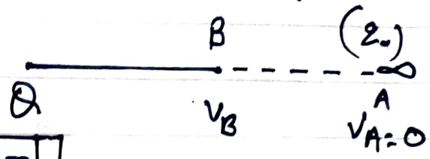
Chapter-2 Electrostatic potential and Capacitance

Electrostatic Potential

Electric potential at a point in the electric field is defined as the amount of work done in moving a unit positive charge from infinity to that point against electrostatic forces.

$$V = V_B = \frac{W}{q_0}$$



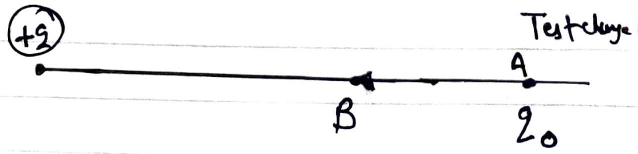
$\boxed{\text{SI unit} = \text{Volt OR Joule coulomb}^{-1}}$

* It is a scalar quantity.
* Potential at infinity is zero.

One Volt ⇒ The electric potential at a point in an electric field is said to be one volt if one joule work has to be done in moving a positive charge of one Coulomb from infinity to that point ~~against~~ against electrostatic force.

Potential difference ⇒ The potential difference b/w two points in an electric field may be defined as the amount of work done in moving a positive charge of unit magnitude from one point to other against electrostatic forces.

$$V = V_B - V_A = \frac{W_{AB}}{q_0}$$



Electric Potential due to Single Point charge

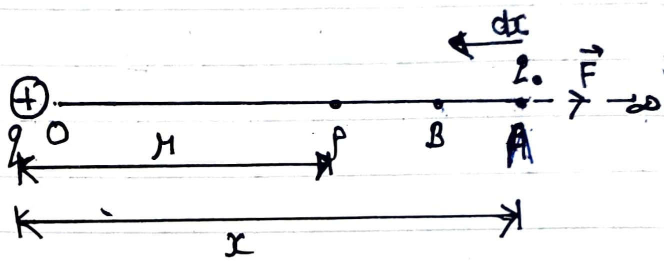
Consider a positive point charge 'q' placed at origin O. We wish to calculate the electric potential at point P at a distance r from it

Suppose a test charge q_0 is placed at point A at distance x from O. Now according to Coulomb's law the force acting on q_0 is

$$F = \frac{Kqq_0}{x^2}$$

{This force acts away from the charge q}

Now let the charge is displaced from A to B by displacement dx. So work done in doing so



$$dw = \vec{F}\cdot\vec{dx}$$

$$dw = F\,dx\cos\theta$$

$$\theta = 180^\circ$$

$$dw = F\,dx\cos 180 = -F\,dx$$

The total work done in bringing the charge from infinity to point P

$$W = \int_0^w dw = -\int_\infty^r F\,dx = -\int_\infty^r \frac{Kqq_0}{x^2}\,dx$$

$$W = \frac{-Kqq_0}{\cancel{x^2}}\left[\frac{-1}{x}\right]_\infty^r = +Kq_0q\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = \frac{Kqq_0}{r}$$

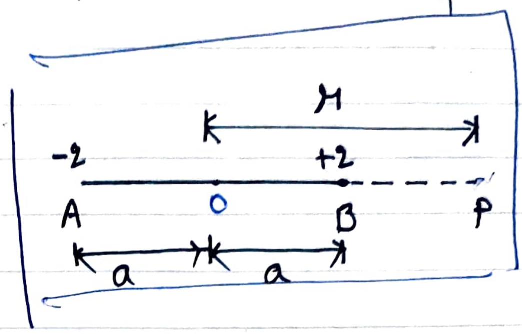
But from defination $V = \frac{W}{q_0} = \frac{Kq\cancel{q_0}}{r\cancel{q_0}} = \frac{Kq}{r}$

$$\boxed{V = \frac{Kq}{r}}$$

Electric potential on axial line of electric dipole

Consider a dipole of charges $+q$ and $-q$ separated by a distance of $2a$. Let P be a point on the axis of dipole at a distance r from its centre O.

Electric potential at point P due to the dipole is



$$V = V_A + V_B$$

$$V = \frac{-Kq}{(r+a)} + \frac{Kq}{(r-a)}$$

$$Kq\left[\frac{1}{(r-a)} - \frac{1}{(r+a)}\right]$$

$$= Kq\left[\frac{(r+a)-(r-a)}{(r-a)(r+a)}\right] = \frac{Kq2a}{(r^2-a^2)} = \frac{Kp}{(r^2-a^2)}$$

$\{p = 2qa\}$

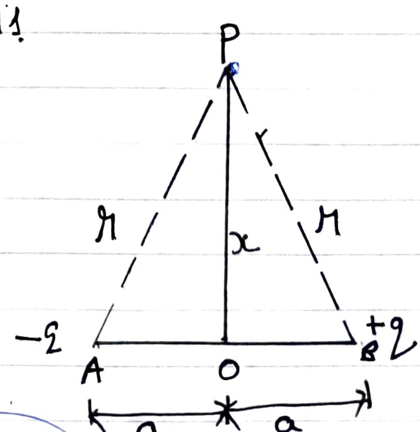
$$V = \frac{Kp}{(r^2-a^2)}$$

$$\boxed{V = \frac{Kp}{r^2}}$$

for short dipole $a^2 << r^2$

Electric potential at equitorial point of a dipole

Electric potential at Point P is



$$V = V_A + V_B$$

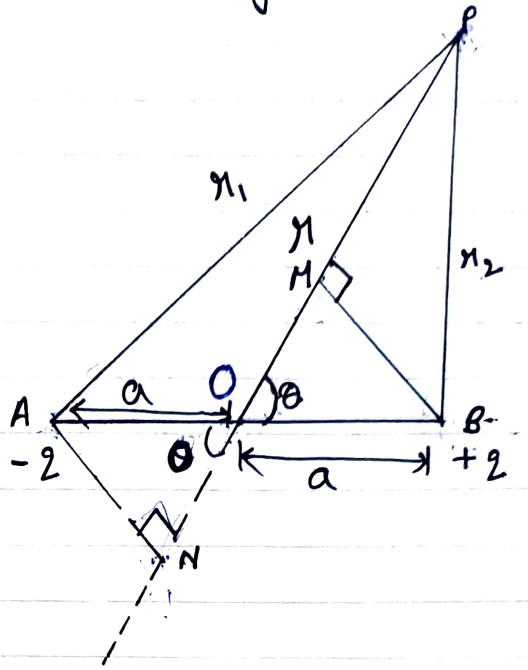
$$V = \frac{-Kq}{r} + \frac{Kq}{r}$$

$$\boxed{V = 0}$$

** ~~Havi~~ Work done in moving charge along equitorial line is always zero

Electric Potential At Any General point Due to a Dipole

Let P be the point and OP makes angle θ with the axis of the dipole



The potential at point P

$V = V_A + V_B$

$V = \frac{-Kq}{r_1} + \frac{Kq}{r_2}$

$V = Kq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

Draw AN ⊥ PN and ~~PA~~. MB ⊥ PM such that

PB ≃ PM and PN ≃ PA

PB = PM PN = PA

{This will be possible when point P is far away}

Now $r_1 = PA = PN = PO + ON$

$r_1 = (r + a\cos\theta)$

{In △ OAN

$\cos\theta = \frac{ON}{AO} = \frac{ON}{a}$

$ON = a\cos\theta$}

Similarly $r_2 = r - a\cos\theta$

$V = Kq\left[\frac{1}{(r - a\cos\theta)} - \frac{1}{r + a\cos\theta}\right]$

$V = Kq\left[\frac{\cancel{r} + a\cos\theta - \cancel{r} + a\cos\theta}{(r^2 - a^2\cos^2\theta)}\right]$

{$a^2 - b^2 = (a+b)(a-b)$}

$V = \frac{Kq(2a\cos\theta)}{(r^2 - a^2\cos^2\theta)}$

$p = 2aq$

$$\boxed{V = \frac{Kp\cos\theta}{(r^2 - a^2\cos^2\theta)}}$$

Spl cases (I) When $\theta = 0$ $\theta = 180$ [Axial line]

$$\boxed{V = \frac{Kp}{r^2}}$$

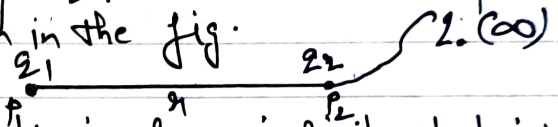
(ii) when $\theta = 90°$ [Equitorial line]

$$\boxed{V = 0}$$

Electric Potential Energy ⇒

The electric potential energy of a system of point charges may be defined as amount of work done in assembling the charges at their location by bringing them in from infinity

Suppose a point charge q_1 is at rest at a point P_1 in space as shown in the fig.



If charge q_2 is moved in from infinity to point P_2 the work done in doing so

$W =$ (Potential due to q_1 at P_2) × Charge

$W = V_1 \times q_1$

$W = \frac{K q_1 q_2}{r}$

$$\boxed{U = \frac{K q_1 q_2}{r}}$$

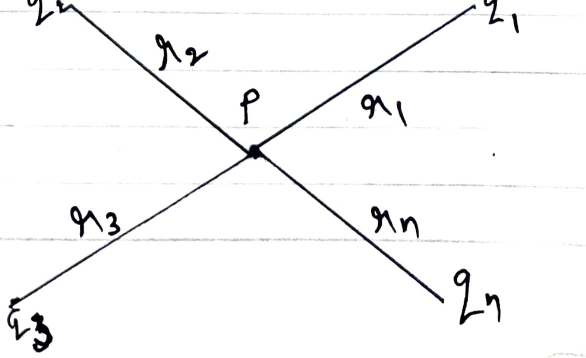
Electric potential due to system of charges

Consider n point charges $q_1, q_2 \cdots q_n$ at a distance $r_1, r_2 \cdots r_n$ respectively from point P as shown in fig. The potential at point P is given by

$V = V_1 + V_2 + \cdots V_n$

$V = \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} \cdots \frac{Kq_n}{r_n}$

$V = Kq\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} \cdots \frac{q_n}{r_n}\right]$



$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}}$$

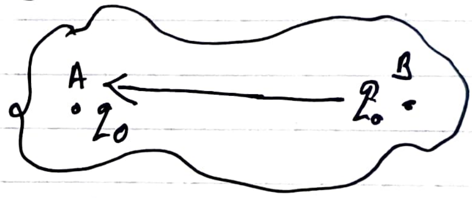
** The potential at a point due to a charge is not affected by the presence of other charge.

Equipotential Surface

Any surface that has same potential at every point on it is called an equipotential surface.

Properties of Equipotential Surface

(i) No work is done in moving a test charge over equipotential surface.



Reason

$W_{AB} = q_0 [V_B - V_A]$

$V_B = V_A$ { Since surface is equipotential }

$\boxed{W_{AB} = 0}$

(ii) Electric field is always perpendicular to the equipotential surface at every point.

Reason ⇒

$dw = \vec{F} \cdot \vec{ds} = F ds \cos\theta$ { Since equipotential $dw = 0$, $F = qE$ }

$0 = qE\, ds \cos\theta$

$q \neq 0$

$E \neq 0$ so $\cos\theta = 0$

$ds \neq$

$\boxed{\theta = 90}$

(iii) Two equipotential surface can not intersect each other.

Reason ⇒ If two equipotential surface intersect each other than there will be two value of ectric potential so it is not possible.

4. Equipotential surface are closer together in the region of strong field and farther away apart in the region of weak field.

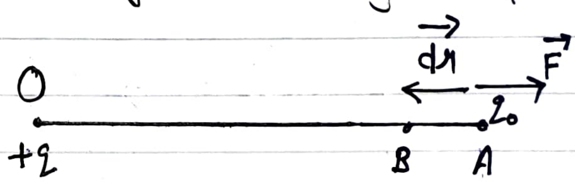
Reason ⇒ $E = -\frac{dV}{dr}$ since dV is constant

$E \propto \frac{1}{dr}$

$\boxed{dr \propto \frac{1}{E}}$

Relation b/w Electric field And potential OR Electric field as a gradient of electric potential

Consider two points A and B seprated by distance dr are so close that electric field intensity is same at both points.



Work done in moving Charge q_0 from A to B is

$dw = \vec{F} \cdot \vec{dr} = F dr \cos\theta$

$dw = -F dr$ $\{\theta = 180°$ and $\cos 180° = -1\}$

$F = q_0 E$

$dw = -q_0 E dr$

$\frac{dw}{q_0} = -E dr$ $\left\{ \frac{dw}{q_0} = dv \right\}$

$dv = -E dr$

$\boxed{-\frac{dv}{dr} = E}$

This is known as potential gradient

* -Ve sign shows that the direction of electric field is in the direction of decreasing potential.
* It is a vector quantity
* Its unit is Vm^{-1}

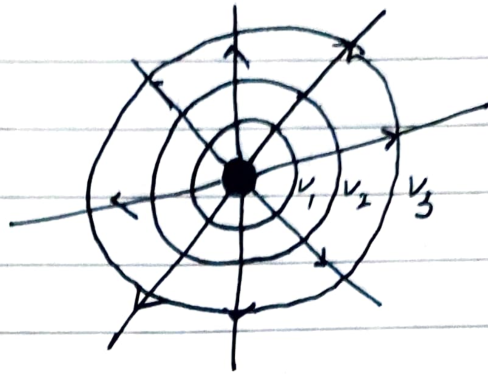
Shapes Of Equipotential Surface

1 Point charge (+q)

$V_1 > V_2 > V_3$

E is radial

V is spherical

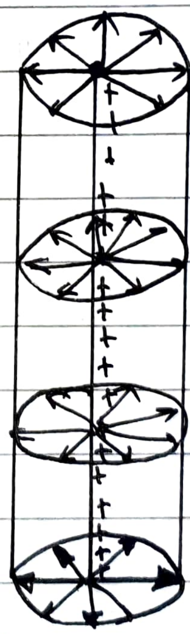


el

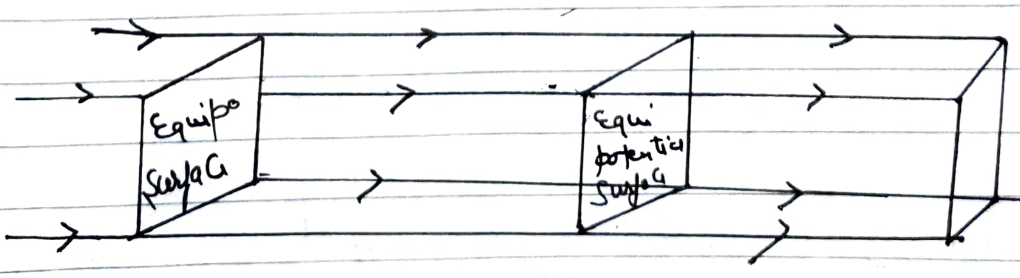
So due to point charge equipotential surfaces are of spherical shape.

2 Line Charge

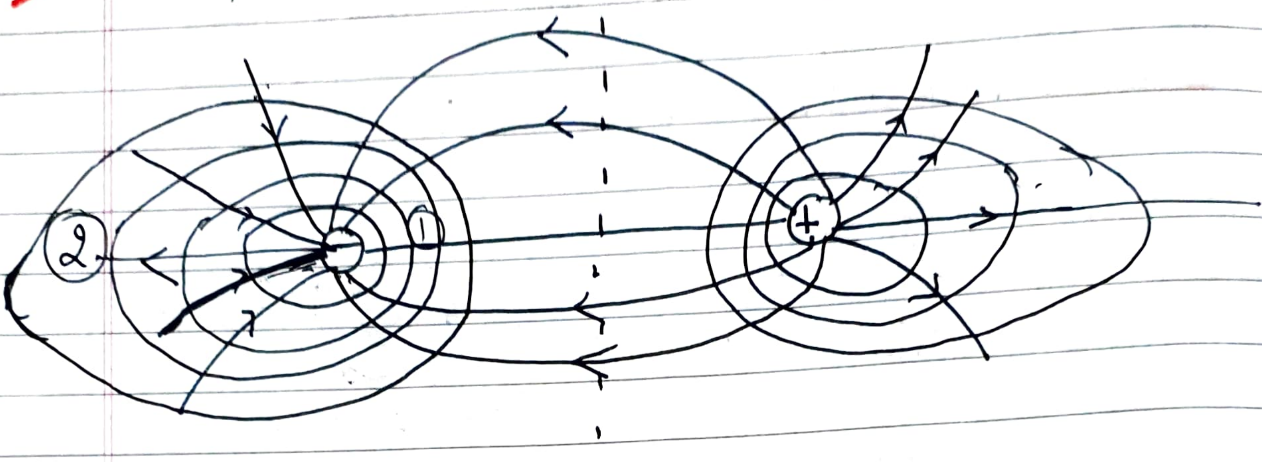
Equipotential surface $\Rightarrow$ Each point on the cylinder is having equipotential



3 Due to charges situated at infinity



III Dipole

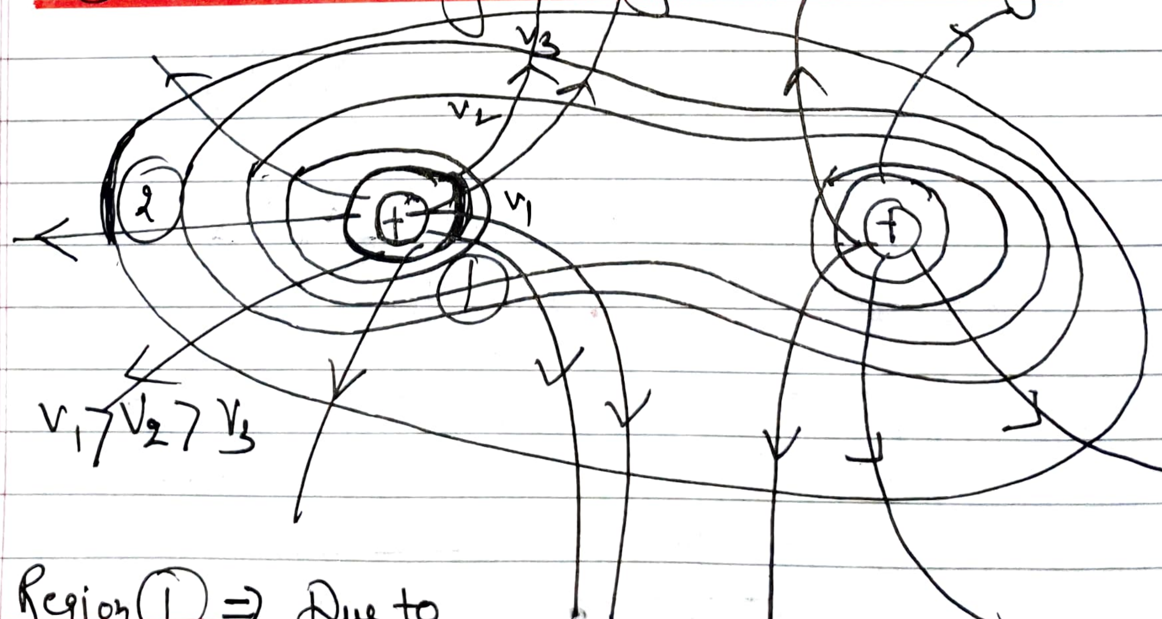


$dV = E\,dr$

at point (1) ⇒ Equipotential surface are closer to each other because in region (1) electric field is strong.

at point (2) ⇒ Equipotential surface are away from each other because of weak electric field.

(IV) Due to two charges having simillar charge

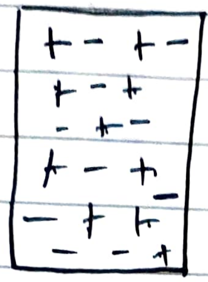


$V_1 > V_2 > V_3$

Region (1) ⇒ Due to repulsion electric field lines are close to each other so they have high value of electric field. and hence potential

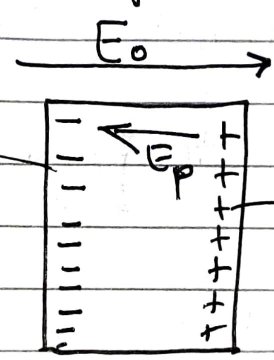
Electrostatic Properties Of Conductor

1. A conductor in static condition is electrically neutral.



$E = 0$ { Inside a conductor electric field is zero.

2. When the conductor is placed inside a electric field still electric field inside the conductor is zero.



Applied direction of electric field

Due to separation of charges electric field is produced internally

Due to external field charges get separated

Net electric field inside conductor $= E_0 - E_p$

$E = E_0 - E_p$ { But $E_0 = E_p$

$\boxed{E = 0}$

3. Any excess charge given to the conductor always reside on to the surface of conductor.

4. Potential Inside the conductor is constant.

$$E = \frac{-dv}{dr}$$

$E = 0$

$$0 = -\frac{dV}{dr}$$

$dV = 0$

integrating both side

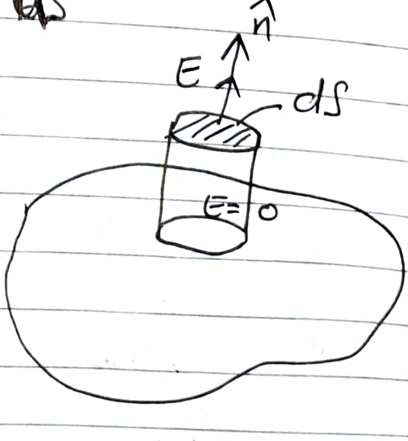
$$\boxed{V = \text{constant}}$$

(5) Electric field is always normal to the conductor

Because if it is not at right angle then the tangential component of electric field is will create surface current which is not there so it is always normal to the conductor.

(6) Electric field at the surface of a charged conductor is proportional to surface charge density.

Let us consider a conductor having charge density σ. Let a cylindrical Gaussian surface having ~~charge~~ area ds



Since electric field inside interior of a conductor is zero so net flux

$\phi = E\, ds \cos\theta$

$\theta = 0°$

$\phi = E\, ds \quad — (1)$

Now from Gauss theorem

$$\phi = \frac{q}{\varepsilon_0}$$

$$q = \sigma\, ds$$

$$\phi = \frac{\sigma\, ds}{\varepsilon_0} \quad \text{— (2)}$$

from (1) and (2)

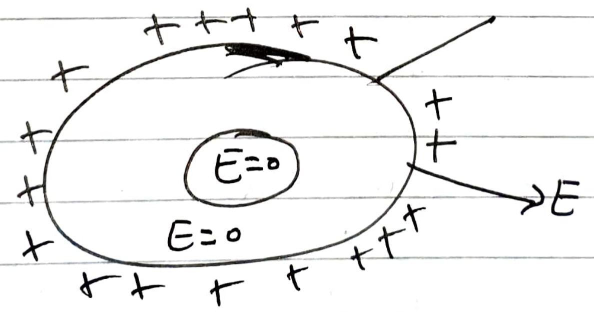
$$\frac{\sigma\, ds}{\varepsilon_0} = E\, ds$$

$$E = \frac{\sigma}{\varepsilon_0}$$

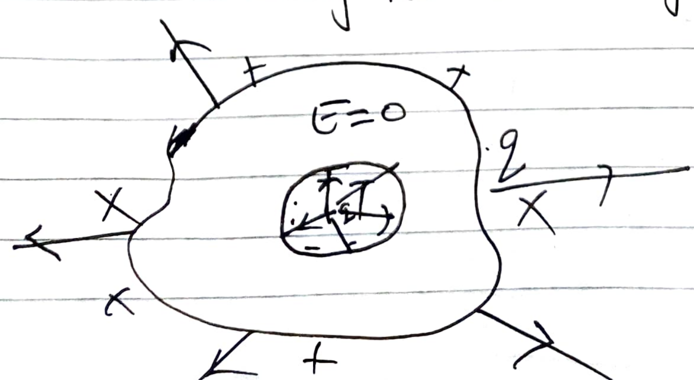
$$\boxed{E = \frac{\sigma}{\varepsilon_0}\hat{n}}$$

here $\hat{n}$ represent that electric field is normal to the surface.

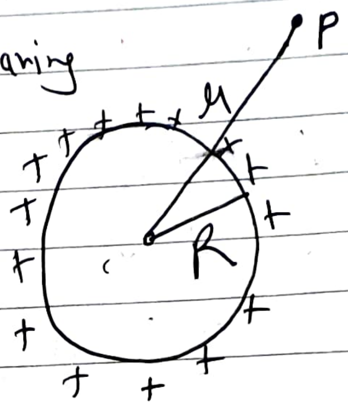
(7) Electric field is zero in the cavity of hollow charged conductor when cavity is not having any charge



Sub case ⟹ But if some +q is given to the cavity then electric field will be like



Electric potential of a shell



Let us consider a shell having Charge q at its surface.

Now electric field due to shell at point P will be

$$E = \frac{Kq}{r^2}$$

Now electric potential at surface

$$E = -\frac{dV}{dr}$$

$$dV = -E\,dr$$

$$dV = -\frac{Kq}{r^2}\,dr$$

integrating both side

$$V = -Kq\int \frac{1}{r^2}\,dr$$

$$\boxed{V = +\frac{Kq}{r}}$$

so at a point $r > R$ electric potential ~~at~~ will be

$$\boxed{V = \frac{Kq}{r}}$$

Now at surface of shell

$r = R$

$$\boxed{V = \frac{Kq}{R}}$$

At any point inside shell

$E = 0$

$E = \frac{dV}{dr}$

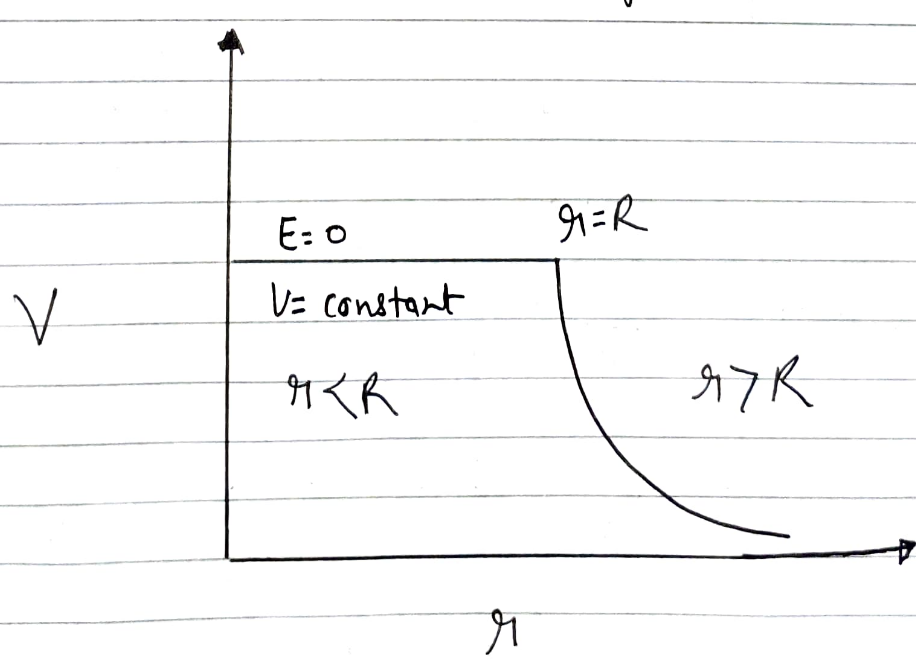
$0 = \frac{dV}{dr} \Rightarrow dV = 0$

integrating both side

$$\boxed{V = \text{constant}}$$

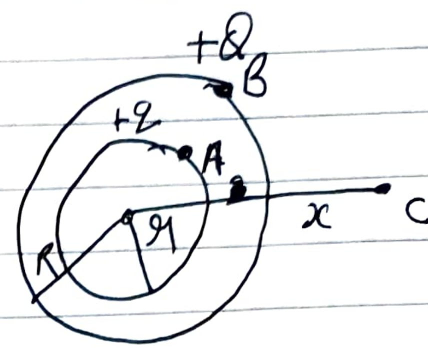
$$\boxed{V = \frac{Kq}{R}}$$

Graph b/w potential and r for a shell



Q→ Find out potential at point A, B and C

Sol At point A



$V_A = V_{+q} + V_{+Q}$

$$\boxed{V_A = \frac{Kq}{r} + \frac{KQ}{R}}$$

At point B

$V_B = V_q + V_Q$

$$\boxed{V_B = \frac{Kq}{R} + \frac{KQ}{R}}$$

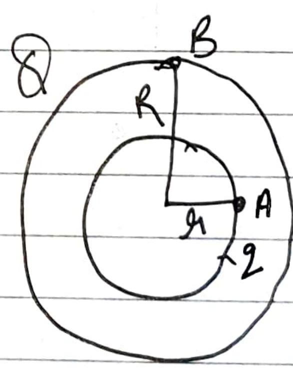
At point C

$V_C = V_{+q} + V_{+Q}$

$$\boxed{V_C = \frac{Kq}{x} + \frac{KQ}{x}}$$

Q→ Prove that inner shell is always at higher potential then outer shell.

Sol



$$V_A = \frac{Kq}{r} + \frac{KQ}{R} \quad \text{—(1)}$$

$$V_B = \frac{Kq}{R} + \frac{KQ}{R} \quad \text{—(2)}$$

$$V_A - V_B = \frac{Kq}{r} + \frac{KQ}{R} - \frac{Kq}{R} - \frac{KQ}{R}$$

$$V_A - V_B = \frac{Kq}{r} - \frac{Kq}{R} = Kq\left[\frac{1}{r} - \frac{1}{R}\right]$$

since $\frac{1}{r} > \frac{1}{R}$

so $V_A - V_B = +ve$

So A is higher potential then B

Capacitance

Capacitance of a conductor is defined as the amount of charge required to increase the potential of the conductor by unit volt.

When the charge given to the insulated conductor its potential increases proportionally.

$$Q \propto V$$

$$Q = CV$$

where C = Capacitance.

Factors on which capacitance depends

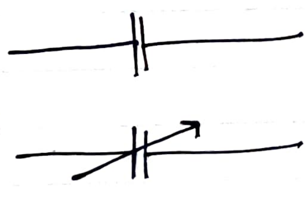
1) Size and shape of the conductor
2) Permitivity of the medium.
3) Presence of other conductor in the neighbourhood.

SI unit of Capacitance = Farad

Dimension of capacitance = $[M^{-1} L^{-2} T^{4} A^{2}]$

One Farad ⇒ The capacitance of a conductor is said to be one farad if the addition of one Coulomb of charge increases its potential by one volt.

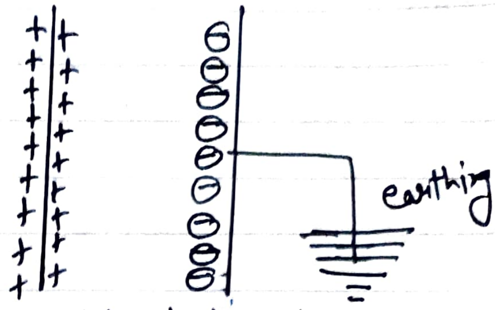
Capacitor ⇒ A capacitor is an arrangement of two conductors seperated by an insulating medium that is used to store electric charge and energy.



Fixed Capacitor

Variable Capacitor

Principle ⇒ The capacitance of an insulated conductor is increased by placing an insulated earthed connected conductor near it. Such system of two conductor is called capacitor.



<u>Parallel Plate Capacitor</u> ⇒ It consist of two large plane parallel conducting plates separated by a small distance.

Let A = Area of each plate
d = distance b/w two plates
$\pm Q = \pm \sigma A$ = total charge on each plate
$\pm \sigma$ = Surface charge density of each plate

Electric field b/w the plate
$E = \frac{\sigma}{\varepsilon_0}$ — (1)

Potential diff b/w Plates = Electric field x distance

$V = E d$ — (2)

$V = \frac{\sigma}{\varepsilon_0} d$ {from (1)

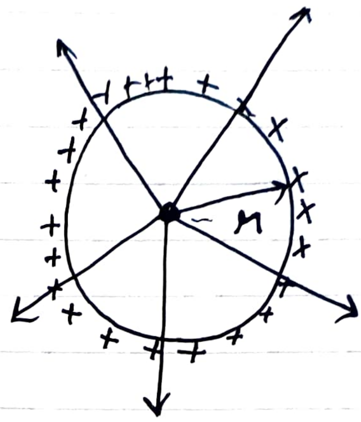
$C = \frac{Q}{V} = \frac{\cancel{\sigma} A \varepsilon_0}{\cancel{\sigma} d} = \frac{\varepsilon_0 A}{d}$ {$Q = \sigma A$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Capacitance of an isolated spherical capacitor ⇒

Consider a isolated spherical conductor of radius r. The charge Q can be assumed to be distributed on its surface

Potential on the surface



$$V = \frac{KQ}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\cancel{Q}\, r}{K\cancel{Q}}$$

$$\boxed{C = 4\pi\varepsilon_0 r}$$ $$K = \frac{1}{4\pi\varepsilon_0}$$

$$\boxed{C \propto r}$$

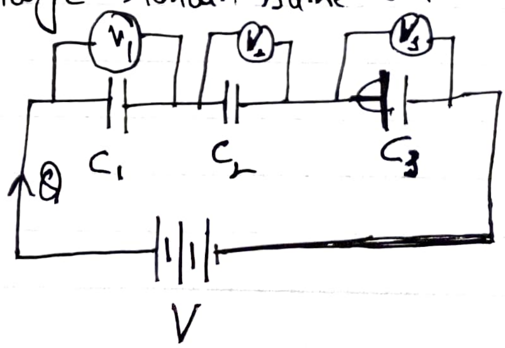
★ for 1 farad Capacitor $r = \frac{C}{4\pi\varepsilon_0} = 9\times10^{9}\,m$

$= 9\times10^{6}\,km$

So in order to make capacitor of 1 farad we need a spherical capacitor of radius $9\times10^{6}\,km$

Combination Of Capacitor

(1) Series Combination ⇒ In series combination charge remain same but voltage divides.



$$V = V_1 + V_2 + V_3$$

$$\frac{\cancel{Q}}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$ $\left\{ Q = CV,\ \frac{Q}{C} = V \right.$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Parallel Connection ⇒ * Potential remain same
* Charge is divided

$Q = Q_1 + Q_2 + Q_3$

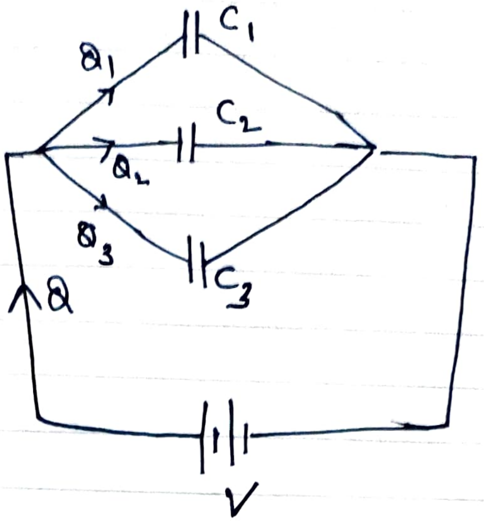
$Q = C_p V$

$Q_1 = C_1 V \quad Q_2 = C_2 V$

$Q_3 = C_3 V$

$C_p V = C_1 V + C_2 V + C_3 V$

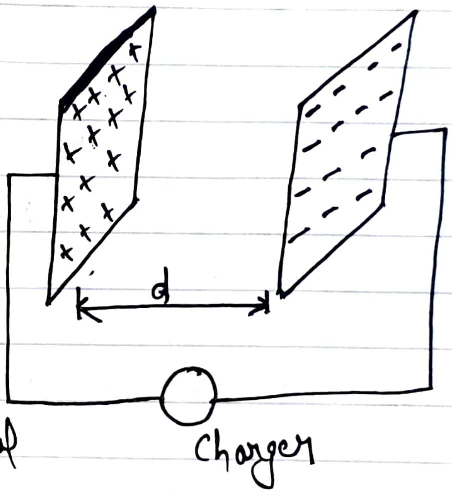
$$\boxed{C_p = C_1 + C_2 + C_3}$$



Energy Stored in a Capacitor

Let q be the initial charge on the capacitor plate.

Now if dq is charge is given to the plate of the capacitor. Then work done in doing so is stored in the form of potential energy



$du = dw = V\,dq$

$q = CV \Rightarrow V = \frac{q}{C}$

$du = \frac{q}{C}\,dq$

total work done in transfering Q charge will be stored in the potential energy form

$$U = \int_0^Q dw = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\boxed{U = \frac{1}{2}\cdot\frac{Q^2}{C}}$$

put $Q = CV$

$$U = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$$

$$\boxed{U = \frac{1}{2}CV^2}$$

Energy density ⇒ The energy stored per unit volume b/w plates of capacitor is known as energy density.

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad}$$

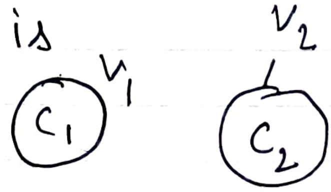
$$u = \frac{1}{2}\frac{\varepsilon_0 A}{d}\frac{(Ed)^2}{Ad}$$

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Loss of Energy in redistribution of charges.

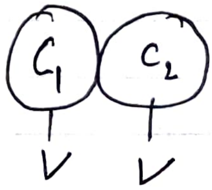
Let C_1 and C_2 be the capacitances and V_1, V_2 be the potential of two conductors before they are connected together

Potential Energy before connection is



$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

After connection Let V be their common potential



$$V = \frac{\text{Total Charge}}{\text{Total Capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2}$$

$$\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1+C_2}}$$

Potential Energy after connection is

$$U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1+C_2)V^2$$

$$U_f = \frac{1}{2}(C_1+C_2)\left[\frac{C_1V_1+C_2V_2}{C_1+C_2}\right]^2 = \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

Loss of Energy = $\Delta U = U_i - U_f$

$$\Delta U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{(C_1+C_2)}\left[V_1^2+V_2^2-2V_1V_2\right]$$

{after solving above equation

$$\boxed{\Delta U = \frac{1}{2}\frac{C_1C_2(V_1-V_2)^2}{C_1+C_2}}$$

★ This loss of energy takes placed due to production of displacement current due to which heat energy is lost in the form of electromagnetic wave

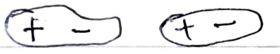
Dielectric

A dielectric is a substance which does not allow the charges to flow through it but allows the electric field to pass through it.

Example ⇒ Wood, glass, Mica, water

Types Of dielectric

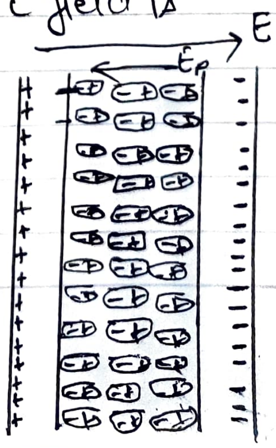
Polar Dielectric	Non polar Dielectric
In this centre of mass of positive charge do not coincide with negative charge	In this centre of mass of negative charge coincide with the positive charge.
Example → Water, HCl, CO_2	Example ⇒ H_2, N_2, O_2, C_6H_6



Polarisation of Dielectric Slab ⇒ It is the process of inducing equal and opposite charges on the two opposite faces of the dielectric slab.

Effect of External Electric field on the dielectric Slab

(1) Non Polar Dielectric ⇒ When the electric field is applied then the charge centre of the dielectric get seperated and charges are induced on the face of the dielectric



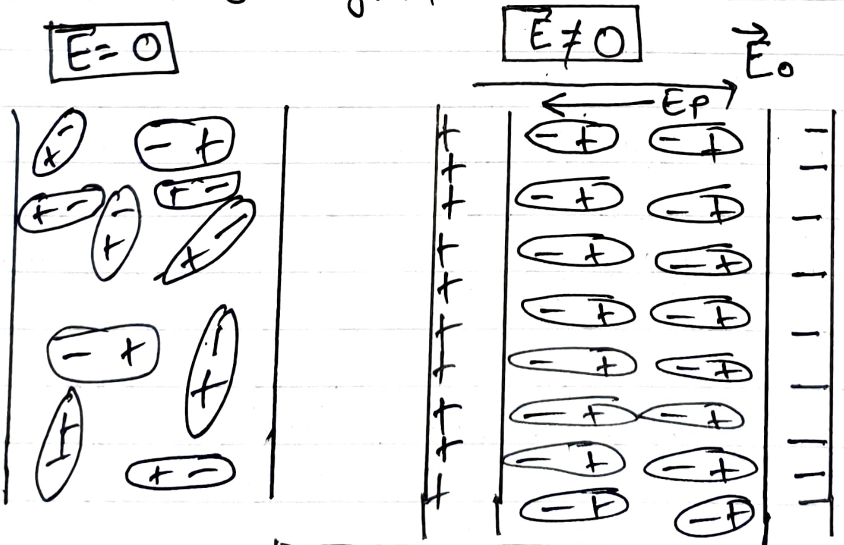
There is no charge induced in the interior part of the dielectric. because they Cancel out each other.

The net electric field inside the dielectric is given by

$$\boxed{\vec{E} = \vec{E}_0 - \vec{E}_p}$$

(ii) <u>Polar Dielectric</u> ⇒ In this charges are initially seperated but they are arranged in random order. So net electric field is zero.

But when the electric field is applied molecules of the dielectric arrange themselves in a particular direction. And now they have induced electric field.



$$\boxed{\vec{E} = \vec{E}_0 - \vec{E}_p}$$

Dielectric Constant ⇒ The ratio of the strength of the applied electric field to the reduced value of electric field on placing the dielectric b/w the plates of the Capacitor is Called dielectric constant

$$K = \frac{E_0}{E}$$

E_0 = Applied electric field

E = Net electric field inside the dielectric

Value of K is always greater than one

Polarisation Density ⇒ The induced dipole moment developed per unit volume in a dielectric slab on placing it inside electric field is called polarisation density. It is denoted by P.

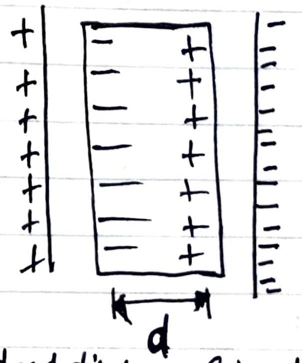
Let p be the dipole moment induced per atom and N be the total of atom per unit volume the

$$\boxed{P = Np}$$

According to defination of Polarising density

$$P = \frac{\text{Total dipole Moment Induced}}{\text{Volume}}$$

$$P = \frac{q_i \cancel{d}}{A \cancel{d}}$$



$$\boxed{P = \sigma_p}$$

$\left\{ \sigma = \frac{q_i}{A} \right.$

Induced dipole = $q_i \times d$

Charge × ⊥ dis

When the dielectric slab is placed b/w the plate of Capacitor then Net electric field developed.

$$E = \frac{\sigma - \sigma_p}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$$

$$\boxed{E = E_0 - \frac{\sigma_p}{\varepsilon_0}}$$

$$\left\{ E_p = \frac{\sigma_p}{\varepsilon_0} \right.$$

Electrical Susceptibility ⇒ The polarisation density of a dielectric slab is ~~dielectric~~ directly proportional to the reduced value of electric field.

$$\boxed{P = \chi \varepsilon_0 E}$$

χ = Electrical susceptibility

$$E = E_0 - E_p$$

$$E = E_0 - \frac{\sigma_p}{\varepsilon_0}$$

$$E = E_0 - \frac{\chi \cancel{\varepsilon_0} E}{\cancel{\varepsilon_0}}$$

$$\left\{ \sigma_p = P = \chi \varepsilon_0 E \right.$$

$$E = E_0 - \chi E$$

$$1 = \frac{E_0}{E} - \chi$$

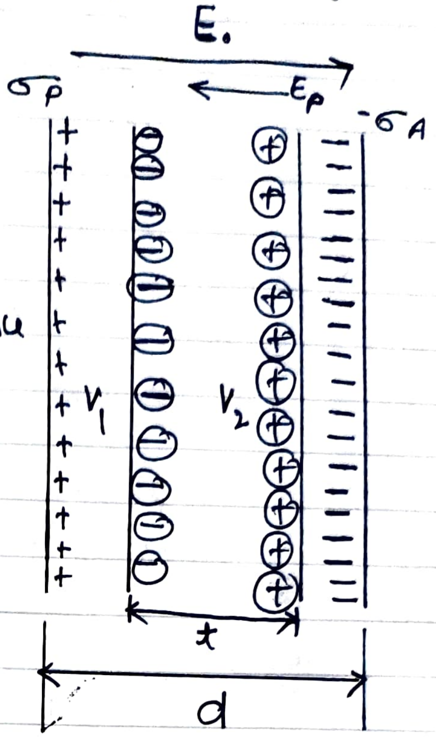
$$1 = K - \chi$$

$$\boxed{1 + \chi = K}$$

Dielectric Strength ⇒ It is the maximum value of electric field that can a dielectric sustain is called dielectric strength.

Effect Of dielectric slab on the capacitance of a capacitor:

Let us consider a parallel plate capacitor having charge density σ. The plate Area is A and the plates are seperated by distance d.



The capacitance of capacitor when vacuum is in b/w the plates of capacitor

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{—} (1)$$

Now suppose the dielectric slab of thickness t is inserted then capacitance will be

$$C = \frac{Q}{V} \quad \text{—} (2)$$

$$Q = \sigma A \quad \text{—} (3)$$

$$V = V_1 + V_2$$

$$V = E_0(d-t) + Et$$

$$V = E_0(d-t) + \frac{E_0 t}{K}$$

$$V = E_0\left[(d-t) + \frac{t}{K}\right]$$

$$V = \frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{K}\right] \quad \text{—} (4)$$

$E = \frac{V}{d}$

$K = \frac{E_0}{E}$

Putting the value of equation 3 and 4 in equation (2)

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{K}\right]}$$

$$C = \frac{\varepsilon_0 A}{\left[(d-t) + \frac{t}{K}\right]} \quad \text{— (5)}$$

Divide the equation (5) by (4)

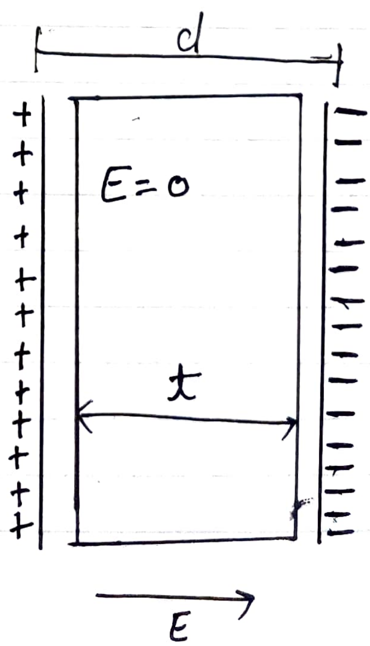
$$\frac{C}{C_0} = \frac{d}{\left[(d-t) + \frac{t}{K}\right]}$$

$$\boxed{C = C_0 \frac{d}{\left[(d-t) + \frac{t}{K}\right]}} \quad \text{—}$$

From above equation we can see that Capacitance increases with increase in thickness of slab.

☆☆ Effect of mettalic ~~the~~ slab on the capacitance of a capacitor

Let us consider a parallel plate capacitor having charge density σ and plate Area A and the plates are seperated by distance d.



Now the capacitance of the capacitor when vacuum is b/w the plates

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{--- (1)}$$

Now when Mettalic slab is inserted then Capacitance will be

$$C = \frac{Q}{V} \quad \text{--- (2)}$$

$$Q = \sigma A \quad \text{--- (3)}$$

$$V = \cancel{V_1} \cancel{+} V_2$$

$$V = E_0 (d-t) + 0$$

$$V = \frac{\sigma}{\varepsilon_0}(d-t) \quad \text{--- (4)}$$

Put the value of 3 and 4 in equation (2)

$$C = \frac{\cancel{\sigma} A}{\frac{\cancel{\sigma}}{\varepsilon_0}(d-t)} = \frac{\varepsilon_0 A}{(d-t)}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d-t}}$$

from this equation we can Conclude that ~~dielectric strength~~ capacitance increase with increase in thickness of mettalic slab.

Effect Of diectric on various Parameters

① Effect of dielectric when the battery is kept disconnected

Charge — Remain same because battery is disconnected

Electric field — Reduces $E = \frac{E_0}{K}$

Potential difference — Reduces = $V = \frac{V_0}{K}$

Capacitance : Increases $C = KC_0$

Energy = Decreases $\left[\because E = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)\left(\frac{V_0}{K}\right)^2 \right]$

② Effect of dielectric when the battery is kept connected

Charge — Increases $Q = KQ_0$

Electric field — Constant $\left\{ E = \frac{V}{d} = \frac{\text{Constant}}{\text{Constant}} \right\}$

Potential difference — Remains constant since battery is connected

Capacitance — Increases $\boxed{C = KC_0}$

Energy — Increase $[E = KE_0]$