ELECTRIC CHARGES AND FIELD

ELECTROSTATIC ⇒ The branch of physics which deals with the study of charges at rest is known as electrostatic.

Application Of Electrostatics ⇒

(I) Xerox Machine (ii) Electrostatic precipitators
(III) Spray Painting

Electric Charge ⇒ It is the intrinsic property of elementary particle due to which they interact with each other.

It is a scalar quantity
Its SI unit is coulomb.
Its C.G.S unit is stat Coulomb.
Its Dimension is [AT]

Type Of Charges

Positive Charge

(I) It is the charge on proton

(II) Earlier positive charge was known as vitreous [Glass Rod]

(III) Its magnitude is $+1.6 \times 10^{-19}$ C

Negative Charges

(I) It is the charge on electron.

(II) Negative Charge was known as resinous [Amber]

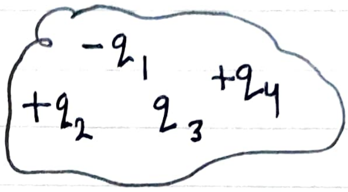
(III) Its magnitude is -1.6×10^{-19} C

PROPERTIES OF CHARGES

(I) Like Charges repell each other and unlike Charges attract each other.

2 Additivity Of Electric Charges ⇒ The total Charge on a body is algebric sum of charges on the body.

$$Q = -q_1 + q_2 - q_4 + q_3$$



3 Conservation Of Charges ⇒ The net charge of an isolated system is conserved.

Example ⇒

(I) Annihilation Of Matter

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

Before Annihilation = $-e + e = 0$

charge after Annihilation = 0

{ γ = Neutral

(II) In pair production ⇒ γ ray photon on interacting with matter transforms into an electron (e^-) and positron (e^+)

$$\gamma \rightarrow e^- + e^+$$

4 Quantization Of Charge ⇒ According to this the total Charge on a body is an integeral multiple of elementary Charge.

$$\boxed{Q = \pm ne}$$

$n = 1, 2, 3, \ldots$

$n \neq \frac{1}{2}, \frac{3}{5}$

$e = 1.6 \times 10^{-19}$ C

Quantization of charge is even valid at quark level. Quark are the constituent of electron, proton and neutron.

Name of Quark	**Name Of Anti Quark**
(I) Up Quark $= +\frac{2}{3}e$ [u]	Antiup $= -\frac{2}{3}e$
(II) Down Quark $= -\frac{e}{3}$ [d]	Antidown $= \frac{e}{3}$
(III) Charm $= \frac{2}{3}e$ [c]	Anticharm $= -\frac{2e}{3}$
(IV) Strange $= -\frac{e}{3}$ [s]	Antistrange $= \frac{e}{3}$
(V) Top $= +\frac{2e}{3}$ [t]	Anti top $= -\frac{2e}{3}$
(VI) Bottom $= -\frac{e}{3}$ [b]	Anti bottom $= \frac{e}{3}$

Constituent Of electron $= \frac{2}{3}e + \frac{2}{3}e - \frac{e}{3} = e$ [uud]

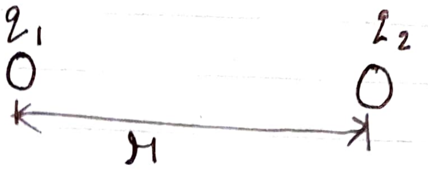
" " ~~Pro~~ Neutron $= \frac{2}{3}e - \frac{e}{3} - \frac{e}{3} = 0$ [udd]

Comparison Between Charge and Mass

Electric Charge	Mass
(1) It can be positive negative or zero	It is always positive
(2) It is considered to be quantized	It is not quantized
(3) Charges are Conserved	Mass is not conserved
(4) Charges are not affected by the velocity of Charge	Mass is affected by the velocity $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$
(5) Charges can not exist without mass.	Mass can exist without Charge.
(6) Electric Charges is derived quantity.	It is a fundamental quantity.

Coulomb's law ⇒ According to Coulombs law, the magnitude of force of interaction b/w any two point charges at rest is directly proportional to the product of magnitude of charges and inversely proportional to the square of the distance b/w them.

(i) $F \propto q_1 q_2$ — ①



(ii) $F \propto \frac{1}{r^2}$ — ⑪

Combining equation ① and ②

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K\frac{q_1 q_2}{r^2}$$

$K = 9 \times 10^9 \, N m^2 C^{-2}$ for vacuum

$K = \frac{1}{4\pi\varepsilon_0}$

$\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

ε_0 = permitivity of free space.

Permitivity ⇒ It is the ability of a medium to allow the electric charges to interact with each other. Its Dimension is $[M^{-1} L^{-3} T^4 A^2]$

Relative Permitivity OR Dielectric Constant ⇒ It is defined as the ratio of force exerted b/w two charges when they are placed in vacuum to the force when they are placed in some medium provided that their magnitude and distance b/w them remain same.

$$\varepsilon_r = K = \frac{F_v}{F_m} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon_m}\frac{q_1 q_2}{r^2}}$$

$$K = \frac{\varepsilon_m}{\varepsilon_0} \Rightarrow \varepsilon_m = K\varepsilon_0$$

* Dielectric constant has no unit and dimension.
* $E_r = \infty$ for metal
* Dielectric constant decreases with increase in temprature

Characteristics of Coulomb's force

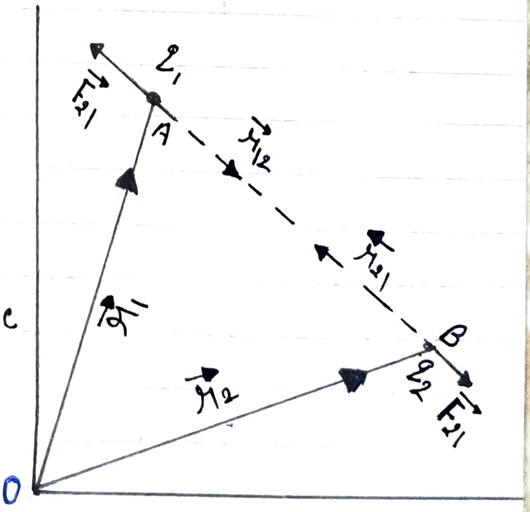
(i) It is central force
(ii) It is spherically symmetric.
(iii) It follows inverse square law.
(iv) It can be attractive or repulsive.

Draw back of Coulomb's law -

(i) It is only valid for point charges.
(ii) It is only valid when the charges are at rest.
(iii) It is only valid when distance b/w two charges is more than 10^{-15} m.

Coulomb's force b/w Charges in terms of position Vector

Let us consider two charges q_1 and q_2 placed in vacuum having their position vector $\vec{r}_1$ and $\vec{r}_2$



Force on charge 2 due to charge 1 is given by

$$\vec{F}_{21} = \frac{K q_1 q_2}{|\vec{r}_{12}|^2} \hat{r}_{12}$$

$\vec{r}_{12}$ = Position vector b/w A and B starting from A and ending at B

Now in $\triangle OAB$ using triangle law of vector addition

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{AB} = \vec{OB} - \vec{OA} \Rightarrow \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{F}_{21} = \frac{K q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$

Similiarly $\vec{F}_{12} = \frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$

Since $\vec{r}_{12} = -\vec{r}_{21}$
so $\vec{F}_{12} = -\vec{F}_{21}$

So Coulomb's law follows Newton's third law of motion.

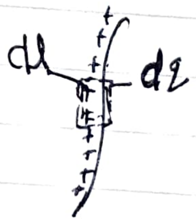
Charge Distribution

There are three type charge distribution

(i) Linear Charge Distribution
OR
Linear charge Density ⇒ It is defined as charge per unit length.

$$\lambda = \frac{dq}{dl}$$

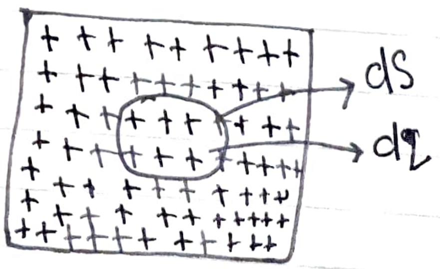
It's SI unit is $C m^{-1}$



(ii) Surface Charge Density ⇒ It is defined as charge per unit area.

$$\sigma = \frac{dq}{dS}$$

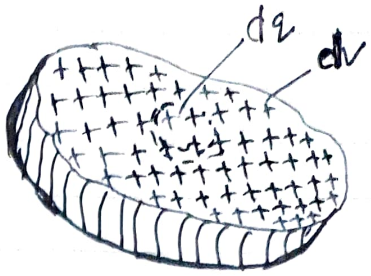
$\sigma = C m^{-2}$



(iii) Volume of Charge Density ⇒ It is defined as Charge per unit volume.

$$\rho = \frac{dq}{dv}$$

$\rho = Cm^{-3}$



ELECTRIC FIELD

It is the space around a charged body within which its influence can be felt by other Charge is called electric field.

Electric Field Intensity ⇒ The electric field intensity due to a source charge at any point in its electric field is defined as the force experienced by a unit positive charge placed at that point.

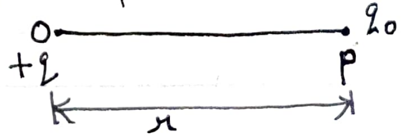
$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

q_0 = Test Charge

SI unit = NC^{-1}
Dimensional formula = $[MLT^{-3}A^{-1}]$
It is a vector quantity.

Electric Field Intensity Due to point Charges

Let us consider a charge at point P.
Now we have to Calculate E.F.I at Point P. To Determine that we will place test Charge at point P.

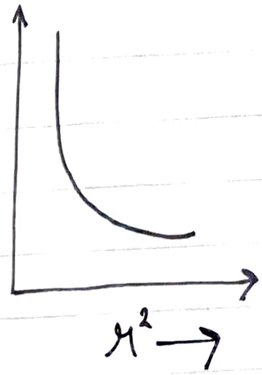


The force due to Charge on test charge

$$F = \frac{K q q_0}{r^2}$$

Now according to defination Of E.F.I

$$E = \frac{F}{q_0} = \frac{K q q_0}{r^2 q_0} = \frac{K q}{r^2}$$



$$\boxed{E = \frac{K q}{r^2}}$$

Electric Field line ⇒ It is the path followed by charge in the field of some other charge.

Properties Of Electric field line ⇒

(1) Electric field lines start from positive charge and ends at negative charge.

(2) The tangent at any point on an any electric field line gives the direction of electric field at that point.

(3) Two electric field lines do not cross each other.

(4) Electric field lines do not form Closed loop.

(5) Electric field lines do not pass through conductor, But they pass through ~~inductor~~ dielectric.

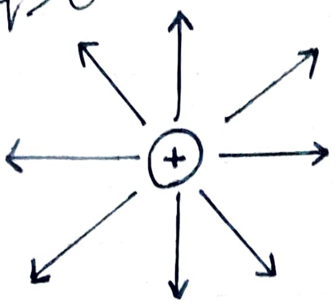
(6) Electric field lines are perpendicular to the surface of positive and negative Charge.

Q→ ★★★ Draw electric field lines due to

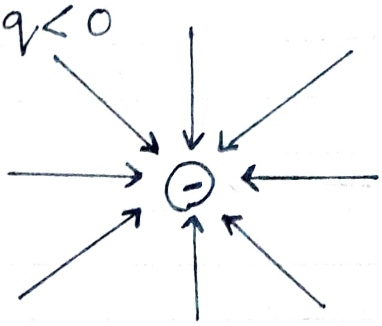
(i) $q > 0$ (ii) $q < 0$ (iii) Due to electric dipole
(iv) Two positive Charge (v) Two negative Charge
(vi) Sheet of Charge (VII) Sheet of positive and a negative Charge.

Current Electricity

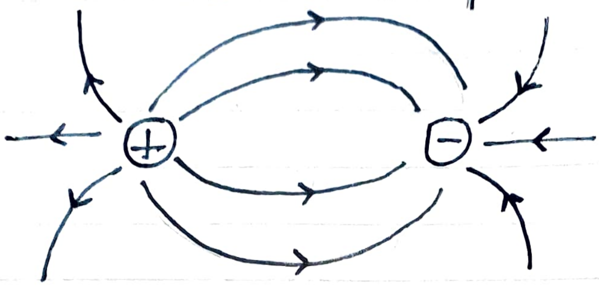
(i) $q > 0$



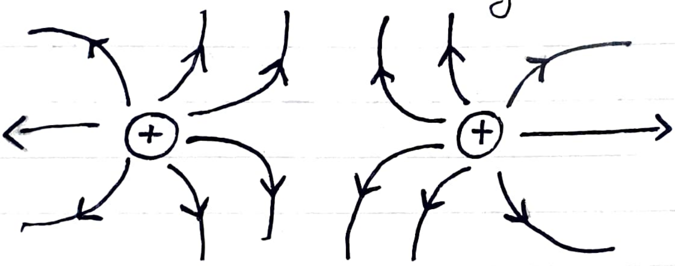
(ii) $q < 0$



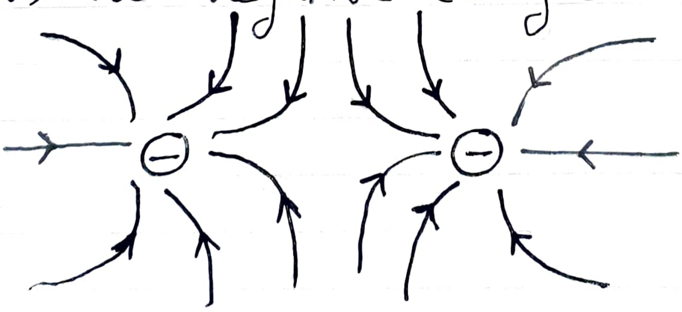
(iii) Due to electric dipole



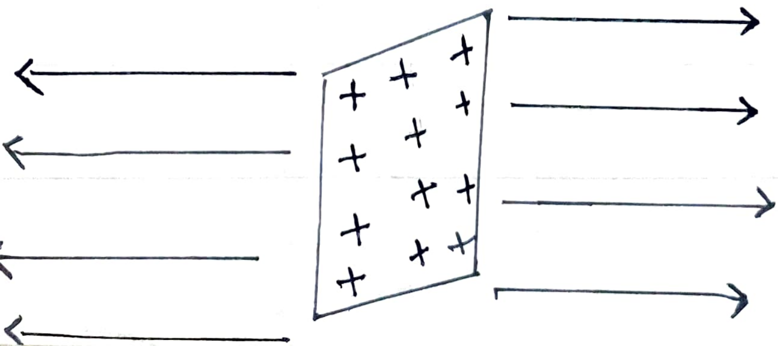
(iv) Two positive charge



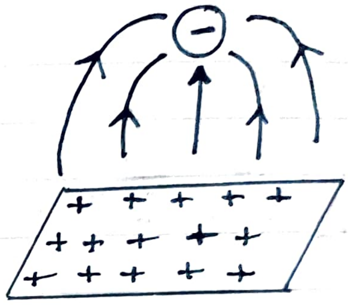
(v) Two negative charges



(VI) sheet of charge



(VII) Sheet of (+ve) and (-ve) charge



Electric Field Due To Circular loop of Charge

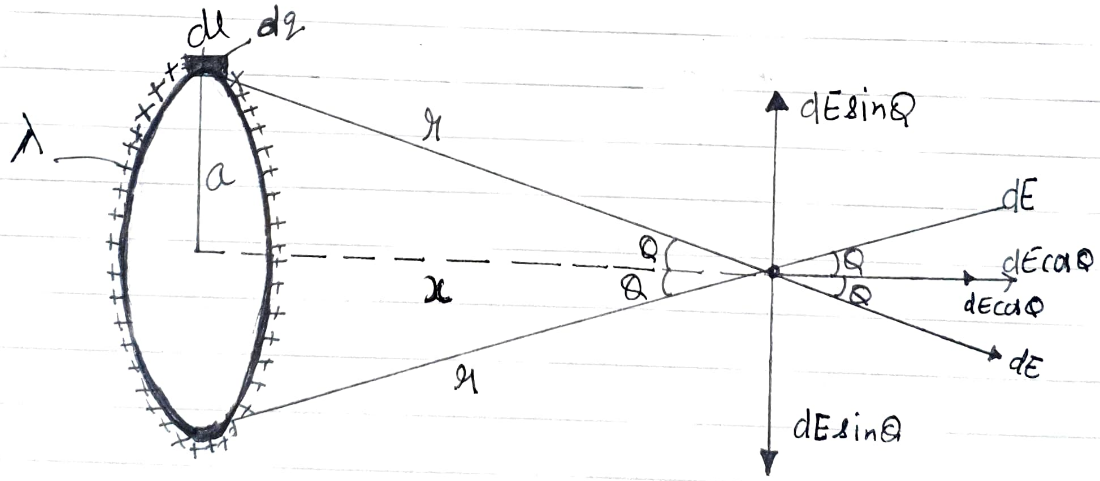
Let us consider an uniformly charged circular loop having Charge density λ

Let the radius of loop be a. Now Let a point P at a distance x from the centre of loop.

Now consider a small loop of the element of length dl having charge dq.

Now due to this element the electric field intensity will be dE. Now the dE can be resolved into two Component

(i) $dE \sin\theta$ along Y axis
(ii) $dE \cos\theta$ along X axis



The $dE \sin\theta$ component of electric field will be cancelled out by $dE \sin\theta$ component of the electric field by diametrically opposite element of the loop.

So net electric field due to element at P = $dE \cos\theta$

Total electric field intensity at point P due to loop $= \int_0^{2\pi a} dE \cos\theta$ — ☆

$\cos\theta = \frac{x}{r}$ — ①

$dE = \frac{K\,dq}{r^2}$

$\lambda = \frac{dq}{dl} \Rightarrow dq = \lambda\, dl$

$dE = \frac{K\lambda\, dl}{r^2}$ — ②

Putting the value of ① and ② in ☆ equation

Total electric field at point P $= \int_0^{2\pi a} \frac{K\lambda\, dl}{r^2}\left(\frac{x}{r}\right)$

$= \frac{K\lambda x}{r^3}\int_0^{2\pi a} dl$

$= \frac{K\lambda x}{r^3}\left[l\right]_0^{2\pi a} = \frac{K\lambda x}{r^3}\left[2\pi a - 0\right]$

T.E.F at P $= \frac{K\lambda x(2\pi a)}{r^3}$

$r = \sqrt{x^2 + a^2}$

$$\text{T.E.F at P} = \frac{K\lambda x 2\pi a}{(x^2+a^2)^{3/2}}$$

Case - I

When the point P lies at the centre $x = 0$

$E = 0$

Case II When the point P lies at a distance $x >> a$ then a^2 can be neglected in comparision to x^2

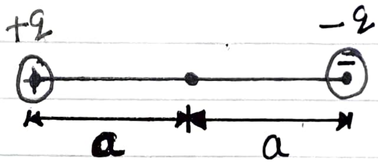
$$E = \frac{K\lambda \times 2\pi a}{(x^2)^{3/2}}$$

$$E = \frac{K\lambda 2\pi a}{x^2}$$

from the result we can see that loop behave as point charge.

Electric Dipole

A system of two equal and opposite charges separated by certain distance is called electric dipole.



Electric Dipole Moment ⇒ It is defined as the product of magnitude of either charge and distance b/w them.

$$\vec{p} = (2\vec{a})q$$

- ☆ Dipole moment is a vector quantity.
- ☆ Its direction is from negative to positive charge.
- ☆ Its SI unit is Coulomb meter [Cm]

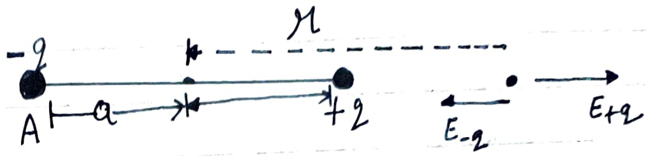
Ideal Dipole ⇒ An electric dipole is said to be ideal if it has following characteristics

(i) The magnitude of two charges should be large.

(ii) The distance b/w the two charges should be very small.

Electric Field on axial line of an electric dipole

The total electric field at point P



$$\vec{E} = \vec{E}_A + \vec{E}_B$$

$$|\vec{E}_A| = \frac{Kq}{(r+a)^2} \qquad |\vec{E}_B| = \frac{Kq}{(r-a)^2}$$

$$|\vec{E}| = |\vec{E}_B| - |\vec{E}_A|$$

Since the direction $\vec{E}_A$ is to the left so it will be taken as negative

$$E = \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2}$$

$$E = Kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E = Kq\left[\frac{(r+a)^2 - (r-a)^2}{(r+a)^2(r-a)^2}\right]$$

$$E = Kq\left[\frac{r^2+a^2+2ar - r^2 - a^2 + 2ar}{[(r+a)(r-a)]^2}\right]$$

$$E = Kq\left[\frac{4ar}{(r^2-a^2)^2}\right]$$

$$E = \frac{2Kpr}{(r^2-a^2)^2}$$

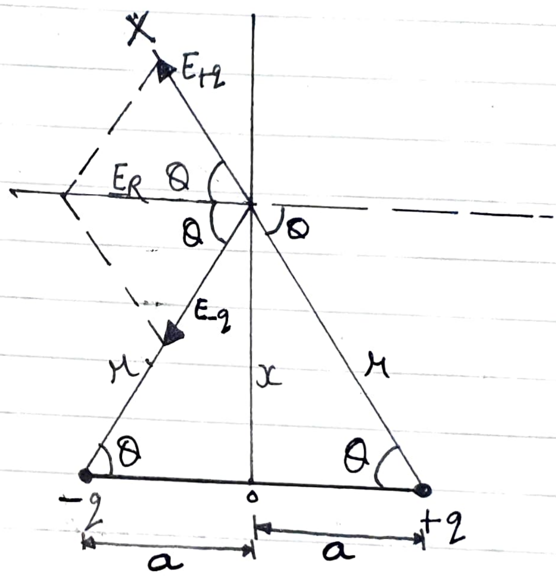
The direction of electric field will be from P to X

When the dipole is very small the a^2 can be ignored so

$$E = \frac{2Kpr}{r^4} \Rightarrow \boxed{E = \frac{2Kp}{r^3}}$$

Electric field On Equitorial line Of an electric dipole

The electric field at point C due to +q charge is from B to X. Similiarly due to (−q) charge is from C to A



The net electric field due to dipole on equitorial line will be

$$E_R = \sqrt{(\vec{E}_{+q})^2 + (E_{-q})^2 + 2E_{+q}E_{-q}\cos 2\theta}$$

$$|\vec{E}_{+q}| = |\vec{E}_{-q}| = E$$

$$E_R = \sqrt{E^2 + E^2 + 2E^2\cos 2\theta}$$

$$E_R = \sqrt{2E^2 + 2E^2\cos 2\theta}$$

$$E_R = \sqrt{2E^2(1+\cos 2\theta)}$$

$$\{(1+\cos 2\theta) = 2\cos^2\theta\}$$

$$E_R = \sqrt{2E^2\, 2\cos^2\theta}$$

$$E_R = 2E\cos\theta \quad \text{—— ☆}$$

$$\cos\theta = \frac{a}{r} \quad \text{— (1)} \qquad E = \frac{Kq}{r^2} \quad \text{— (2)}$$

Putting the value of ① and ② in ☆ equation

$$E_R = \frac{2Kq}{r^2}\frac{a}{r} = \frac{2Kqa}{r^3}$$

$\{ 2qa = p$

$$\boxed{E_R = \frac{Kp}{r^3}}$$

$r = (a^2+x^2)^{1/2}$

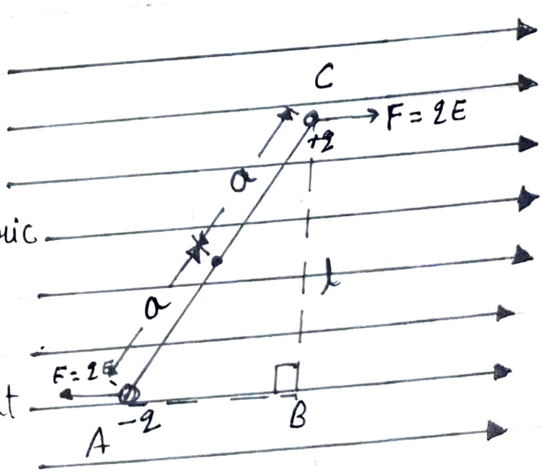
$$\boxed{E_R = \frac{Kp}{(a^2+x^2)^{3/2}}}$$

☆ When the length of dipole is very small in that case we can ignore a^2

$$E_R = \frac{Kp}{(x^2)^{3/2}} = \frac{Kp}{x^3} \Rightarrow \boxed{E_R = \frac{Kp}{x^3}}$$

Electric Dipole placed In Uniform Electric Field

The force on (+q) charge is in the direction of electric field.



The force on the negative charge is in opposite direction of the electric field.

The magnitude of force is same but direction is opposite

$$|\vec{F}_{+q}| = |\vec{F}_{-q}| = qE$$

So $F_{+q} + F_{-q} = 0$

☆☆ Net force on electric dipole is zero if it is placed in uniform electric field

Net force on dipole is not zero if it is placed in non uniform electric field

Net force on dipole is zero but this equal and opposite force will form couple and will exert Torque

$\tau =$ either force $\times \perp$ distance

$\tau = qE \times l$

In $\triangle ABC$ $\sin\theta = \frac{l}{2a}$

$l = 2a\sin\theta$

$\tau = qE\,2a\sin\theta$

$\tau = pE\sin\theta$ $\quad p = (q\,2a)$

$\tau = (\vec{p} \times \vec{E})$

Case I when $\theta = 0$

$\tau = 0$

This is known as stable Equilibrium

Case II when $\theta = 90°$

$\tau = max$

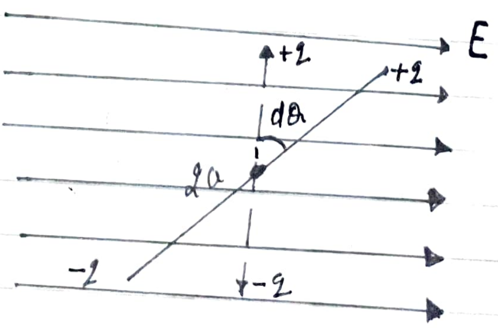
Case III when $\theta = 180°$

$\tau = 0$

This is known as unstable equilibrium

Potential Energy stored in an electric dipole when rotated in a uniform electric field.

Let us consider an electric dipole placed in a uniform electric field at an angle θ. Now suppose the dipole is rotated by an angle $d\theta$



So work done in rotating the dipole is

$$dw = \tau\, d\theta$$
$$dw = pE \sin\theta\, d\theta$$

total work done in rotating the dipole from θ_1 to θ_2

$$\int_0^w dw = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta$$

$$w = pE \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$w = -pE \left[\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$w = -pE\left[\cos\theta_2 - \cos\theta_1\right] \qquad \{w = u\}$$
$$u = -pE\left[\cos\theta_2 - \cos\theta_1\right]$$

case when $\theta_1 = 90^\circ$ $\theta_2 = \theta$

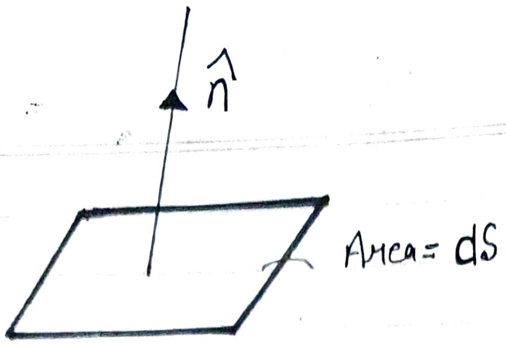
$$U = -pE\cos\theta$$

Area Vector ⇒ Some time the magnitude of area is not important only but its orientation is also important.

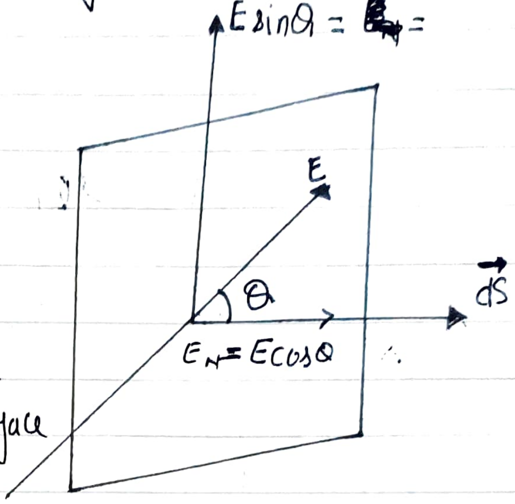
For Example The amount of water flowing through a ring depend upon the orientation of ring.

An area vector is represented by $\vec{ds}$

$\therefore$ $\vec{ds} = ds\,\hat{n}$ where $\hat{n}$ is a unit vector normal to the area.



Electric Flux ⇒ It is defined as the total number of electric lines of force crossing the surface in a direction normal to the surface.



Let electric lines of force are entering at angle θ with area vector so flux will be

$$\phi = E_N \, ds$$

$$\phi = E \, ds \cos\theta$$

$$\phi = \vec{E} \cdot \vec{ds}$$

{ ds = Area of the surface

For total flux ~~with~~ we will take surface integral of electric field of that surface.

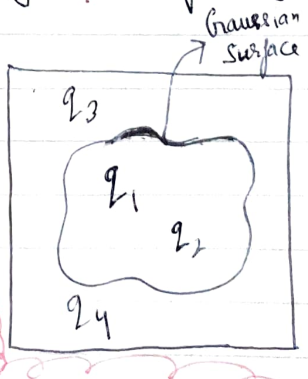
$$\phi_T = \oint \vec{E} \cdot \vec{ds}$$

☆ Electric field is a scalar quantity
☆ Its SI unit is $N m^2 C^{-1}$

Gauss' Theorem

It state that total flux through a closed surface enclosing a ~~the~~ charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of charge enclosed.

$$\phi = \frac{q_{net}}{\varepsilon_0}$$



$$\phi = \oint \vec{E} \cdot \vec{ds}$$

$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{net}}{\varepsilon_0}$$

E on left side is due to charges q_1, q_2, q_3 and q_4

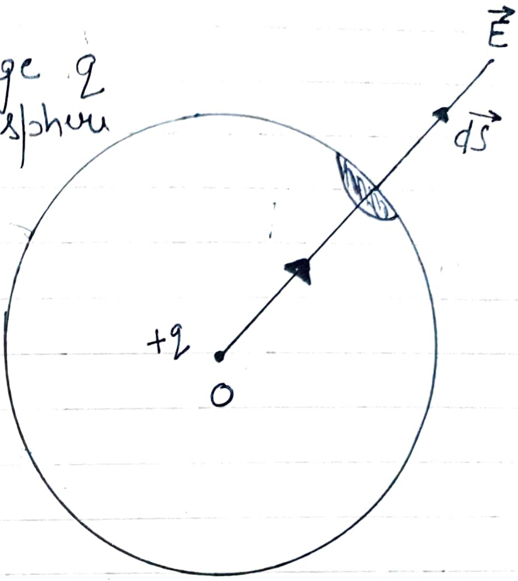
q_{net} on Right side is due to q_1 and q_2

Gauss' Theorem can also be defined as

" Surface integral of electric field over the closed surface is equal to $\frac{1}{\varepsilon_0}$ times the charge enclosed."

Proof of Gauss' Theorem ⇒

Let us consider a point charge q is placed at the centre of a sphere of radius r.



Let E be the value of electric field at any point on the surface of sphere.

$$|\vec{E}| = \frac{Kq}{r^2} \quad \text{—— (1)}$$

The flux coming out through the small area ds is

$$d\phi = \vec{E}\cdot\vec{ds} = E\,ds\cos\theta$$

{ $\theta = 0°$ Since $\vec{E}$ and $\vec{ds}$ are in same line

$\therefore$ $d\phi = E\,ds\cos 0°$

$d\phi = E\,ds$

$\therefore$ total flux coming out

$$\oint d\phi = E\oint ds$$

$$\phi = E \times 4\pi r^2 \quad \text{—— (2)}$$

Putting the value of (1) in equation (2)

$$\phi = \frac{Kq}{r^2} \times 4\pi r^2$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \times 4\pi r^2 \Rightarrow \phi = \frac{q}{\varepsilon_0}$$

This theorem holds for any shape of surface

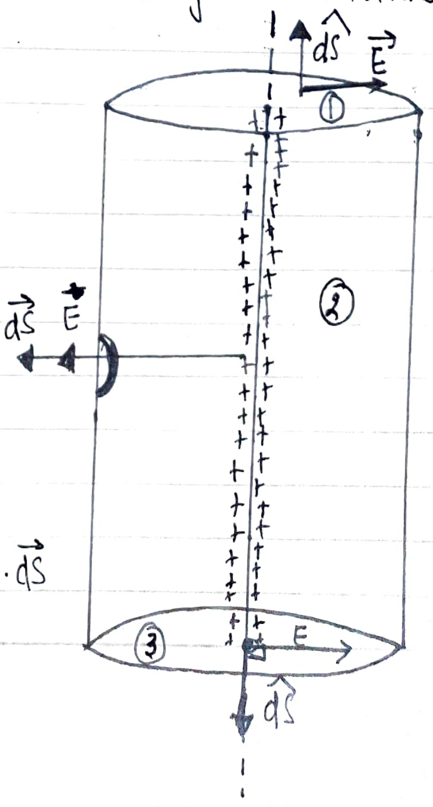
Gaussian Surface ⇒ Gaussian surface around a charge distribution is closed surface such that electric field intensity at all the points on the surface is same and the electric flux through the surface is along the normal to the surface.

APPLICATION OF GAUSS' THEOREM

1. Electric Field Due To Infinitely long Wire having Uniform Charge Density

Let us consider a straight wire having positively charge with charge density λ

Now since we have to calculate the electric field at point P which is at a distance r. To do so we will draw Gaussian surface which is of cylindrical shape with height l and radius r. Now According to Gauss' Theorem



$$\phi = \frac{q}{\varepsilon_0} \quad \text{—— (1)}$$

$$\lambda = \frac{q}{l} \Rightarrow q = \lambda l \quad \text{—— (2)}$$

from equation (1) and (2)

$$\phi = \frac{\lambda l}{\varepsilon_0} \quad \text{—— (A)}$$

$$\phi = \oint_1 \vec{E}\cdot\vec{ds} + \oint_2 \vec{E}\cdot\vec{ds} + \oint_3 \vec{E}\cdot\vec{ds}$$

$$\phi = \oint_{1} \cancel{E\,ds\cos\theta_1}^{\,0} + \int_{2} E\,ds\cos\theta_2 + \int_{3} \cancel{E\,ds\cos\theta_3}^{\,0}$$

$$\theta_1 = \theta_3 = 90 \qquad \theta_2 = 0^\circ$$

$$\phi = \oint E\,ds\cos 0^\circ$$

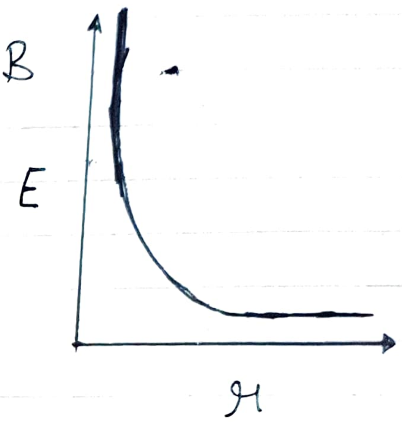
$$\phi = E\oint ds \qquad \{\text{Area of cylinder} = 2\pi r l\}$$

$$\phi = E\,2\pi r l \quad \text{——} \quad (B)$$

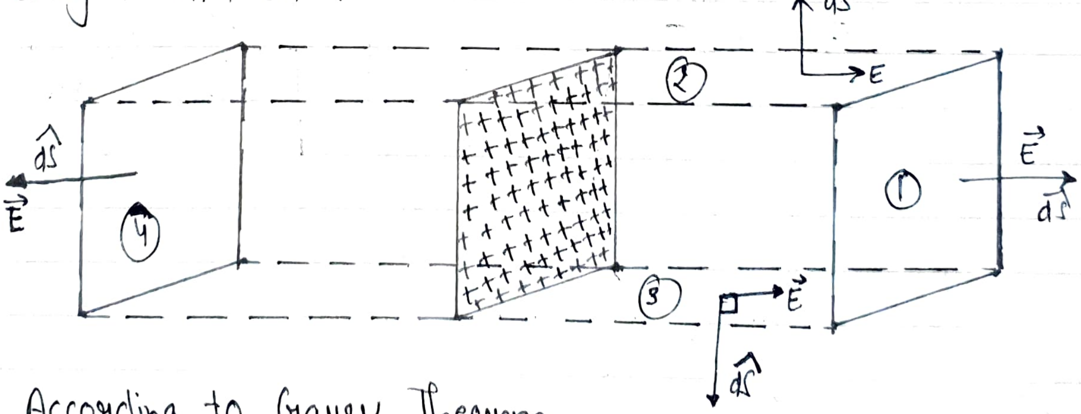
from equation (A) and B

$$E\,2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0} \qquad E \propto \frac{1}{r}$$



2. Electric Field Intensity At Any Point Due to Uniformly Charged Thin sheet.



According to Gauss' Theorem

$$\phi = \frac{q}{\varepsilon_0} \quad \text{——} \quad (1)$$

Let the surface charge density be σ

$$\sigma = \frac{q}{A} \Rightarrow q = \sigma A \quad - (2)$$

from equation (1) and (2)

$$\phi = \frac{\sigma A}{\varepsilon_0} \quad \text{—— (A)}$$

$$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4$$

$$\phi = \oint_1 E ds \cos\theta_1 + \oint_2 E ds \cos\theta_2 + \oint_3 E ds \cos\theta_3 + \oint_4 E ds \cos\theta_4$$

$$\theta_1 = \theta_4 = 0^\circ$$
$$\theta_2 = \theta_3 = 90^\circ$$

$$\phi = \oint E ds + \oint E ds$$

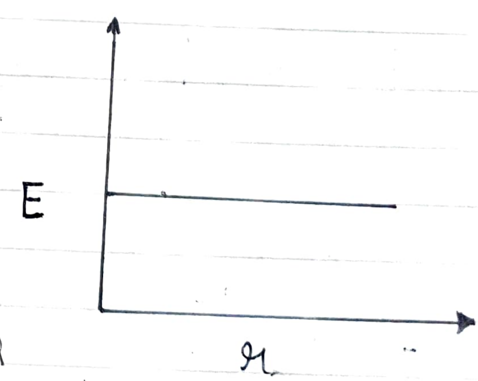
$$\phi = 2E \oint ds \qquad \left\{ \oint ds = A \right.$$

$$\phi = 2EA \quad \text{—— (B)}$$

from equation (A) and (B)

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$



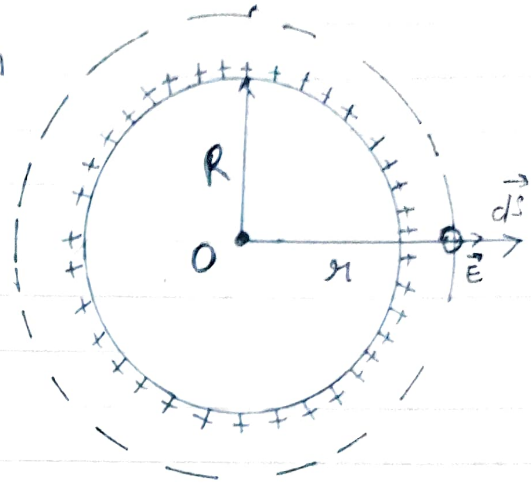
Electric field intensity will have same value at each and every point.

Electric Field Intensity due to Uniformly charged shell of radius R at

(i) $r > R$ (ii) $R = r$ (iii) $R > r$

Case I When $r > R$

According to Gauss' Theorem



$$\phi = \frac{q}{\varepsilon_0}$$

$$\sigma = \frac{q}{4\pi R^2}$$

$$q = \sigma 4\pi R^2$$

$$\phi = \frac{\sigma 4\pi R^2}{\varepsilon_0} \quad \text{—(1)}$$

$\cancel{\phi = E\cos\theta}$ $\quad \phi = \oint E\,ds\cos\theta$

$\theta = 0^\circ$ { Since area vector and electric field are in same line

$$\phi = \oint E\,ds$$

$$\phi = E\oint ds \Rightarrow \phi = E \times 4\pi r^2 \quad \text{—(2)}$$

from equation (1) and (2)

$$\frac{\sigma 4\pi R^2}{\varepsilon_0} = E \times 4\pi r^2$$

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

$$\boxed{E \propto \frac{1}{r^2}}$$

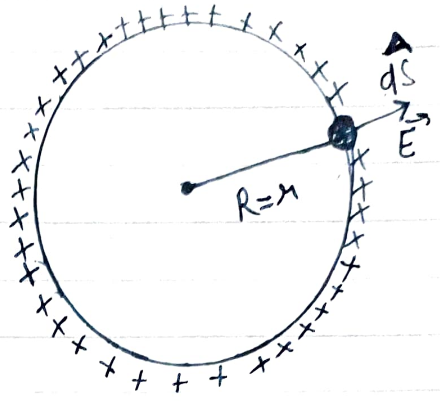
Case-II When $R = r$

According to Gauss' theorem

$$\phi = \frac{q}{\varepsilon_0} \quad \text{—(1)}$$

$$\phi = \oint \vec{E} \cdot \vec{ds}$$

$$\phi = E \times 4\pi r^2 \quad \text{—(2)}$$



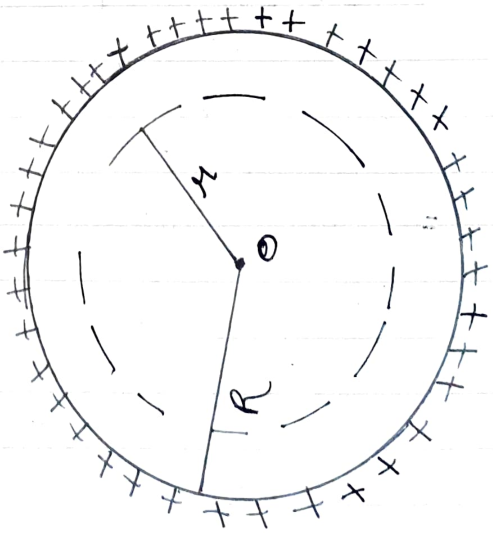
from equation (1) and (2)

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 4\pi R^2}$$

$$\left\{ \frac{q}{4\pi R^2} = \sigma \quad \text{since } r = R \right.$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$



Case-III

$$\phi = \frac{q}{\varepsilon_0}$$

$$\phi = \frac{0}{\varepsilon_0}$$

$$\phi = 0 \quad \text{—(1)}$$

$$\phi = \oint \vec{E} \cdot \vec{ds} \quad \text{—(2)}$$

from (1) and (2)

$$\oint \vec{E} \cdot \vec{ds} = 0$$

$$\boxed{E = 0}$$

as $ds \neq 0$