Alternating Current - 7

An electric current whose magnitude changes continuously with time and direction reverses periodically is called alternating current.

$\boxed{I = I_0 \sin\omega t}$

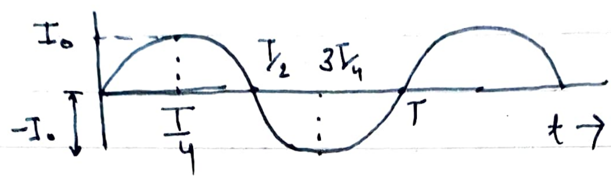
$\boxed{V = V_0 \sin\omega t}$

where V_0 = Peak Voltage.
I_0 = Peak Current

$\omega = 2\pi\nu$

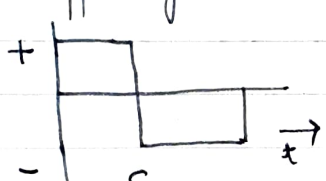
$\boxed{I = I_0 \sin 2\pi\nu t}$

ν = frequency.
In India frequency of AC = 50 hertz.

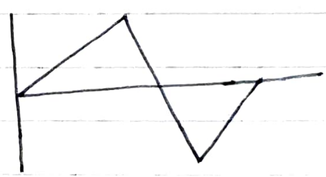


Symbol of AC = 

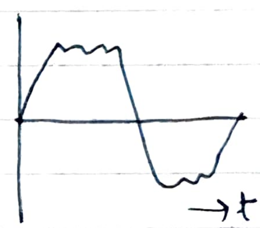
Types of Alternating current



Square



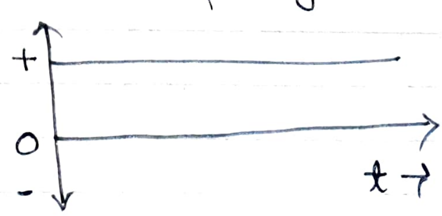
Triangular



Mixed

Direct Current ⇒ It is that current whose magnitude does not change with change in time.
It also does not change its polarity.

$\boxed{\nu_{DC} = 0}$



AC	DC
(i) It is cheap	(i) It is costly
(ii) AC is more dangerous	(ii) It is less dangerous.
(iii) It can be converted into DC	(iii) It cannot be converted into AC

Average Value of Alternating Current ⇒ It is that value of steady current which sends the same amount of charge through a circuit in a certain time interval as is sent by an alternating current through the same circuit in same time.

Let an alternating current is applied across a circuit is given by

$$\boxed{I = I_0 \sin\omega t} \quad \text{—— (1)}$$

$$I_{av} = \frac{\int_0^{T/2} I\,dt}{T/2} = \frac{\text{Total charge}}{\text{time taken}}$$

$$\left\{ I = \frac{dQ}{dt} \qquad dQ = I\,dt \right.$$

$$\left\{ I_{av} = \frac{\text{total charge}}{\text{time}} \right.$$

$$I_{av} = \frac{2}{T}\int_0^{T/2} I_0 \sin\omega t\,dt$$

$$I_{av} = \frac{2I_0}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2}$$

$$I_{av} = -\frac{2I_0}{T\omega}\left[\cos\frac{2\pi}{T}\times\frac{T}{2} - \cos\frac{2\pi}{T}\times 0\right] \qquad \left\{ \omega = \frac{2\pi}{T} \right.$$

$$I_{av} = \frac{-2I_0\times T}{T(2\pi)}\left[-1-1\right]$$

$$I_{av} = \frac{-2I_0\cancel{T}}{\cancel{2}\pi\cancel{T}}\times(-\cancel{2})$$

$$I_{av} = \frac{2I_0}{\pi}$$

$$\boxed{I_{av} = 0.637\,I_0}$$

Similiarly $\boxed{\mathcal{E}_{av} = 0.637\,\mathcal{E}_0}$

\# Mean Value of alternating current or voltage over a complete cycle is zero

7

2

Root Mean Square Value of Alternating Current ⇒ It is defined as that value of steady current which produces same amount of heat in a conductor in a certain time interval as is produced by A.C. in the same conductor in same interval of time.

Derivation for I_{rms}

Let an alternating current $\boxed{I = I_0 \sin\omega t}$ is passed through the conductor of resistance R. Now heat produced in the ~~semicond~~ conductor in small time dt will be given by

$$dH = I^2 R\,dt$$

$$dH = (I_0^2 \sin^2 \omega t) R\,dt \qquad \{ I = I_0 \sin\omega t$$

for total heat generated in T time

$$\int_0^H dH = I_0^2 R \int_0^T \sin^2 \omega t\,dt$$

$$H = I_0^2 R \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt \qquad \{ \cos 2\theta = 1 - 2\sin^2\theta$$

$$H = \frac{I_0^2 R}{2}\left[T - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$H = \frac{I_0^2 R}{2}\left[(T-0) - \frac{1}{2\omega}\left[\sin 2 \times \frac{2\pi}{T} \times T - \sin \frac{2\pi}{T} \times 0\right]\right]$$

$$H = \frac{I_0^2 R}{2}[T - 0]$$

$$\boxed{H = \frac{I_0^2 R T}{2}} \quad \text{—— (1)}$$

Let I_{rms} be the value of steady current passed through resistance R

$$\boxed{H = I_{rms}^2 R T} \quad \text{—— (2)}$$

Imp Integration of sin over a complete cycle will always gives zero

According to defination ~~Eqn~~ Heat produced in both the equation are same

$$I^2_{rms} RT = \frac{I_0^2 RT}{2}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$\boxed{I_{rms} = 0.707 I_0}$$

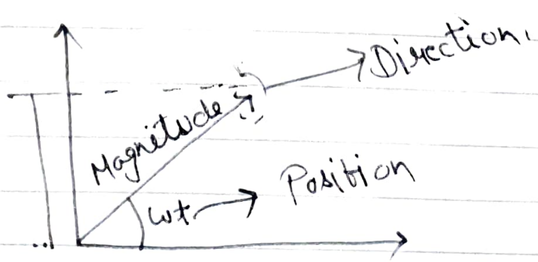
※ Alternating current ammeter and voltmeter reads R.M.S value i.e if in numerical you are given that voltmeter reading is x volt then x volt will be V_{rms}

Phasor ⇒ A phasor is a vector which rotates about origin with angular velocity ω.

Alternating current is a scalar quantity but its some property ~~with~~ matches with rotating vector.

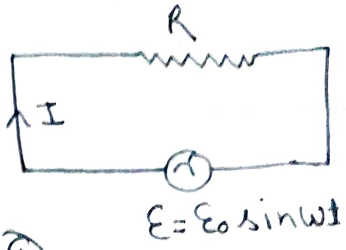
※ In phasor diagram length of phasor gives magnitude of alternating quantity and arrow head gives direction.

Angle (ωt) gives the position.



Alternating Current Applied to a resistor

Let an alternating current is passed through resistor of resistance R



$$\boxed{\mathcal{E} = \mathcal{E}_0 \sin \omega t} \quad -(1)$$

Applying Kirchhoff's law to the circuit

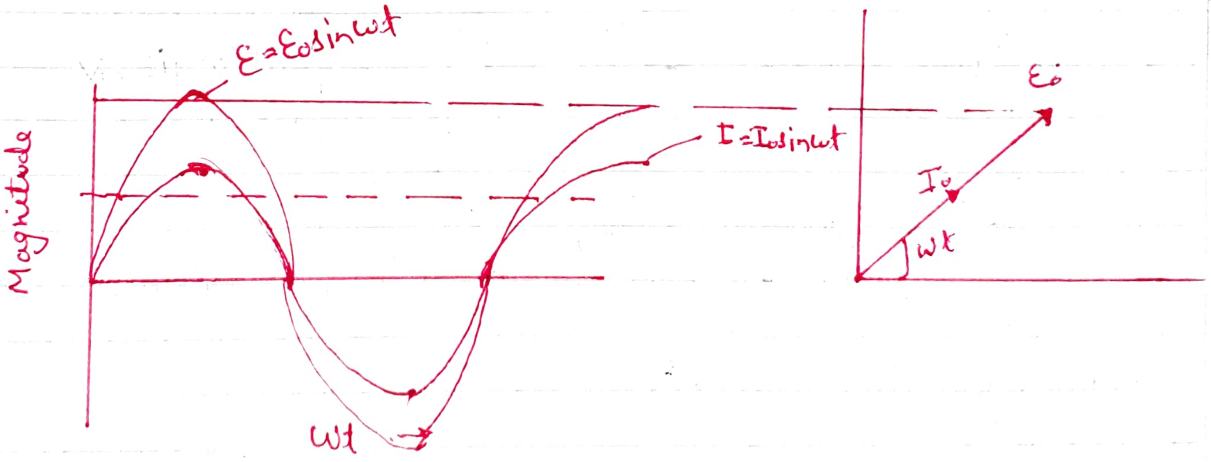
$$\mathcal{E} - IR = 0$$
$$\mathcal{E} = IR$$
$$\mathcal{E}_0 \sin \omega t = IR$$

$\left\{ \begin{array}{l} \mathcal{E} = \mathcal{E}_0 \sin \omega t \\ V = IR \end{array} \right.$

$$I = \frac{\mathcal{E}_0 \sin \omega t}{R}$$

$$\boxed{I = I_0 \sin \omega t} \quad -(2)$$

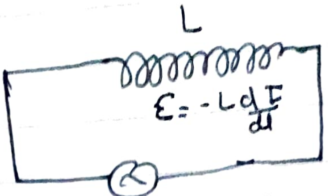
from equation (1) and (2) we can see that current and voltage are in same phase.



* The value of resistance offered by a conductor doesnot depend upon frequency of alternating current passed through it
* Average power of a resistor is not zero.

Alternating current applied to an Inductor

Let an Inductor of Inductance L across which an alternating EMF is applied



$$\mathcal{E} = \mathcal{E}_0 \sin\omega t \quad \text{— (1)}$$

Applying Kirchhoff's law to the circuit

$$\mathcal{E} + \left(-L\frac{dI}{dt}\right) = 0$$

$$\mathcal{E} = L\frac{dI}{dt}$$

$$dI = \frac{\mathcal{E}\,dt}{L}$$

$$dI = \frac{\mathcal{E}_0 \sin\omega t\,dt}{L}$$

integerating both side

$$I = \frac{\mathcal{E}_0}{L}\int \sin\omega t\,dt$$

$$I = -\frac{\mathcal{E}_0}{\omega L}\cos\omega t$$

$$\boxed{I = \frac{\mathcal{E}_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)} \quad \left\{ X_L = \omega L \right.$$

$$\boxed{I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{— (2)}$$

{ from equation (1) and (2) ~~Erdit~~ we can see that current lags voltage by 90° }

Concept

$-\cos\theta = -\sin(90 - \theta)$

$-\cos\theta = \sin(\theta - 90°)$

$\boxed{X_L = \omega L}$

X_L = Inductive Reactance

$X_L = 2\pi\nu L$

for DC $\boxed{\nu = 0}$

$\boxed{X_L = 0}$

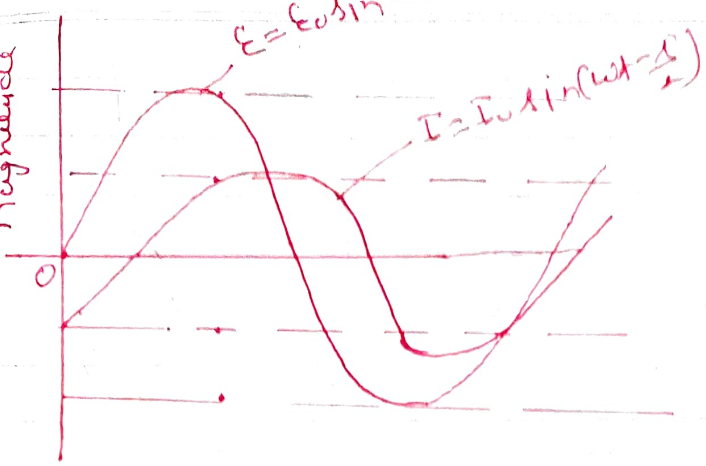
So it does not offer any resistance to DC current

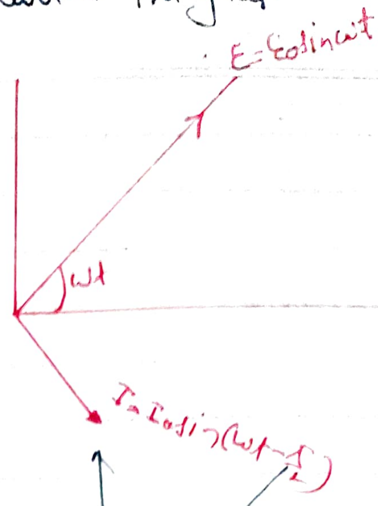
for AC

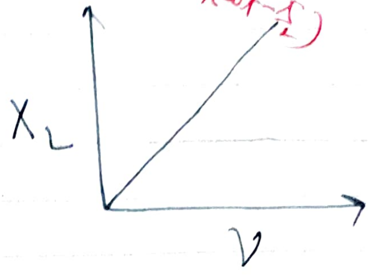
$\boxed{X_L = 2\pi\nu L}$

it offers resistance to AC.

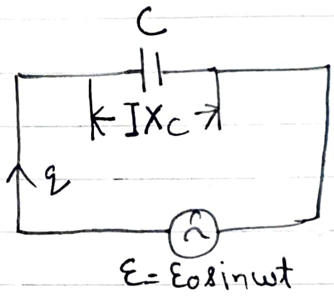
Inductive Reactance (X_L) ⇒ It is the resistance offered by inductor to flow of current through it.







Alternating Voltage Applied to Capacitor



Let us consider a circuit containing ~~inductor~~ capacitor. An alternating emf is applied across it

$$\boxed{\mathcal{E} = \mathcal{E}_0 \sin \omega t} \quad \text{— (1)}$$

According to Kirchhoff's law

$$\mathcal{E} = \frac{q}{C}$$

$$q = \mathcal{E} C$$

$$\frac{dq}{dt} = \frac{d(C\mathcal{E}_0 \sin \omega t)}{dt}$$

$$I = C\mathcal{E}_0 \frac{d}{dt}(\sin \omega t)$$

$$I = \omega C \mathcal{E}_0 \cos \omega t$$

$$I = \frac{\mathcal{E}_0}{\frac{1}{\omega C}} \cos \omega t$$

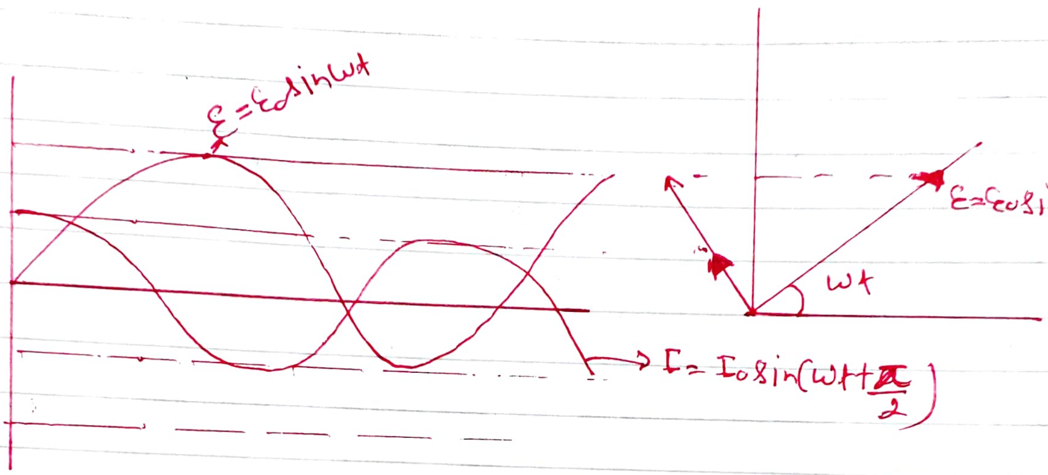
$$I = \frac{\mathcal{E}_0}{X_C} \cos \omega t$$

X_C = Capacitive Reactance

$$I = I_0 \cos \omega t$$

$$\boxed{I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)} \quad \text{—(2)}$$

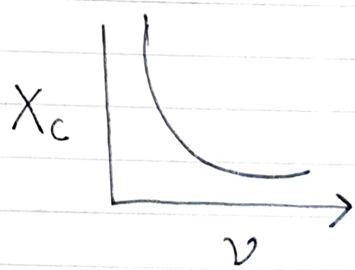
from equation (1) and (2) we can see that current leads the voltage by 90°



Capacitive Reactance [X_C] → It is the resistance offered to the flow of current by capacitor.

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{2\pi \nu C}$$



for AC

$$X_C \propto \frac{1}{\nu}$$

it offers low resistance

for DC

$$\nu = 0$$

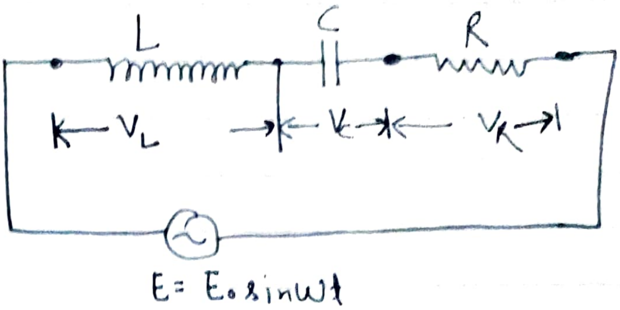
$$X_C = \frac{1}{0}$$

$$\boxed{X_C = \infty}$$

It blocks DC current

Alternating Current Applied Across the LCR circuit

Let a circuit containing L, C and R. An alternating Emf is applied across it.



(i) Inductor

* voltage drop across it $= V_L = I X_L$
* Voltage leads current by $\frac{\pi}{2}$

(ii) Capacitor

* Voltage drop across it $= V_C = I X_C$
* Voltage lags current by $\frac{\pi}{2}$

(iii) Resistance

* Voltage drop across it $= V_R = IR$
* Voltage and current are in same phase.

from the above information drawing phasor diagram considering $\boxed{V_L > V_C}$

In ΔOAB

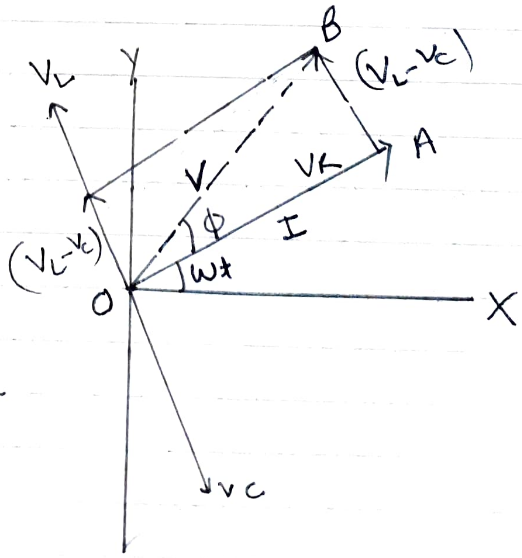
$$(OB)^2 = (OA)^2 + (AB)^2$$

$$V^2 = (V_R)^2 + (V_L - V_C)^2$$

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$



$$\boxed{\frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = I} \implies \frac{V}{Z} = I$$

Z = impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Phase Angle for LCR

$$\tan\phi = \frac{V_L - V_C}{R} = \frac{IX_L - IX_C}{IR}$$

☆ $$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

Impendance [Z] ⇒ Total effective opposition offered by LCR circuit to the alternating current is known as impendance.

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$ ☆

Electrical Resonance → Series LCR Circuit

Electrical resonance is said to take place in a series LCR circuit when the circuit allows maximum current for a given frequency of source of alternating supply.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

※ for max current Z should be minimum.

for Z to be minimum

$$\boxed{X_L = X_C}$$

Resonant frequency ⇒ It is the frequency at which current through LCR circuit is maximum.

At electrical resonance.

$X_L = X_C$

$\omega L = \frac{1}{\omega C}$

$\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}} \Rightarrow 2\pi\nu = \frac{1}{\sqrt{LC}}$

$$\boxed{\nu_r = \frac{1}{2\pi\sqrt{LC}}}$$

※ Resonant frequency is ~~to~~ independent of resistance.

Band Width Of LCR Circuit ⇒ Let ω_0 be resonant angular frequency.

Let $\boxed{\omega_1 = \omega_0 - \Delta\omega}$ —(1) such that circuit circuit behave as capacitive and value of current is $\boxed{I = \frac{I_0}{\sqrt{2}}}$

$\boxed{\omega_2 = \omega_0 + \Delta\omega}$ —(2) such that circuit behave as inductive and value of current it is $\boxed{I = \frac{I_0}{\sqrt{2}}}$

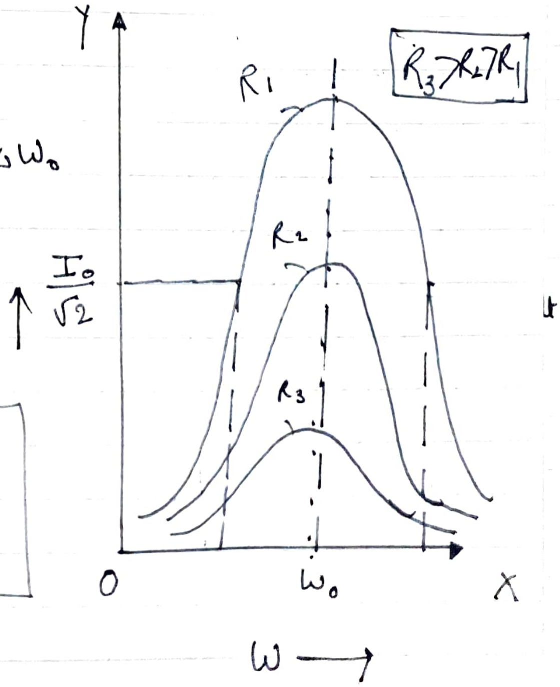
~~Adding equation (1) and (2)~~

Substracting (1) from (2)

$\omega_2 - \omega_1 = \omega_0 + \Delta\omega - \omega_0 + \Delta\omega_0$

$$\boxed{\omega_2 - \omega_1 = 2\Delta\omega}$$

At higher resistance the value of maximum current is low and circuit is ~~more~~ less sharp



Sharper the resonance curve greater the value of Q

Q Measure the ability to differentiate signal of different and nearly same

Quality Factor [Q] ⇒ For a series LCR circuit it is defined as 2π times the ratio of the energy stored in the circuit to the energy dissipated in the resistance per cycle of A.C supply.

$$Q = 2\pi \times \frac{\text{Energy stored in the circuit per cycle}}{\text{Energy dissipated per cycle.}}$$

At resonance Energy stored in the circuit in inductor when current is max

$$E = \frac{1}{2} L I_0^2$$

At resonance Energy dissipated per cycle in resistance $= I^2 RT$

$$Q = \frac{2\pi \frac{1}{2} L I_0^2}{I^2 RT} = \frac{\pi L I_0^2 \nu_0}{\left(\frac{I_0}{\sqrt{2}}\right)^2 R} \qquad \left\{ \nu_0 = \frac{1}{T} \right.$$

$$Q = \frac{2\pi L \cancel{I_0^2} \nu_0}{\cancel{I_0^2} R}$$

$$Q = \frac{2\pi \nu_0 L}{R}$$

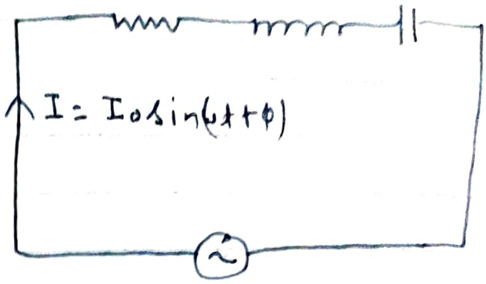
$$Q = \frac{\omega_0 L}{R} \qquad \left\{ \omega_0 = \frac{1}{\sqrt{LC}} \right.$$

$$Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Power Consumed In a series LCR Circuit

Let in a series LCR circuit the phase angle b/w current and voltage be ϕ



$E = E_0 \sin \omega t$

~~Instan~~ $E = E_0 \sin \omega t$

$I = I_0 \sin(\omega t + \phi)$

Instantaneous power ~~fo~~ input to LCR circuit is given by

$$P_i = EI = E_0 I_0 \sin \omega t \sin(\omega t + \phi)$$

$$P_i = E_0 I_0 \sin \omega t [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

$$P_i = E_0 I_0 \left[\sin^2 \omega t \cos \phi + \frac{2}{2} \sin \omega t \cos \omega t \sin \phi\right]$$

$$P_i = E_0 I_0 \left[\sin^2 \omega t \cos \phi + \frac{\sin 2\omega t \sin \phi}{2}\right]$$

$$P_{av} = \frac{\int_0^T P_i \, dt}{T}$$

$$P_{av} = \frac{E_0 I_0}{T} \int_0^T \left[\sin^2 \omega t \cos \phi + \frac{\sin 2\omega t \sin \phi}{2}\right] dt$$

$$P_{av} = \frac{E_0 I_0}{T} \Bigg[\cos \phi \underbrace{\int_0^T \sin^2 \omega t \, dt}_{A} + \frac{1}{2} \sin \phi \underbrace{\int_0^T \sin 2\omega t \, dt}_{B} \Bigg]$$

$$A = \int_0^T \sin^2 \omega t \, dt = \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt = \frac{1}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$$

$$A = \frac{1}{2}\left[T - \frac{1}{2\omega} \sin 2\omega t\right]_0^T = \frac{T}{2}$$

$$B = \int_0^T \sin 2\omega t = 0$$

{ integration of sin function ~~after~~ to a complete cycle is alway zero.

$$P_{av} = \frac{E_0 I_0}{T}\left[\cos\phi\left(\frac{T}{2}\right) + \frac{1}{2}\sin\phi(0)\right]$$

$$P_{av} = \frac{E_0 I_0}{\cancel{T}}\cos\phi\left(\frac{\cancel{T}}{2}\right)$$

$$P_{av} = \frac{E_0 I_0}{2}\cos\phi$$

$$P_{av} = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}\cos\phi$$

$$\boxed{P_{av} = E_{rms} I_{rms} \cos\phi}$$

$$\left\{ I_{rms} = \frac{I_0}{\sqrt{2}} \quad E_{rms} = \frac{E_0}{\sqrt{2}} \right.$$

Case

1 Power consumed in Resistance

Phase angle in resistance = $\phi = 0°$

$$\boxed{P_{av} = E_{rms} I_{rms}}$$

2 Inductor

$\phi = 90°$

$$\boxed{P_{av} = 0}$$

3 Capacitor

$\phi = 90°$

$$\boxed{P_{av} = 0}$$

Power factor ⇒ It is defined as cosine angle b/w current voltage.

$$\boxed{\text{Power factor} = \cos\phi}$$

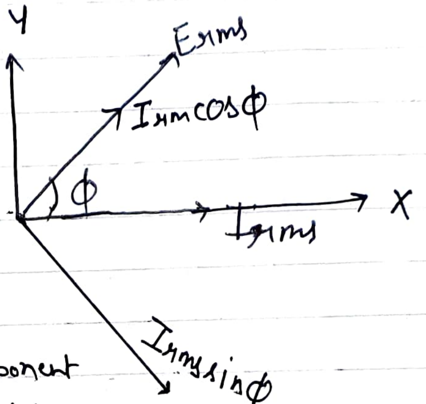
$$\boxed{P_f = \frac{P}{E_{rms} I_{rms}} = \frac{\text{true power}}{\text{Apparent Power}}}$$

Power factor

Resistance	Inductor	Capacitor
Power factor = $\cos\phi$	$\phi = 90°$	$\phi = 90°$
$\phi = 0°$	$P.f = \cos 90°$	$P.f = \cos 90°$
$\boxed{P.f = 1}$	$\boxed{P.f = 0}$	$\boxed{P.f = 0}$

Wattless Current ⇒ Wattless current is that component of the circuit current due to which power consumed in the circuit is zero

★ $I_{rms}\sin\phi$ component of current will not consume any power



★ ~~I_{rms}~~,

Wattfull Current ⇒ It is that component of circuit current due to which power is consumed in the circuit.

LC- Oscillations

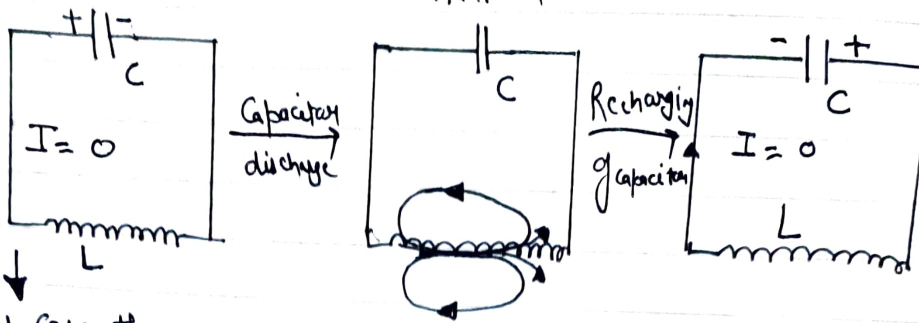
Electrical oscillations produced by the exchange of energy b/w a Capacitor which stores electrical energy and inductor which stores energy in the form of magnetic field.

Tank circuit ⇒ A circuit containing inductor and capacitor connected in parallel is known as tank circuit.

$U_E = \frac{1}{2}\frac{Q_0^2}{2C}$ — Charge Maximum

$U_B = \frac{1}{2}LI^2$ — Current Maximum

$U_B = 0$, $U_E = \frac{1}{2}\frac{Q_0^2}{C}$ — Charge Maximum



In this case the Capaciter is fully Charged and stores energy in the form electric field

In this condition due to discharge of capacitor current start flowing and electrical energy gets converted into magnetic energy

In this capacitor starts ~~dis~~ recharging and energy gets fully converted into electrical energy from magnetic energy

So this conversion of energy b/w inductor and capacitor Continues and this is called LC oscillations.

TRANSFORMER

It is a device used to convert low alternating voltage at higher current into high alternating voltage at low current.

OR

It is a device which changes the alternating voltage.

Principle ⇒ It is based upon principle of mutual induction.

Construction ⇒ (i) It consist of two separate coil of insulated wire wound wound on same iron core

(ii) The coil to which input voltage is applied is called primary coil and the coil across which output is taken is known as secondary coil.

Working ⇒ When an alternating Emf is applied across primary coil, an alternating current flows through it. Due to the flow of alternating current alternating magnetic flux is produced.

This alternating flux will be linked with primary and secondary coil and will induce Emf

$$E_p = -N_p \frac{d\phi}{dt} \quad -(1)$$

{ N_p = No. of turns in primary coil
N_s = No. of turns in secondary coil

$$E_s = -N_s \frac{d\phi}{dt} \quad -(2)$$

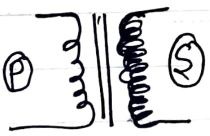
→ Divide equation (2) by (1)

$$\cancel{\frac{E_s}{E_p}} = \boxed{\frac{E_s}{E_p} = \frac{N_s}{N_p}}$$

[$K = \frac{N_s}{N_p}$ = Transformation Ratio] → ★

Types Of transformer

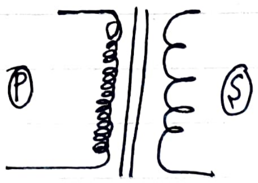
Step Up Transformer



$K > 1$

$\boxed{N_s > N_p}$

Stepdown transformer



$K < 1$

$\boxed{N_s < N_p}$

Ideal Transformer ⇒ A transformer is said to be ideal if there is no power loss takes place.

Input power = Output power

$$E_p I_p = E_s I_s$$

$$\boxed{\frac{E_s}{E_p} = \frac{I_p}{I_s}}$$

from ~~x~~ equation

$$\boxed{\frac{N_s}{N_p} = \frac{I_p}{I_s}}$$

Non Ideal transformer ⇒ If there is energy loss taking place in the transformer then it is known as non ideal transformer

efficiency of Non Ideal transformer $= \eta = \frac{\text{Output power}}{\text{Input power}}$

$$\boxed{\eta = \frac{E_s I_s}{E_p I_p}}$$

η varies from 90 to 99%

Energy loss in Transformer

(i) Copper loss ⇒ Energy loss in the copper wiring due to its resistance.

$$\boxed{\text{Copper loss} = I^2RT}$$

Method to Reduce ⇒ It can be reduced by using wire of large cross-sectional area.

(ii) Flux leakage loss ⇒ It is the loss due to flux ~~not~~ leakage i.e total flux produced by primary coil does not reach to secondary coil.

Method to Reduce ⇒ flux leakage can be reducing by winding secondary coil over primary coil.

(iii) **Iron Losses**

Eddy Current loss

It is the loss caused due to heating effect by eddy current in core

* It can be reduced by using laminated core

Hysteresis losses.

When alternating current is passed through primary coil coil alternating magnetic field is produced due to which iron core gets magnetised and demagnetised which causes loss of energy.

* It can be reduced by using material having narrow hysteresis loop.

(iv) **Humming Loss** ⇒ A transformer produces humming noise due to expansion and contraction of core due to magnetisation and demagnetisation.