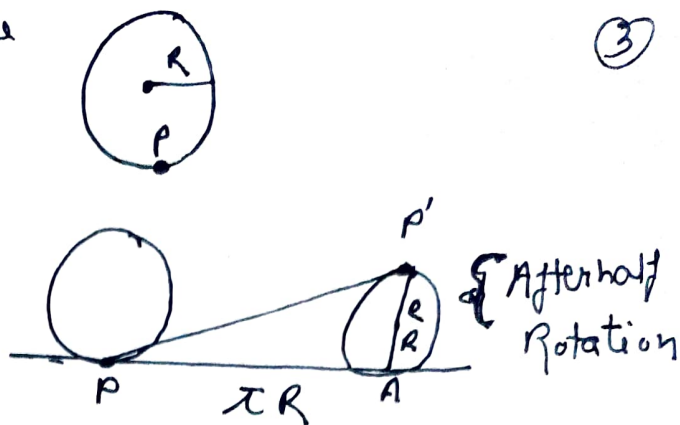
(IV) What is displacement of particle after half rotation

(3)

Sol In  $\Delta PP'A$

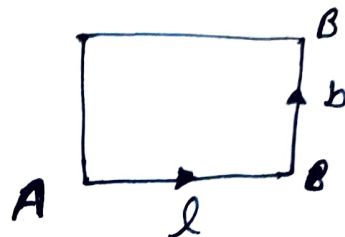
$$PP' = \sqrt{(AR)^2 + (QR)^2}$$

$$PP' = R\sqrt{\pi^2 + 4}$$



(V) Distance =  $l + b$

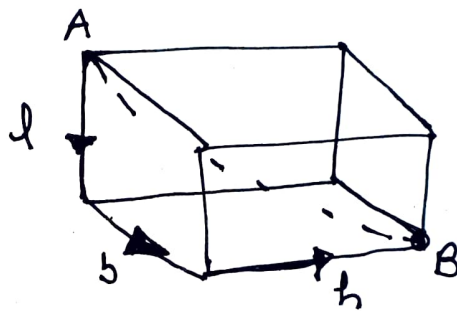
$$\text{Displacement} = \sqrt{l^2 + b^2}$$



(VI) Distance =  $l + b + h$

$$\text{Displacement} = \sqrt{l^2 + b^2 + h^2}$$

(Body diagonal of the cuboid)



# Speed

Rate of change of distance is called speed.

It is scalar quantity

Its SI unit is m/s

Speed  $\geq$  Velocity

## \* Speed

→ Uniform Speed

→ Non Uniform speed

→ Average Speed  $\Rightarrow$  It is the ratio of total distance travelled by body to the total time taken by the body.

$$\text{Average Speed} \Rightarrow \frac{\text{Total distance}}{\text{total time}}$$

$$\text{Average Speed} = \frac{\Delta S}{\Delta T}$$

It is generally for large time interval

→ Instantaneous Speed  $\Rightarrow$  It is the speed of a body at any instant of time.

$$\text{Instantaneous speed} = \frac{dS}{dt}$$

$$\frac{dS}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t}$$

## Velocity $\Rightarrow$

→ Average Velocity  $\Rightarrow$  It is the total displacement by total time.

$$\text{Average Velocity} = \frac{\text{total displacement}}{\text{total time}}$$

# Velocity

(5)

Rate of change of displacement is called velocity.

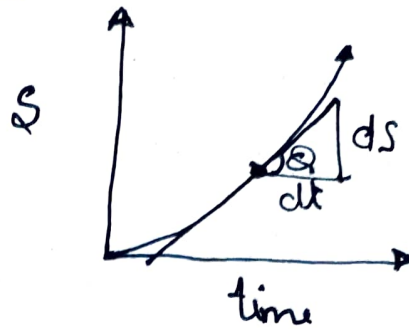
It is vector quantity

Its SI unit is m/s

Instantaneous Velocity  $\Rightarrow$  It is the velocity of the body at any instant of time.

Instantaneous Velocity =  $\frac{ds}{dt}$

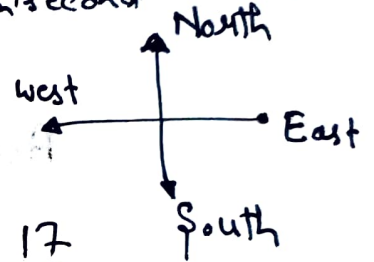
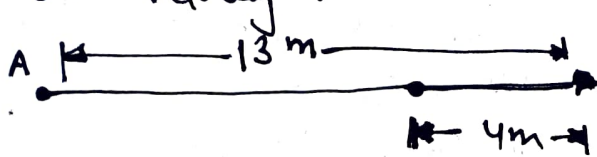
Slope of Displacement time graph gives instantaneous velocity.



Numerical Based Upon Average Speed/Velocity

Q  $\rightarrow$  A boy walks 13 m east then 4 m west. Find his average speed and velocity.   
In 6 sec      4 sec

Sol



① Average Speed =  $\frac{\text{Total distance}}{\text{Total time}} = \frac{13+4}{6+4} = \frac{17}{10}$

② Average Velocity =  $\frac{9}{10} = 0.9 \text{ m/s}$

1.7 m/s

we can see Average speed > Average velocity

Q  $\rightarrow$  A man goes from A to B at  $v_1$  speed and returns from B to A at  $v_2$  speed. Calculate Average speed and velocity

Displacement = 0

Average velocity = 0

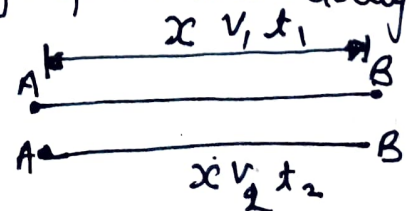
Distance =  $x + x = 2x$

total time =  $t_1 + t_2$

Distance = speed  $\times$  time

$x = v_1 \times t_1$

$t_1 = \frac{x}{v_1}$        $t_2 = \frac{x}{v_2}$



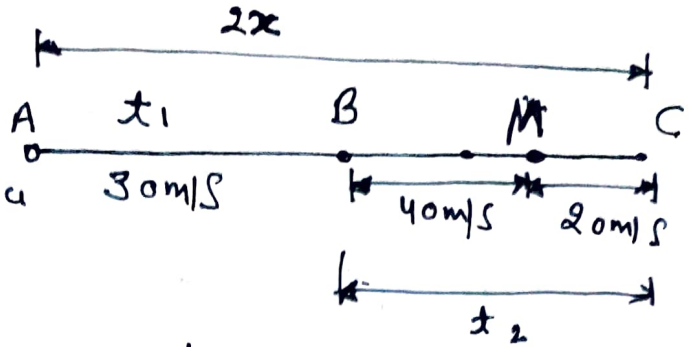
Average speed =  $\frac{T.D}{T.T} = \frac{x+x}{t_1+t_2}$

A.S =  $\frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}} = \frac{2x}{x(\frac{1}{v_1} + \frac{1}{v_2})} = \frac{2v_1 v_2}{v_1 + v_2}$

Q → A man covers half distance with speed 30 m/s and the second half distance for 40 m/s for half time 20 m/s for second half time. (2)

Sol

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{time}}$$



$$\text{Total distance} = 2x$$

$$t_1 = \frac{x}{30}$$

$$t_2 = \frac{x}{40 + 20}$$

$$A.S = \frac{2x}{\frac{x}{30} + \frac{x}{60}} = \frac{2x}{x(\frac{1}{30} + \frac{1}{60})}$$

$$A.S = \frac{2x}{x(\frac{1}{30} + \frac{1}{60})}$$

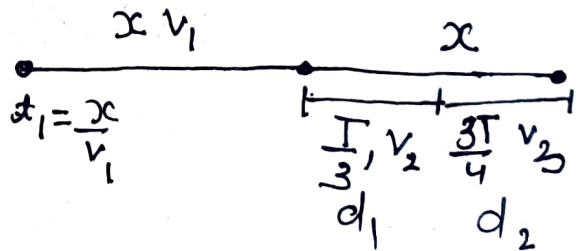
$$A.S = \frac{2 \times 30 \times 60}{30 + 60} = \frac{360}{90} = 40$$

A.S = 40 m/s

Q → A bus travels first half of the distance with speed  $v_1$  and for the next half it travels with speed  $v_2$  for one third of the time and  $v_3$  for the remaining time. Calculate average speed.

Sol

$$\text{Average speed} = \frac{2x}{t_1 + t}$$



$$t_1 = \frac{x}{v_1}$$

$$x = d_1 + d_2$$

$$x = \frac{T v_2}{3} + \frac{3T v_3}{4}$$

$$x = T \left( \frac{v_2}{3} + \frac{3v_3}{4} \right) \Rightarrow x = T \left( \frac{4v_2 + 9v_3}{12} \right)$$

$$A.S = \frac{2x}{\frac{x}{v_1} + \frac{12x}{4v_2 + 9v_3}}$$

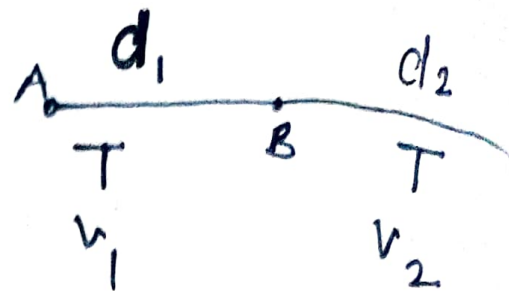
$$T = \frac{12x}{4v_2 + 9v_3}$$

A.S =  $\frac{2v_1(4v_2 + 9v_3)}{4v_2 + 9v_3 + v_1}$

Q → A man travels a distance with speed  $v_1$  and  $v_2$  two equal time periods. Find average speed.

Sol

$$\text{Average speed} = \frac{d_1 + d_2}{T + T}$$



$$d_1 = v_1 T \quad d_2 = v_2 T$$

$$AS = \frac{v_1 T + v_2 T}{2T} = \frac{v_1 + v_2}{2}$$

### NUMERICAL ON INSTANTANEOUS VELOCITY

Q →

$S = t^3 - 6t^2$  find velocity at  $t = 1$  s and 3 sec.

Sol

$$S = t^3 - 6t^2$$

$$\frac{dS}{dt} = v = 3t^2 - 12t$$

$$V = 3t^2 - 12t$$

$$V(1) = 3(1)^2 - 12(1) = -9 \text{ m/s}$$

$$V(3) = 3(3)^2 - 12(3) = -6 \text{ m/s}$$

Q →

Find the time when the body was at rest if the position of the body was given by  $s = t^3 + 6t^2 + 10$

$$s = t^3 + 6t^2 + 10$$

$$\frac{ds}{dt} = v = 3t^2 + 12t + 0$$

Since we have to find out time when the body was at rest that is time when  $v = 0$

$$0 = 3t^2 + 12t + 0$$

$$3t^2 + 12t = 0$$

$$3t^2 = -12t$$

$$t = \dots$$

$$t = \dots$$

Q →  $3t^2 + 4t + 9$  Find velocity at  $t = 2$  second. And average velocity b/w 1sec to 4sec.

Sol  $V = \frac{ds}{dt} = 6t + 4$

$V(2) = (6 \times 2 + 4) = 12 + 4 = 16 \text{ m/s}$

$S = 3t^2 + 4t + 9$

$S(1) = 3(1)^2 + 4(1) + 9 = 16$

$S(4) = 3(4)^2 + 4 \times 4 + 9 = 73$

Net disp =  $73 - 16 = 19 \text{ m}$

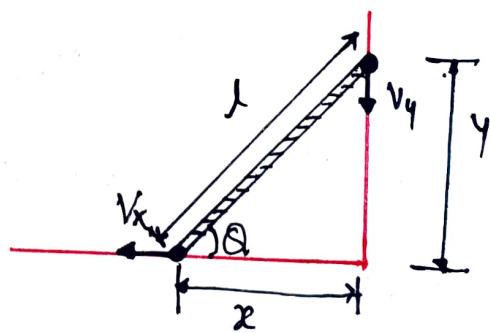
total time =  $4 - 1 = 3 \text{ sec}$

Var =  $\frac{\text{total disp}}{\text{total time}} = \frac{19}{3} = 6.33 \text{ m/s}$

Average velocity b/w 1 and 4 sec  
 $V(1) = 6 \times 1 + 4 = 10 \text{ m/s}$   
 $V(4) = 6 \times 4 + 4 = 28 \text{ m/s}$   
 $\text{Var} = \frac{V(1) + V(4)}{2} =$   
 Do not use this method for calculating average ~~speed~~ velocity.

Q → A ladder is placed along a wall. The wall and the floor are frictionless. If the bottom part of the ladder is sliding with speed  $V$  then Calculate the speed of the top of the ladder at angle  $\theta$ .

Sol  $V_x = \frac{dx}{dt}$   
 $V_y = -\frac{dy}{dt}$  since  $y$  is decreasing



Now since the length of ladder will remain constant so

using Pythagorean theorem

$x^2 + y^2 = l^2$

Differentiating both side w.r.t  $t$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$2x V_x - 2y V_y = 0$

$2x V_x = 2y V_y$

$V_y = \frac{x}{y} V_x$

$\cot \theta = \frac{x}{y}$

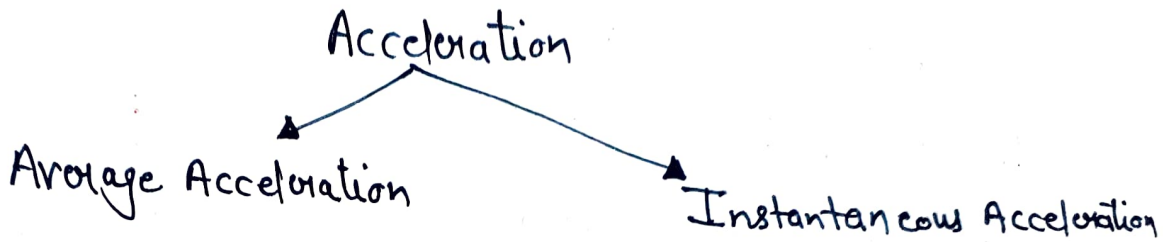
$V_y = V_x \cot \theta$

# Acceleration

Acceleration  $\Rightarrow$  Rate of change of velocity is known as acceleration.

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{m/s}}{\text{s}} = \text{m/s}^2 \quad \text{SI unit of acceleration.}$$

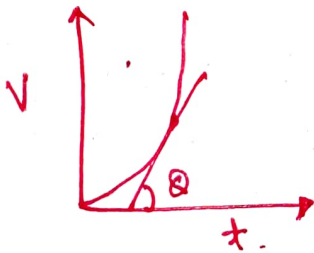
Acceleration is a vector quantity



$$a_{\text{av}} = \frac{\Delta v}{\Delta t}$$

$$a_{\text{in}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

$$a_{\text{in}} = \frac{dv}{dt}$$



Slope of v-t graph gives acceleration

## NUMERICAL

Q  $\rightarrow v = 2t^2 - 4t$  Find acceleration at  $t = 3\text{ sec}$ . Also find average b/w 1 to 3 sec.

Sol  $a = \frac{dv}{dt} = 4t - 4$

$$a(3) = 4(3) - 4 = 8 \text{ m/s}^2$$

$$a(3) = 8 \text{ m/s}^2$$

$$a(1) = 4(1) - 4 = 0$$

For average acceleration

$$\Delta a = \frac{\Delta v}{\Delta t} = \frac{v(3) - v(1)}{3 - 1}$$

$$\Delta a = \frac{8 - 0}{2}$$

$$\Delta a = 4 \text{ m/s}^2$$

Q Identify type of Motion why

(I)  $x = 4t^2 + 6t$

Sol  
Body is moving with changing velocity but having constant ~~zero~~ acceleration  
quadratic equation

(II)  $x = 4t^3 + 6t^2 + 11t$

Body will move with ~~zero~~ variable velocity with ~~zero~~ acceleration changing ~~into~~ with time  
Cubic equation

(III)  $x = 6t + 11$

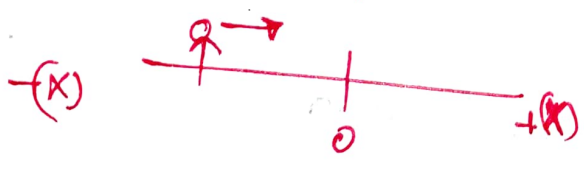
Moves with constant velocity and having acceleration zero  
Linear equation.

☆☆ DIRECTION OF MOTION

We can not determine the direction of motion by displacement and acceleration. It can be only determined by Velocity

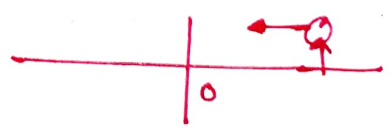
Velocity = +ve

Body will be moving in positive x direction



Velocity (-ve)

Negative x direction



Q →  $s = t^3 - 6t^2$  . Find acceleration when body is at rest? (10)

Sol  $v = \frac{ds}{dt} = 3t^2 - 12t$   
 when body is at rest then  $v = 0$   
 $0 = 3t^2 - 12t$   
 $3t^2 = 12t$   
 $t = 4 \text{ sec}$

acceleration  
 $a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 12t)$   
 $a = 6t - 12$   
 $a_4 = 6 \times 4 - 12$   
 $a_4 = 12 \text{ m/s}^2$   
 ∴ acceleration  $12 \text{ m/s}^2$  when body was at rest.

Q →  $x = 3t \hat{i} + 4t \hat{j}$  . Find  $v$  at  $t = 1 \text{ s}$   
 $a$  at  $t = 1 \text{ s}$   
 speed at  $t = 1 \text{ s}$

Sol  $s_x = 3t$        $s_y = 4t$   
 $\frac{dx}{dt} = \frac{d(s_x)}{dt} = v = 3$        $\frac{ds_y}{dt} = 4$   
 $v_x = 3$        $v_y = 4$

$v = 3\hat{i} + 4\hat{j}$

∴ velocity at  $t = 1$

$\vec{v} = 3\hat{i} + 4\hat{j}$

speed =  $|\vec{v}| = \sqrt{3^2 + 4^2}$   
 $|\vec{v}| = \sqrt{9 + 16}$   
 $|\vec{v}| = 5$

Acceleration

$a = \frac{dv}{dt} = 0$

$a = 0$

Since velocity was constant acceleration will be zero.

Q →  $\vec{s} = t^2 \hat{i} + (3t - 1) \hat{j}$

• Find  $v$  and speed at  $t = 1 \text{ s}$

Sol  $\frac{d\vec{s}}{dt} = \vec{v} = 2t \hat{i} + 3 \hat{j}$

$\vec{v}(1) = 2\hat{i} + 3\hat{j}$

speed =  $|\vec{v}| = \sqrt{2^2 + 3^2}$   
 $= \sqrt{4 + 9}$   
 $|\vec{v}| = \sqrt{13}$

acceleration

$\frac{d\vec{v}}{dt} = 0$

$a = 0$

Q → If  $S = t^3 - 6t^2 + 9t$  Find velocity and acceleration at  $t = 1$  sec. And also find out the direction of motion at  $t = 2$  sec. (13)

Sol

$$S = t^3 - 6t^2 + 9t$$

$$V = \frac{dS}{dt} = 3t^2 - 12t + 9$$

$$a = \frac{dV}{dt} = 6t - 6$$

~~$$V(1) = 3(1)^2 - 12(1) + 9 = 0$$~~

$$V(2) = 3(2)^2 - 12(2) + 9$$

$$V(2) = -3 \text{ m/s}$$

Body was moving in negative direction.

$$a(2) = 6(2) - 6$$

$$a(2) = 6$$

Direction of body can not be determined by acceleration

Q → Find the time when the body was moving in negative direction and when the body was at rest.  $S = t^3 - 6t^2 + 9t$

Sol

$$V = \frac{dS}{dt} = 3t^2 - 12t + 9$$

For body to be at rest

$$V = 0$$

~~$$3t^2 - 12t + 9 = 0$$~~

$$3t^2 - 12t + 9 = 0$$

$$t^2 - 4t + 3 = 0$$

$$t^2 - 3t - t + 3 = 0$$

$$t(t-3) - 1(t-3) = 0$$

$$(t-1)(t-3) = 0$$

So body will be at

rest when  $t = 1$  sec

$t = 3$  sec

For body to move in negative direction

$$V < 0$$

$$3t^2 - 12t + 9 < 0$$

$$(t-1)(t-3) < 0$$

From above we can see that

$$1 < t < 3$$

So velocity to be negative when time is more than 1 sec and less than 3 sec

## Displacement of a body in $n^{\text{th}}$ second.

(14)

Q → If  $S = 4t^2 + 2t + 1$  then find the displacement covered by body in 3<sup>rd</sup> second

Sol The displacement in third second means the displacement covered by body b/w 2<sup>nd</sup> second & 3<sup>rd</sup> second.

$$S(3) = 4(3)^2 + 2(3) + 1$$

$$S(3) = 36 + 6 + 1$$

$$S(3) = 43$$

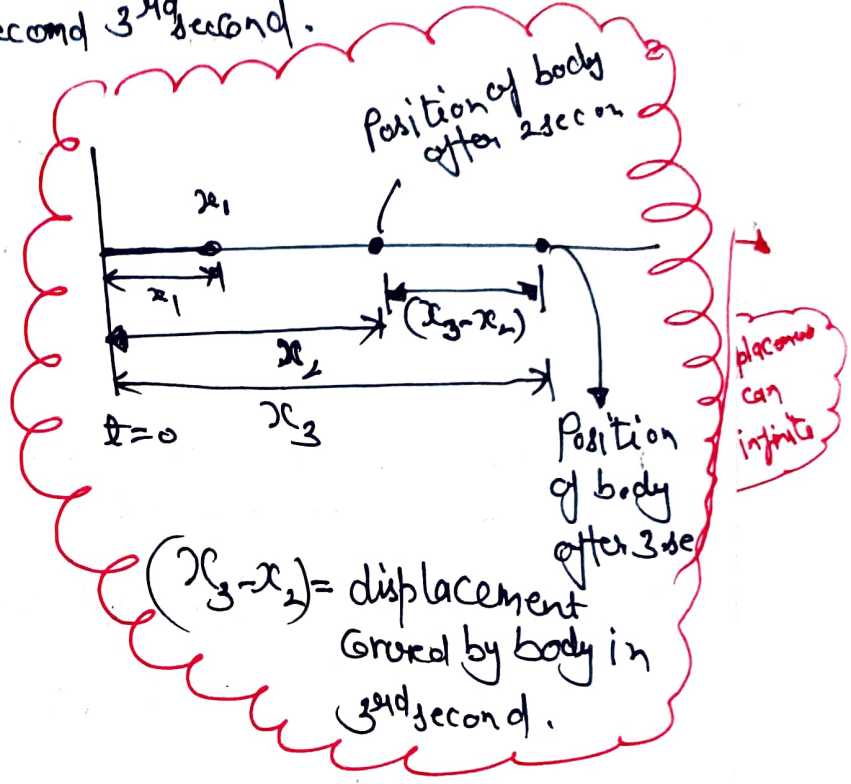
$$S(2) = 4(2)^2 + 2(2) + 1$$

$$S(2) = 16 + 4 + 1$$

$$S(2) = 21 \text{ m}$$

$$S_{3\text{rd}} = S_3 - S_2 = 43 - 21$$

$$S_{3\text{rd}} = 22$$



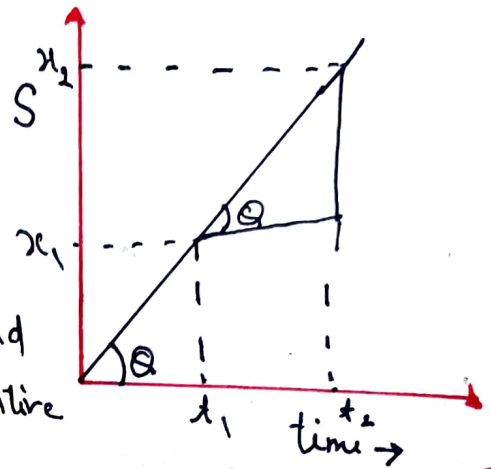
# GRAPH

## I Displacement v/s time

Slope of the displacement time graph gives velocity

\* slope =  $\tan \theta = m = \frac{x_2 - x_1}{t_2 - t_1}$

\* If slope is +ve velocity will be positive and if it is negative then velocity will be negative

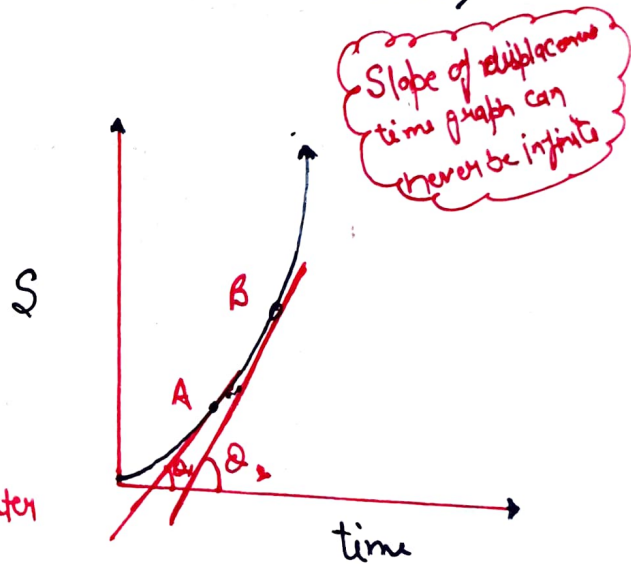


II \* This graph represent non uniform motion

\* Velocity at different point will be different since slope is different at different point.

Since  $\theta_1 < \theta_2$

∴ velocity at point B is greater than velocity at point A

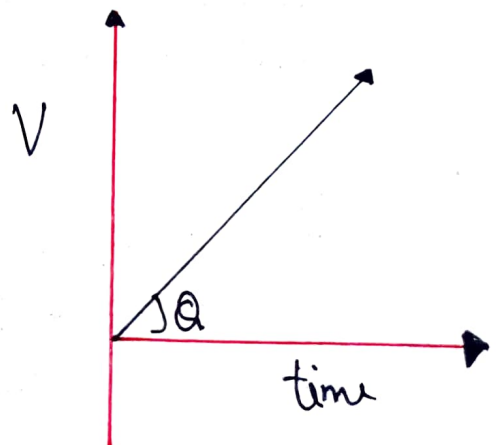


## Velocity v/s time

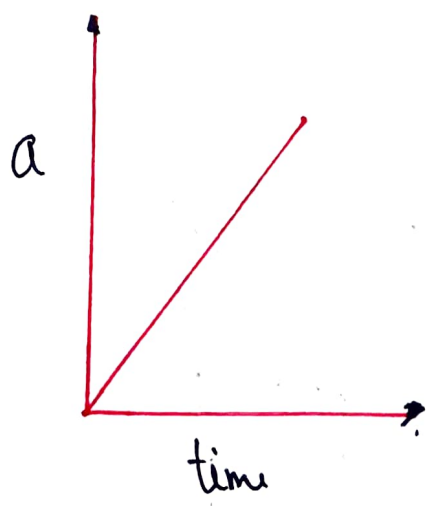
1 The slope of velocity time graph will give acceleration

2 Area under the curve of velocity time will give you displacement covered.

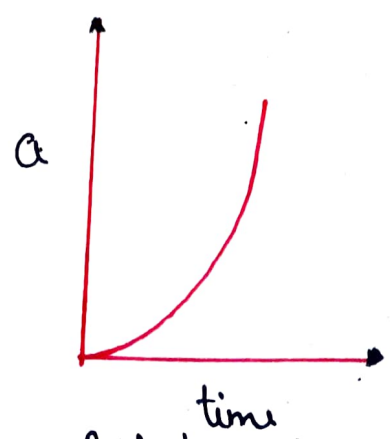
Slope of velocity time can never be infinite



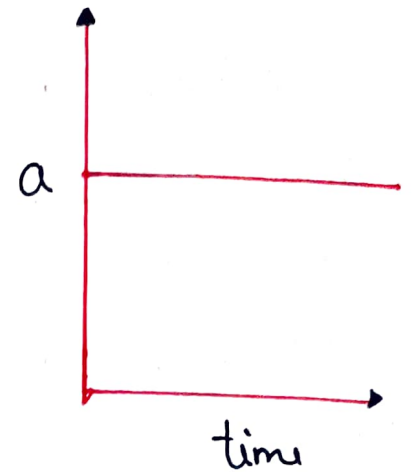
# Acceleration time.



Body is under uniformly accelerated motion



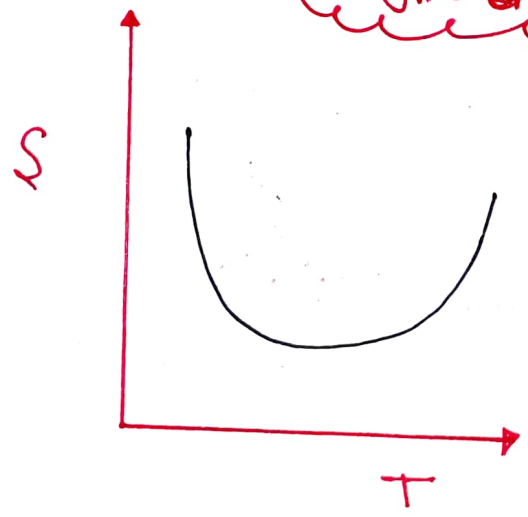
Body is under non uniformly accelerated motion



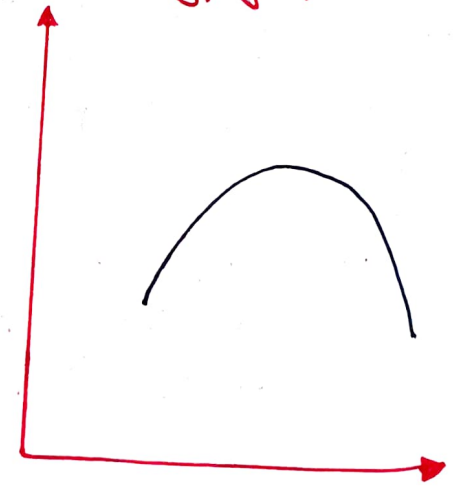
Body is moving with constant acceleration

Area of acceleration time gives ~~change~~ velocity

Slope of acceleration gives jerk



If the graph is in upward direction then acceleration will be positive



If the acc-graph is in downward direction then acceleration will be negative

if velocity and acceleration have same sign then speed will increase

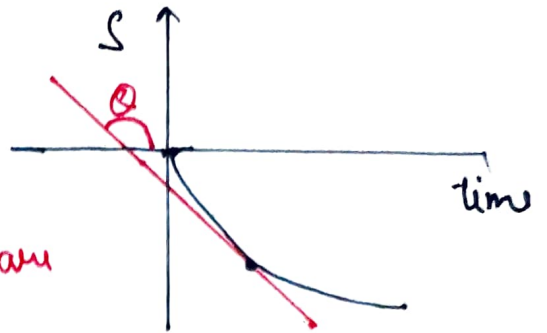
if velocity and acceleration are having different sign speed will decrease.

✱ ✱ ✱

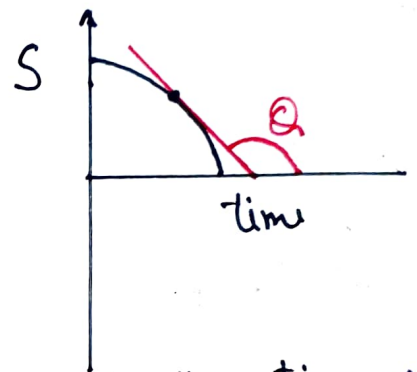
# Question Based Upon Graph

Q → 1 (a) What will be the ~~type~~ nature of velocity, acceleration and speed

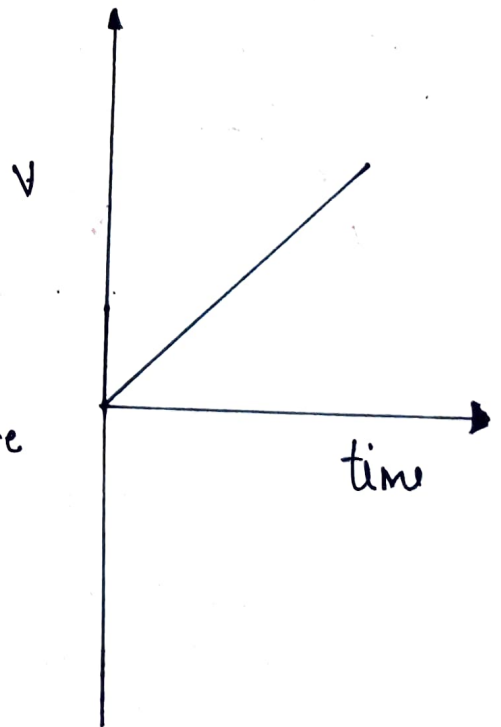
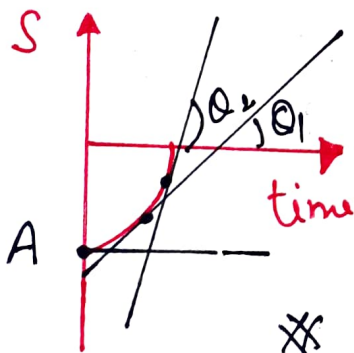
Sol Since  $\theta > 90^\circ$   
 Velocity will be negative  
 Acceleration is positive  
 Speed will decrease since  $v$  and  $a$  are of opposite nature.



Q → 1 (b) Since  $\theta > 90^\circ$   
 Velocity = -ve  
 Acceleration = -ve  
 Speed will increase.



Q → 2 Draw velocity-time graph ~~velocity-time~~ position-time graph.



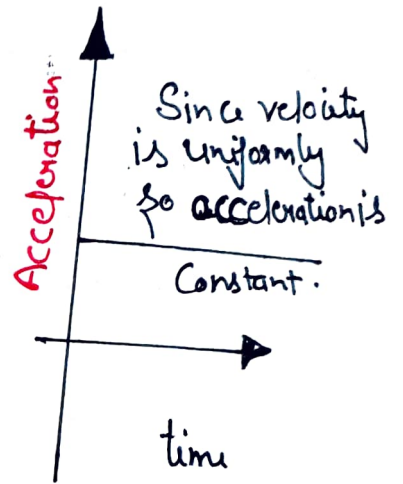
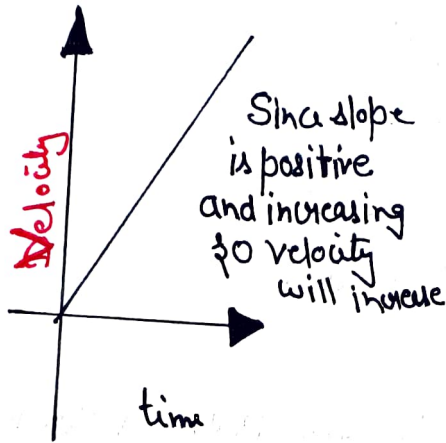
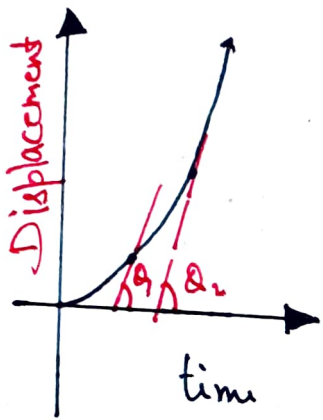
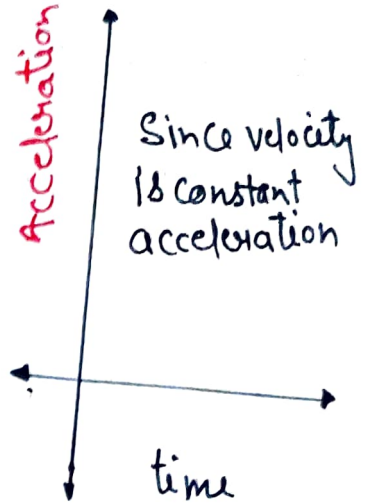
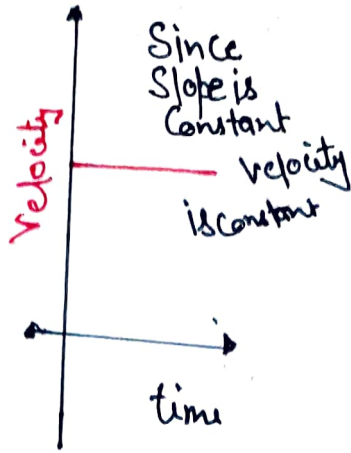
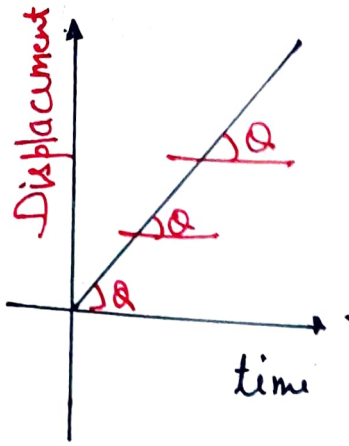
\* Since ~~the~~ slope of the graph is positive so velocity will be positive ( $\theta < 90^\circ$ )

\* Since slope is increasing with time so velocity is also increasing.

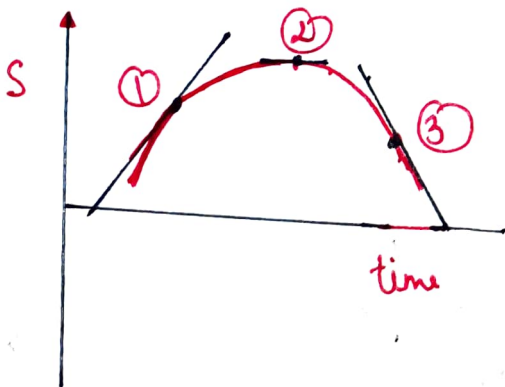
\* Since at point A of the graph slope is zero so velocity is zero

# Conversion Of Graph

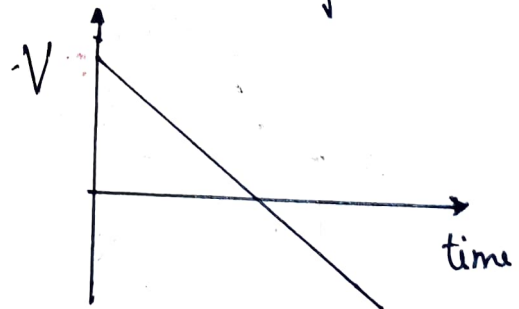
(4)



## Question



Convert this graph into velocity time



Sol

at point (1) the slope is ~~neg~~ positive so the velocity is positive and slope is ~~decreasing~~ increasing till point (2) so velocity is ~~decreasing~~ increasing

(i) at point two slope is zero so the velocity is zero.

(ii) At point (3) ~~the~~ slope is negative and increasing so velocity is negative and increasing.

Q → CONVERT V-T into S-T Graph

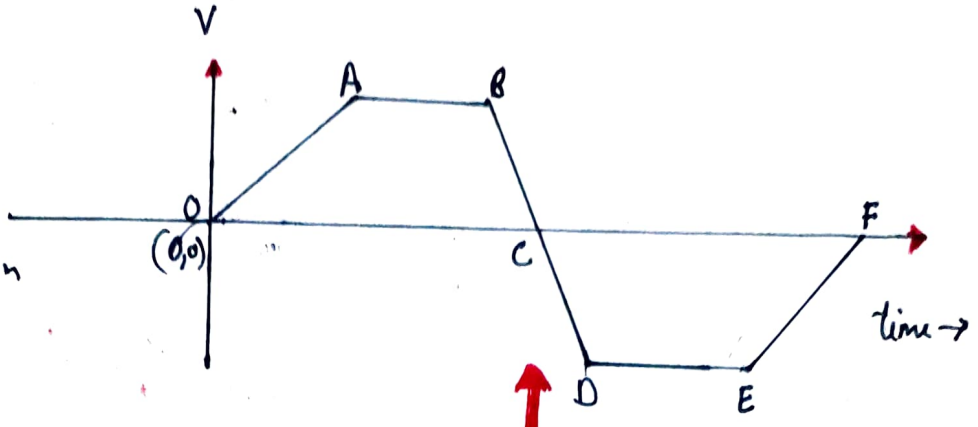
Sol<sup>n</sup> I From 0 to A

Slope of the graph is positive so acceleration is positive so

$$s = ut + \frac{1}{2}at^2$$

$$y = ax^2 + bx + c$$

so S-T graph should be parabolic upto point A



From A to B

Slope of the graph is zero

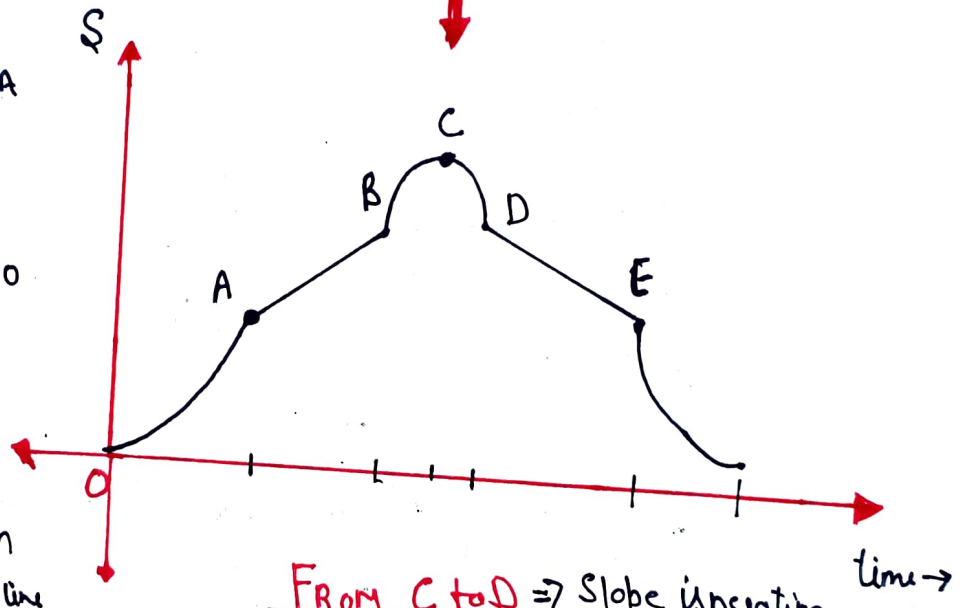
$$a = 0$$

$$s = ut + \frac{1}{2}(0)t^2$$

$$s = ut$$

$y = mx + c$  so graph should be a straight line in upward direction.

The slope of S-t graph should be constant.



From C to D ⇒ Slope is negative acceleration is negative  
 $s = ut - \frac{1}{2}at^2$  graph will be parabolic downward

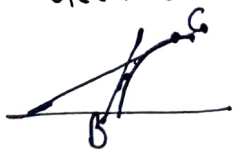
The slope of the graph must be negative as velocity is negative

From B to C

Slope of the graph is negative so acceleration is negative

$$s = ut - \frac{1}{2}at^2$$

so the graph must be parabolic downward and velocity must decrease. But velocity is positive

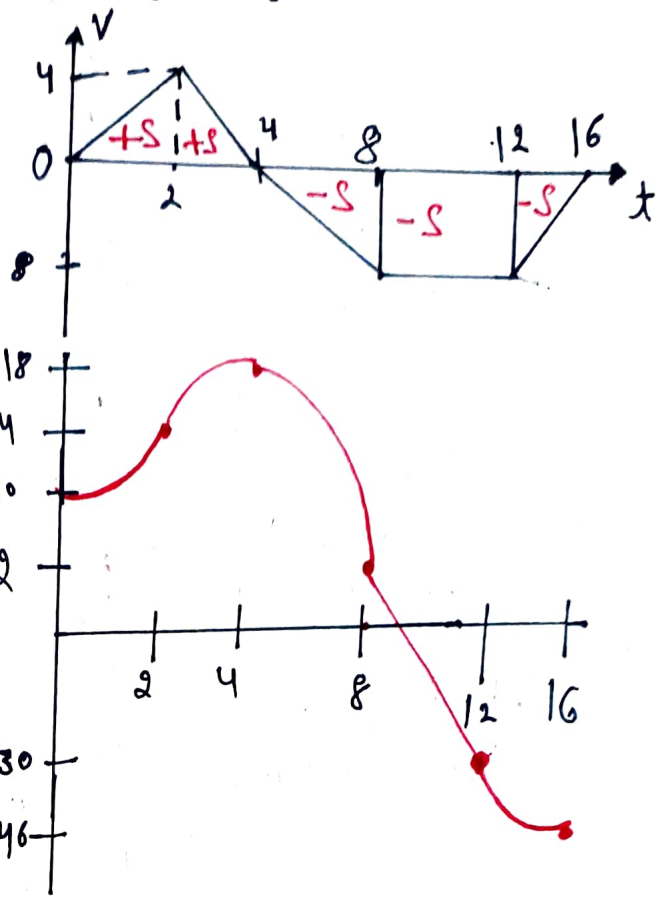


From D to E Slope is zero so acceleration is zero  
 $s = ut$  so graph will be straight line and velocity should be negative

From E to F Slope is positive so acceleration is positive  
 $s = ut + \frac{1}{2}at^2$  graph will be parabolic and it should have negative and decreasing slope

Q → Velocity time graph of a particle moving along x axis is a shown. At  $t=0$   $x=10$

- (a) Plot x-T graph
- (b) Find average velocity and average speed of the particle during complete journey
- (c) Average acceleration b/w 2 to 8 sec



Sol For Average Velocity

$$A.V = \frac{\text{Total disp}}{\text{Total time}}$$

$$A.V = \frac{4+4-16-32-16}{16} = -\frac{56}{16}$$

Average velocity = -3.5 m/s

Average ~~Distance~~ Speed

$$A.S = \frac{\text{Total distance}}{\text{Total time}}$$

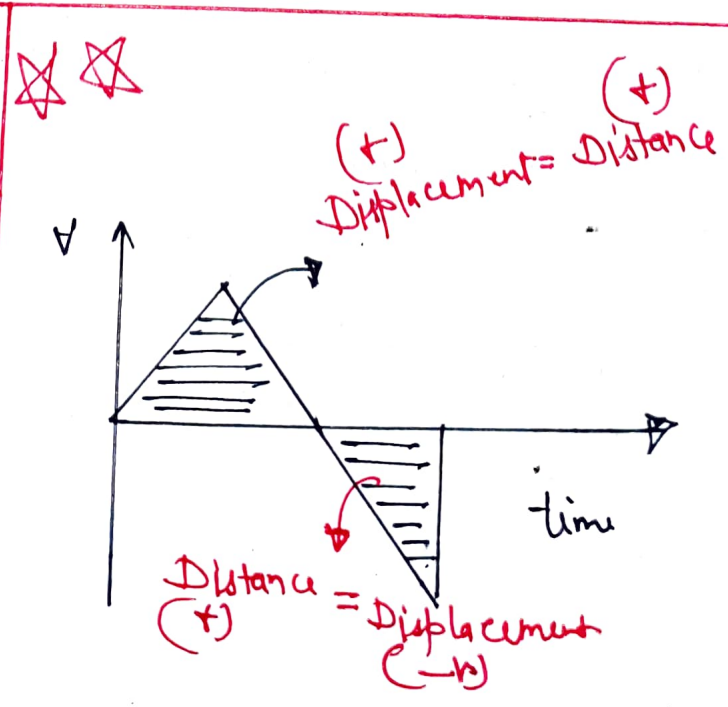
$$A.S = \frac{4+4+16+32+16}{16}$$

$$A.S = \frac{72}{16} = 4.5 \text{ m/s}$$

Time Interval	Area and displacement	Final X coordinate at the end of interval $X = X_i + S$
0-2	4	$X = 10 + 4 = 14$
2-4	4	$14 + 4 = 18$
4-8	-16	$18 - 16 = 2$
8-12	-32	$2 - 32 = -30$
12-16	-16	$-30 - 16 = -46$

\* Distance will be always positive and will be equal to area of velocity time graph

\* Displacement will be positive if velocity is positive and negative if ~~area~~ velocity is negative.



# RELATIVE MOTION

**RELATIVE VELOCITY**  $\Rightarrow$  Relative Velocity is defined as rate of change of relative position of one object with respect to other.

A Relative when two objects are moving in same direction  $\Rightarrow$

(i) Relative Velocity of A with respect to B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

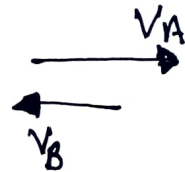


(ii) Relative Velocity of B with respect to A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

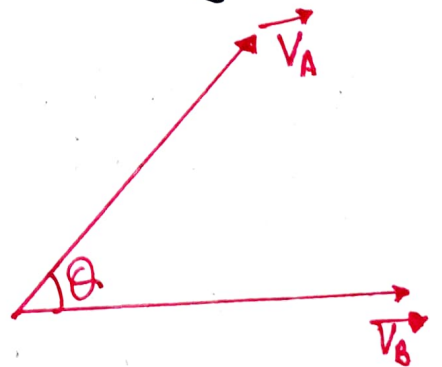
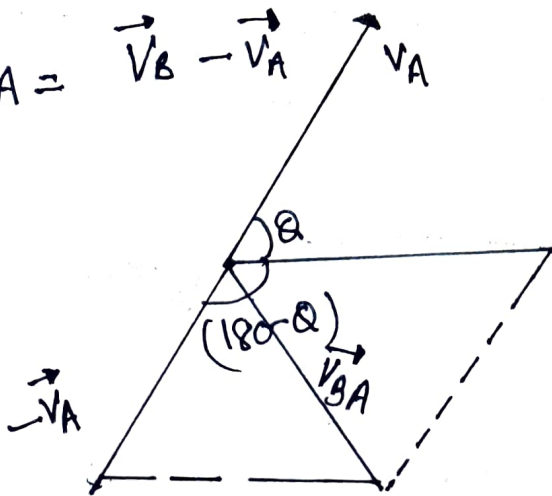
B Relative Velocity when two objects are moving in opposite direction

(i)  $\vec{V}_{AB} = \vec{V}_A + \vec{V}_B$       (ii)  $\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$



C Relative Velocity when two objects are moving at angle  $\theta$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$



$$|\vec{V}_{AB}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos(180 - \theta)}$$

$$|\vec{V}_{AB}| = \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos \theta}$$

$$\vec{V}_{AB} = \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos \theta}$$

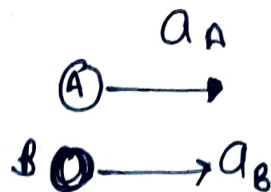
$$\vec{V}_{BA} = \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos \theta}$$

**Relative Acceleration**  $\Rightarrow$  It is defined as the rate of change of velocity of one object with respect to another object is called relative acceleration.

Case-1 When acceleration is in same direction

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

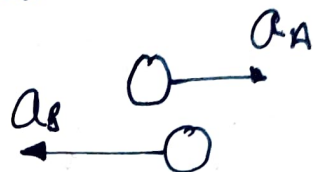
$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$



Case-2 When Acceleration is in opposite direction

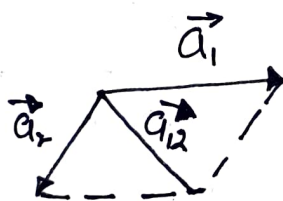
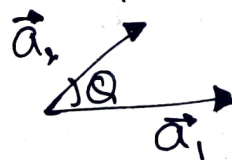
$$\vec{a}_{AB} = \vec{a}_A + \vec{a}_B$$

$$\vec{a}_{BA} = \vec{a}_B + \vec{a}_A$$



Case-3 When two Bodies are accelerating at some angle

$$\vec{a}_{12} = \sqrt{a_1^2 + a_2^2 - 2a_1a_2 \cos \theta}$$



**Numerical** When two object are having  $a=0$

$\Rightarrow$  A person (A) in train is having 4 m/s with respect to train and the train is having speed of 10 m/s with respect to ground. A person (B) is on the ground. Find.

(1) Velocity A with respect to B

$$\vec{V}_{AB} = \vec{V}_{Aq} - \vec{V}_{Bq}$$

$$\vec{V}_{Bq} = 0 \quad V_{AT} = 4 \text{ m/s}, \quad V_{Tq} = 10 \text{ m/s}$$

$$V_{AT} = V_{Aq} - V_{Tq}$$

$$4 - 10 = V_{Aq} - 10$$

$$V_{Aq} = 14 \text{ m/s}$$

$$\vec{V}_{AB} = 14 - 0$$

$$V_{AB} = 14 \text{ m/s}$$

$$V_{BA} = -14 \text{ m/s}$$



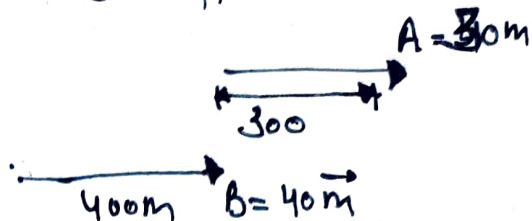
Q → Two trains are having length 300m and 400m are moving with speed 30m/s and 40m/s. Then if they are moving on parallel track then Find

(i) Time taken to overtake (when moving in same direction)

(ii) Time taken to cross (when moving in opposite direction)

Sol<sup>n</sup> ⇒ Case-1 Overtake -

The time of overtake start when the head of one train is at and tail of other train are at same position =

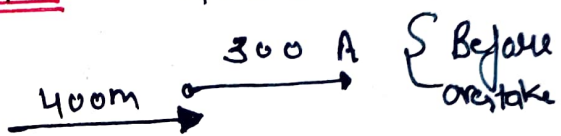


Distance to be covered for overtake =  $400 + 300\text{m} = 700$

$$\text{Time} = \frac{700}{V_{BA}}$$

$$V_{BA} = 40 - 30 = 10\text{m/s}$$

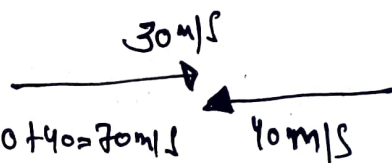
$$T = \frac{700}{10} = 70\text{ sec}$$



Case-II Cross

Distance = 700m  $V_{AB} = 30 + 40 = 70\text{m/s}$

$$\text{time} = \frac{700}{70} = 10\text{ s}$$



Q → Velocity of car with respect to Ground =  $V_{CG} = 15\text{m/s}$

Velocity of truck with Ground  $V_{TG} = 10\text{m/s}$



Velocity of person A on the truck with respect to truck =  $V_{AT} = 10\text{m/s}$

Velocity of monkey on the ~~truck~~ Car =  $V_{MC} = -1\text{m/s}$

Find  $V_{MA} = ?$

$$V_{MA} = V_{MG} - V_{AG}$$

$$V_{MA} = +14 - 11$$

$$V_{MA} = 3 - 15\text{m/s}$$

$$V_{AT} = V_{AG} - V_{TG} \Rightarrow V_{AG} = V_{AT} + V_{TG}$$

$$V_{AG} = 1 + 10 = 11\text{m/s}$$

$$V_{MC} = V_{MG} + V_{CG}$$

$$V_{MG} = V_{MC} + V_{CG}$$

$$V_{MG} = -1 + 15 = +14\text{m/s}$$

$$V_{AG} = 11\text{m/s}$$

## NUMERICAL On Relative Velocity when $a \neq 0$

Q → Two car A and B are moving in same direction. When graphs two car was 150m and the velocity of car A was 20m/s and acceleration is zero while car B was at rest and start with acceleration  $1\text{m/s}^2$ . When the two cars will overtake each other

Sol

$$u_{AB} = u_A - u_B = 20 - 0 = 20\text{m/s}$$

$$u_{AB} = 20\text{m/s}$$

$$a_{AB} = a_A - a_B = 0 - 1 = -1\text{m/s}^2$$

$$a_{AB} = -1\text{m/s}^2$$

$$s = u_{AB}t + \frac{1}{2}a_{AB}t^2$$

$$150 = 20t - \frac{1}{2} \times (1)t^2$$

$$300 = 40t - t^2$$

$$t^2 - 40t + 300 = 0$$

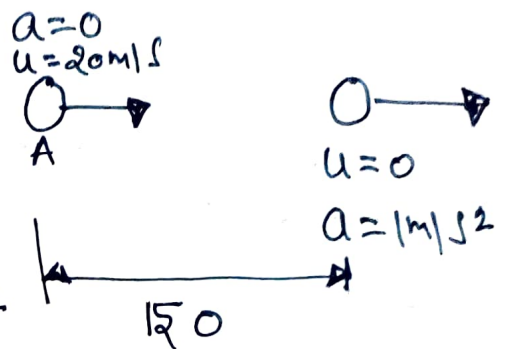
$$t^2 - 30t - 10t + 300 = 0$$

$$t(t-30) - 10(t-30)$$

$$(t-10)(t-30)$$

$t = 10\text{sec}$  at this time car A will overtake car B

$t = 30\text{sec}$  In this car B will overtake car A



Q → Find the time when person A catches person B. If the person A is not being able to catch person B then find the minimum distance b/w them.

Sol

$$U_{AB} = U_A - U_B = 10 - 0 = 10 \text{ m/s}$$

$$a_{AB} = 0 - 2 \text{ m/s} = -2 \text{ m/s}^2$$

$$S_{AB} = 50 \text{ m}$$

$$S = ut + \frac{1}{2}at^2$$

$$50 = 10t - \frac{1}{2}(2)t^2$$

$$50 = 10t - t^2$$

$$t^2 - 10t + 50 = 0$$

$$b^2 - 4ac$$

$$100 - 4 \times 50 \times 1$$

$$= -300$$

So there will be no real value of time so he will not be able to catch

To Find Minimum distance find the distance covered when relative velocity becomes zero

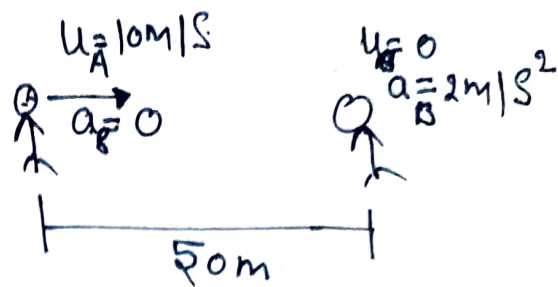
$$V^2 - U^2 = 2aS$$

$$0^2 - (10)^2 = -2(2)S$$

$$+100 = +4S$$

$$S = 25 \text{ m}$$

Minimum distance will be  $50 - 25 = 25 \text{ m}$



### Explanation of Above Question

Time	$V_A$ $a=0$	$V_B$ $a=2$	$V_{AB}$ $(V_A - V_B)$
0	10	0	10
1	10	2	8
2	10	4	6
3	10	6	4
4	10	8	2
5	10	10	0
6	10	12	-2
7	10	14	-4
8	10	16	-6

Relative velocity is decreasing with time mean distance b/w them will decrease slowly

at  $t=5$  this will be the minimum gap b/w them

After this point the gap b/w two person will increase