**QUESTION BANK**
**MATHEMATICS**
**CLASS- XII**
**CH-1 RELATIONS AND FUNCTIONS**

1 A function f is defined from R→ R as f(x) = ax + b, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one - one and onto or not.

2 Let A = {x: x∈ Z : 0 < x < 12}. Show that R = {(a, b): | a - b | is a multiple of 4} is

1. reflexive.
2. symmetric.
3. transitive and alsofind the set of elements related to 1.

3 Let R be the set of a non - zero real number. Then, show that f : R→ R, given by f(x) $= \frac{1}{x}$ is one - one and onto.

4 Let L be the set of all lines in a plane and R be the relation in L defined as R = {($L_1$ , $L_2$ ) : $L_1$ is perpendicular to $L_2$ }. Show that R is symmetric but neither reflexive nor transitive.

5 Let R be a relation on $N \times N$ , defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ $N \times N$ .Show that R is an equivalence relation.

6 Let $A = R - \{5\}$ and $B = R - \{1\}$ . Consider the function $f: A \rightarrow B$ , defined by $f(x) = \frac{x-3}{x-5}$ . Show that f is one - one and onto.

7 Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

1. R = {(x, y): x is wife of y}
2. R = {(x, y): x is father of y}

8 Let A = [ - 1, 1] . Then, discuss whether the following functions defined on A are one - one, onto or bijective:

1. $f(x) = \frac{x}{2}$
2. g(x) = |x|
3. h(x) = x|x|
4. k(x) = $x^2$

9 **Read the following text carefully and answer the questions that follow:**

Sohan is confused in the Mathematics topic ’Relation and equivalence relation’. To clear his concepts on the topic, he took help his elder brother. He has following notes on this topic.

**Relation:** A relation R from a set A to a set B is a subset of the cartesian product A × B obtained by describing a relationship between first element x and the second element 'y' of the ordered pairs in A × B. A relation R in a set A is called. :

**Reflexive:** If (a, a) ∈ $R\ \forall a \in A$ .

**Symmetric:** If ($a_1$ , $a_1$ ) ∈ $R$ ⇒ ($a_2$ , $a_1$ ) ∈ $R$ ∀ $a_1$ , $a_2$ ∈ R.

**Transitive:** If ($a_1$ , $a_1$ ) ∈ R and ($a_2$ , $a_3$ ) ∈ R ⇒ ($a_2$ , $a_3$ ) ∈ $R$ ∀ $a_1$ , $a_2$ , $a_3$ ∈ A

**Equivalence Relation:** A relation R in a set A is an equivalence relation if R is reflexive, symmetric and transitive.

1. Find the maximum number of equivalence relations on the set A = {1, 2, 3}. (1)
2. Consider the non - empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then show that R is transitive but not symmetric. (1)
3. Show that relation defined by $R_1$ = {(x, y) |$x^2$ = $y^2$ } x, y ∈ R is an equivalence relation. (2)

   **OR**

   Check whether the relation (R) **x is greater than y** for all x, y ∈ N is reflexive, symmetric or transitive. (2)

10 A school is organizing a debate competition with participants as speakers$S = \{S_1,\ S_2,\ S_3,\ S_4\}$ and these are judged by judges $J = \{J_1,\ J_2,\ J_3\}$ . Each speaker can be assigned one judge. Let $R$ be a relation from set $S$ to $J$ defined as $R = \{(x, y)$ : speaker $x$ is judged by judge $y, x \in\ S, y \in J\}$ .


Based on the above, answer the following:

1. How many relations can be there from S to J? **(1)**
2. A student identifies a function from S to J as$f = \{(S_1,\ J_1), (S_2,\ J_2)\ , (S_3,\ J_2), (S_4,\ J_3)\}$ Check if it is bijective. **(1)**
3. 
   a. How many one - one functions can be there from set S to set J? **(2) OR**

   b. Another student considers a relation$R_1 = \{(S_1,\ S_2), \{S_2,\ S_4)\}$ in set S. Write minimum ordered pairs to be included in $R_1$ so that $R_1$ is reflexive but not symmetric. **(2)**

## CH-2 TRIGONOMETRIC FUNCTIONS

1 Find the domain of $\sin^{-1}(x^2 - 3)$ .

2 Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$ .

3 Evaluate: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

4 Write the interval for the principal value of function and draw its graph: $\text{cosec}^{-1}$ x.

5 For the principal values, evaluate $\sin^{-1}[\cos\{2 cosec^{-1}(-2)\}]$

6 For the principal value, evaluate $\sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ .

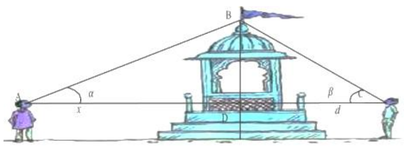
7 Find the value of x, if $\tan\left[\sec^{-1}\left(\frac{1}{x}\right)\right]$ = sin ($\tan^{-1}$ 2), x > 0.

8 Find the principal value of $\tan^{-1}\left(\cos\frac{\pi}{2}\right)$ .

9 Find the set of values of $\text{cosec}^{-1}\left(-\frac{1}{2}\right)$ .

10 **Read the following text carefully and answer the questions that follow:**

Two men on either side of a temple of 30 meters high observe its top at the angles of elevation $\alpha$ and $\beta$ respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters.



1. Find the measure of ∠ CAB = $\alpha$ in terms of sin and cos. (1)
2. Find the measure of ∠ BCA = $\beta$ . (1)
3. Find the measure of ∠ ABC. (2)

**OR**

What is the Domain and Range of $\cos^{-1}$ x. (2)

## CH-3 MATRICES

1 If A = $[a_{ij}]$ is a skew - symmetric matrix, then write the value of $\sum_i \sum_j a_{ij}$ .

2 Find the value of (x - y) from the matrix equation $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

3 Find the values of a and b for which $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

4 Prove that the product of matrices

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} and \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is the null matrix, when $\theta$ and $\phi$ differ by an odd multiple of $\frac{\pi}{2}$

5 If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that A satisfies the equation $A^3 - 4A^2 - 3A + 11I = 0$

6 Give an example of each of the matrices :A, B, and C such that AB = AC but B≠ C, A ≠ O

**CH-4 DETERMINANTS**

1 Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB . Hence, solve the system of linear equations:

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

2 If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is written as B + C, where B is a symmetric matrix and C is a skew - symmetric matrix, then find B.

3 **Read the following text carefully and answer the questions that follow:**

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:


| Market | Products (in numbers) | | |
|---|---|---|---|
| | Pencil | Eraser | Sharpener |
| I | 10,000 | 2,000 | 18,000 |
| II | 6,000 | 20,000 | 8,000 |

If the unit Sale price of Pencil, Eraser and Sharpener are₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively, based on the information given above, answer the following questions:

1. What is the total revenue collected from Market - I? (1)
2. What is the total revenue collected from Market - II? (1)
3. What is the gross profit from both markets considering the unit costs of the three commodities as₹ 2.00, ₹ 1.00, and 50 paise respectively? (2)

**OR**

If matrix A = $[a_{ij}]_{2\times 2}$ , where $a_{ij}$ = 1, if i ≠ j and $a_{ij}$ = 0 if i = j, then what is the value of $A^2$ ? (2)

## CH-3 DETERMINANTS

4 Find the inverse of $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ matrices and verify that $A^{-1}A = I_3$

5 Using matrices, solve the system of equations: $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$ ; $\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$ ; $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$ (x, y, z ≠ 0)

6 If A = $\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$, verify that $(\text{adj } A)^{-1} = (\text{adj } A^{-1})$.

7 If A = $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find $A^{-1}$ and use it to solve the following system of the equations:

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$

8 Using matrices, solve the following system of linear equations:

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 3z = 11$$

9 If A= $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that A (adj A) = (adj A) A= $|A| I_3$ .

10 Using matrices, solve the following system of equations:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$ ,

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ , $\frac{6}{x} + \frac{9}{y} + \frac{-20}{z} = 2$

11 For what values of a and b, the system of equations

2x + ay + 6z = 8

x + 2y + bz = 5

x + y + 3z = 4

has:

1. a unique solution

2. infinitely many solutions
3. no solution

12 Area of a triangle whose vertices are ($x_1$ , $y_1$ ), ($x_2$ , $y_2$ ) and ($x_3$ , $y_3$ ) is given by the determinant

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since, area is a positive quantity, we always take the absolute value of the determinant A. Also, the area of the triangle formed by three collinear points is zero.

1. Find the area of the triangle whose vertices are ( - 2, 6), (3, - 6), and (1, 5). (1)
2. If the points (2, - 3), (k, - 1) and (0, 4) are collinear, then find the value of 4k. (1)
3. If the area of a triangle ABC, with vertices A(1, 3), B(0, 0) and C(k, 0) is 3 sq. units, then find the value of k. (2)

   **OR**

   Using determinants, find the equation of the line joining the points A(1, 2) and B(3, 6). (2)

13 **Read the following text carefully and answer the questions that follow:**

Reena wants to donate a rectangular plot of land for a school of her village. When she was asked by construction agency to give dimensions of the plot, she said that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 $m^2$ .


1. In a rectangular plot of land, if the length is represented as "x" meters and the breadth is represented as "y" meters, how can the situation described be expressed in the form of a system of linear equations? (1)
2. Given the system of linear equations:(1)

x -  y = 50

2x + y = 550

How can we express this system of equations in the form of a matrix equation?

3. If A = $\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$ then what is the value of A $^{-1}$ ?(2)

**OR**

Reena donated a rectangular plot of land to the school. Can you please provide the dimensions of the plot in terms of its length and breadth? (2)

## CH-5 CONTINUITY AND DIFFERENTIABILITY

1 Differentiate the functionw.r.t. x: $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$.

2 Differentiate the functionw.r.t. x: $\tan^{-1}$ (cot x).

3 Differentiate the function with respect to x: $\log_x 2$

4 Differentiating the function w.r.t. x: $\tan^{-1}\left(\frac{3-2x}{1+6x}\right)$.

5 Find the value of k sothat the function f is continuous at the indicated point:$f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, & if\ x \neq 2 \\ k, & if\ x = 2 \end{cases}$ at x = 2.

6 Find$\frac{dy}{dx}, if x^y \cdot y^x = x^x$ .

7 If x = a (cos$\theta$ + $\theta$ sin $\theta$ ) and y = a (sin $\theta$ -  $\theta$ cos $\theta$ ), then find $\frac{d^2y}{dx^2}$

8 Differentiate the functionw.r.t. x: $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$.

9 Check the differentiability of function$f(x) = x|x|$ at $x = 0$ .

10 If$y\sqrt{x^2+1} - \log\left(\sqrt{x^2+1} - x\right) = 0$ prove that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

11 If$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$ , then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

12 Find$\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$

13 **Read the following text carefully and answer the questions that follow:**

Logarithmic differentiation is a powerful technique to differentiate functions of the form f(x) = [ u(x)] $^{v(x)}$ ] , where both u(x) and v(x) are differentiable functions and f and u need to be positive functions.

Let function y = f(x) = (u(x)) $^{v(x)}$ , then y’ = $y\left[\frac{v(x)}{u(x)}u'(x) + v'(x) \cdot \log[u(x)]\right]$

1. If x =$e^{\frac{x}{y}}$ , then find $\frac{dy}{dx}$ . (1)

2. If y = $(2 - x)^3 (3 + 2x)^5$ , then find $\frac{dy}{dx}$. (1)

3. If y = $x^x e^{(2x+5)}$ , then find $\frac{dy}{dx}$. (2)

**OR**

Differentiate $x^x$ w.r.t. x (2)

14 **Read the following text carefully and answer the questions that follow:**

If a relation between x and y is such that y cannot be expressed in terms of x, then y is called implicit function of x.

Assume a function, y = $6x^2 - 11e^y$

This function can be rewritten as

$y + 11e^y = 6x^2$

But it is not possible to completely separate and represent it as a function of y. This type of function is known as an implicit function.

To differentiate an implicit function, we consider y as a function of x and then we use the chain rule to differentiate any term consisting of y.

Now to differentiate the above function, we differentiate directly w.r.t. x the entire function. This step basically indicates the use of chain rule.

i.e., $\frac{dy}{dx} + \frac{d(11e^y)}{dx} = \frac{d(6x^2)}{dx}$

$\Rightarrow \frac{dy}{dx} + 11e^y \frac{dy}{dx} = 12x$

$\Rightarrow \frac{dy}{dx}(1 + 11e^y) = 12x$

$$\Rightarrow \frac{dy}{dx} = \frac{12x}{(1 + 11e^y)}$$

1. If $x^3 + x^2 y + xy^2 + y^3 = 81$, then find $\frac{dy}{dx}$. (1)

2. Find the slope of the tangent to the curve y = $x^2 + 6y^2 + xy$. (1)

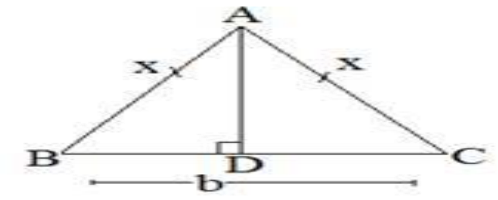
3. Find $\frac{dy}{dx}$ at x = 1, y = $\frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$. (2)

**OR**

If y = $(\sqrt{x})^y$ , then find $\frac{dy}{dx}$. (2)

## CH-6 APPLICATION OF DERIVATIVES

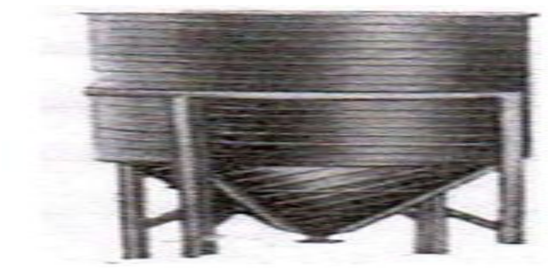
1. The two equal sides of an isosceles△ with fixed base b are decreasing at the rate of 3cm/s. How fast is the area decreasing when the two equal sides are equal to the base?




2. A kite is moving horizontally at a height of 151.5 meters. If the kite speed is 10 m/s, how fast is the string being let out; when the kite is 250 m away from the boy who is flying the kite? The height of boy is 1.5 m.

3 **Read the following text carefully and answer the following questions:**

A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat-bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 $cm^3$/s. The semi - vertical angle of the conical tank is $45^o$ .

1. Find the volume of water in the tank in terms of its radius r. (1)
2. Find rate of change of radius at an instant when r =$2\sqrt{2}$ cm. (1)
3. Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius r =$2\sqrt{2}$ cm. (2)

   **OR**

   Find the rate of change of height **h** at an instant when slant height is 4 cm.(2)

4 Find the values of b for which the function $f(x) = sin\,x - bx + c$ is a decreasing function on R.

5 What are the values of 'a' for which f(x) = $a^x$ is decreasing on R?

6 Find the intervals on which the function f(x) = $x^3 + 2x^2 - 1$ is

1. increasing
2. decreasing.

7 Find the interval in function f(x) = 10 - 6x - $2x^2$ isincreasing or decreasing.

8 Find the intervals in which the function f(x) = 20 - 9x + $6x^2$ - $x^3$ is (i) strictly increasing. (ii) strictly decreasing.

9 Show that the function f(x) = $\cot^{-1}$ (sin x + cos x) is decreasing on (0, $\frac{\pi}{4}$) and increasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

11 

A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point $x$ metres from the start of the street can be modelled by $f(x) = e^x \sin x$ , where $x$ is in metres.

Based on the above, answer the following:

1. Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$ . **(2)**
2. Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. **(2)**

12 The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

13 Prove that the area of a right angled triangle of given hypotenuse is maximum, when the triangle is isosceles.

14 Divide 15 into two parts such that the square of one multiplied with the cube of the other is minimum.

15 The rate of working of an engine is given byR =$15v + \frac{6000}{v}$ , where0<v<30and υ is the speed of the engine. Show that R is the least when υ = 20.

16 A square tank of capacity 250 cubic meters has to be dug out. The cost of the land is₹ 50 per square metre. The cost of digging increases with the depth and for the whole tank, it is ₹ ($400 \times h^2$ )where h metres is the depth of the tank. What should be the dimensions of the tank so that the cost is minimum?

17 **Read the following text carefully and answer the questions that follow:**

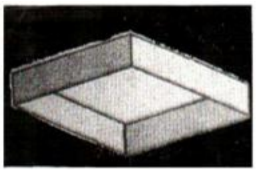
Ankit wants to construct a rectangular tank for his house that can hold 80 $ft^3$ of water.He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keepwidth of tank constant 5ft,but the length and height are variables. The top of the tank is open. Building the tank cost ₹ 20 per sq. foot for the base and ₹ 10 per sq. foot for the side.

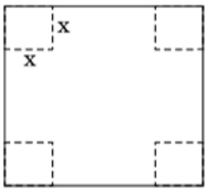
1. Express cost of tank as a function of height(h). (1)
2. Verify by second derivative test that cost is minimum at critical point. (1)
3. Find the value of h at which c(h) is minimum. (2)

   **OR**

   Find the minimum cost of tank? (2)

18 A factory makes an open cardboard box for a jewellery shop from a square sheet of side 18 cmby cutting off squares from each corner and folding up the flaps.






Based on the above information, answer **any four** of the following **five** questions, if xis the length of each square cut from corners: 4 × 1=4

1. The volume of the open box is: (1)
   a. $4x(x^2 - 18x + 81)$
   b. $2x(2x^2 + 36x + 162)$
   c. $2x(2x^2 + 36x - 162)$
   d. $4x(x^2 + 18x + 81)$
2. The condition for the volume (V) to be maximum is:(1)
   a. $\frac{dV}{dx} = 0 and \frac{d^2 V}{dx^2} < 0$
   b. $\frac{dV}{dx} = 0 and \frac{d^2 V}{dx^2} > 0$
   c. $\frac{dV}{dx} > 0 and \frac{d^2 V}{dx^2} = 0$

d. $\frac{dV}{dx} < 0 and \frac{d^2 V}{dx^2} = 0$

3. What should be the side of square to be cut off so that the volume is maximum?(1)

   a. 6 cm

   b. 9 cm

   c. 3 cm

   d. 4 cm

4. Maximum volume of the open box is:(1)

   a. 423 $cm^3$

   b. 432 $cm^3$

   c. 400 $cm^3$

   d. 216 $cm^3$

5. The total area of the removed squares is:(1)

   a. 324 $cm^2$

   b. 144 $cm^2$

   c. 36 $cm^2$

   d. 64 $cm^2$

19 **Read the following text carefully and answer the questions that follow:**

Mrs. Mayais the owner of a high - rise residential society having 50 apartments. When he set rent at ₹ 10000/month, all apartments are rented. If he increases rent by ₹ 250/ month, one fewer apartment is rented. The maintenance cost for each occupied unit is ₹ 500/month.

1. If P is the rent price per apartment and N is the number of rented apartments, then find the profit. (1)

2. If x represents the number of apartments which are not rented, then express profit asa function of x. (1)

3. Find the number of apartments whichare not rented so that profit is maximum. (2)

**OR**

Verify that profit is maximum at critical value of x by second derivative test. (2)

20 **Read the following text carefully and answer the questions that follow:**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$ , the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the lightintensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

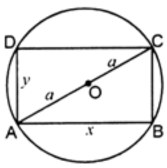
1. If l(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum. (1)
2. Find the darkest spot between the two lights. (1)
3. If you are in between the lamp posts, at distance x feet from the stronger light, thenwrite the combined light intensity coming from both lamp posts as function of x. (2)

**OR**

Find the minimum combined light intensity? (2)

21 **Read the following text carefully and answer the following questions:**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)




1. Find the perimeter of rectangle in terms of any one side and radius of circle. (1)
2. Find critical points to maximize the perimeter of rectangle? (1)

3. Check for maximum or minimum value of perimeter at critical point. (2)

**OR**

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region. (2)

22 A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

1. Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. **(1)**
2. Find $\frac{dS}{dx}$. **(1)**
3. 
   a. Find a relation between x and y such that the surface area (S) is minimum. **(2) OR**

   b. If surface area (S) is constant, the volume (V)= $\frac{1}{4}(Sx - 2x^3)$ , x being the edge of base. Show that volume ( V ) is maximum for $x = \sqrt{\frac{S}{6}}$. **(2)**

## Ch-7 INTEGRAL

1 Evaluate the integral: $\int \frac{1}{1+2\cos x} dx$

2 Evaluate: $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} dx$

3 Evaluate: $\int \frac{x^2}{(x-1)(x+1)^2} dx$

4 Evaluate: $\int (2\tan x - 3\cot x)^2 dx$

5 Evaluate: $\int_0^{\pi} \cos 2x \log \sin x \, dx$

6 Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$

7 Evaluate: $\int \frac{x}{\sqrt{x+z}\sqrt{x+z}} dx$

8 Evaluate: $\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$

9 Evaluate the integral: $\int x^2 \tan^{-1} x dx$

10 Evaluate: $\int \frac{(3\sin x-2)\cos x}{13-\cos^2 x-7\sin x} dx$

11 Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x} dx$

12 Evaluate: $\int \frac{1}{\sin x-\sin 2x} dx$

13 If f'(x) = a sin x + b cos x and f'(0) = 4, f(0) = 3, $f\left(\frac{\pi}{2}\right)$ = 5, find f(x).

14 Find: $\int \frac{dx}{\sin x+\sin 2x}$

15 Evaluate the integral: $\int (2x + 3)\sqrt{4x^2 + 5x + 6}dx$

16 Evaluate: $\int \frac{(1-3x)}{(3x^2+4x+2)} dx$

17 Integrate the (rational) function $\frac{3x-1}{(x+2)^2}$

18 Evaluate: $\int \frac{1}{5+7\cos x+\sin x} dx$

19

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

20 Evaluate the definite integral $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x+4\sin^2 x}$

## Ch-8 APPLICATION OF INTEGRAL

1 Using integration, find the area of the region given below: $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$

2 Sketch the region $\{(x, 0): y = \sqrt{4 - x^2}\}$ and x - axis. Find the area of the region using integration.

3 Find the area under the given curves and given lines: y = $x^4$ , x = 1, x = 5 and x - axis

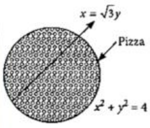
4 Find the area of theregion between the parabola x - $y^2$ - 6 y and the line x = - y.

5 Using integration, find the area of the region bounded by the line y - 1 = x, the x - axis and the ordinates x = - 2 and x = 3.

6 Using integration, find the area of △ ABC whose vertices are A(3, 2), B(5, 7) and C(7, 5).

7 **Read the following text carefully and answer the questions that follow:**

Siya loves to eat pizza. On her birthday, she invites her friends for pizza party in a famous pizza shop. She decided to cut pizza as her birthday cake. Siya cuts the pizza with a knife. Pizza is a circular in shape which is represented by $x^2 + y^2 = 4$ and the sharp edge of knife represents a straight line given by x = $\sqrt{3}y$




4. Find the point of intersection of line and pizza as shown in the given figure. (1)

5. Write the expression for the area bounded by the circular pizza and the edge of the knife in the first quadrant. (1)

6. Find the value of the region bounded by the circular pizza and the edge of the knife in the first quadrant. (2)

   **OR**

   Find the area of the each slice of pizza when Siya cuts the pizza into 4 equal pieces. Also, find area of whole pizza. (2)

8 **Read the following text carefully and answer the questions that follow:**

Consider the curve $x^2 + y^2 = 16$ and line y = x in the first quadrant.

7. Find the point of intersection of both the given curves. (1)

8. Evaluate $\int_0^{2\sqrt{2}}$ xdx. (1)

9. Find the value of the integral $\int_{2\sqrt{2}}^{4} \sqrt{16 - x^2}\, dx$ . (2)

   **OR**

   Find the area bounded by the two given curves. (2)

**Ch-9 DIFFERENTIAL EQUATIONS**

1 Find the general solution of the differential equation $\frac{dy}{dx} + \frac{2}{x}$ y = x

2 Verify that the given function is a solution of the corresponding diff eq. y = cosx + c; $y^1$ + sinx = 0

3 Verify that the given functions is a solution of the corresponding differentialequation $y = \cos x + c; y' + \sin x = 0$

4 Find the general solution of $(x + 2y^3)\frac{dy}{dx}$ = y

5 In the differential equation show that it is homogeneous and solve it: $\frac{dy}{dx} + \frac{x^2 - y^2}{3xy} = 0$ .

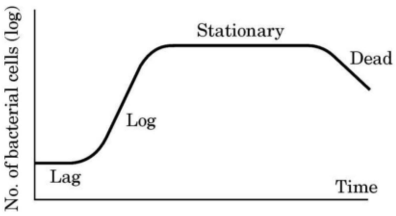
6 Solve the following differential equation.

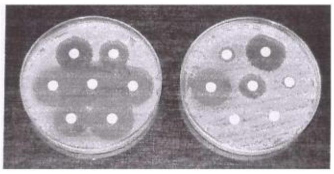
$$xy\log\left|\frac{y}{x}\right| dx + \left[y^2 - x^2\log\left|\frac{y}{x}\right|\right] dy = 0$$

7 Solve the differential equation: $\frac{dy}{dx} = \frac{1+y^2}{y^3}$

8 Solve the differential equation: $(x^2 - 1)\frac{dy}{dx} + 2(x + 2)y = 2(x + 1)$

9 A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.



The differential equation representing the growth of bacteria is given as: $\frac{dP}{dt} = kP$ , where $P$ is the population of bacteria at any time ' $t$ '.

Based on the above information, answer the following questions:

10. Obtain the general solution of the given differential equation and express it as an exponential function of 't'. **(2)**

11. If population of bacteria is 1000 at$t = 0$ , and 2000 at $t = 1$ , find the value of k. **(2)**

9 During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature ($25°C$ ). Initially the processor's temperature is $85°C$ . The rate of cooling is defined by the equation $\frac{d}{dt}\left(T(t)\right) = -k(T(t) - 25)$ ,

where $T(t)$ represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions:

12. Find the expression for temperature of processor,$T(t)$ given that $T(0) = 85°C$ . **(2)**

13. How long will it take for the processor's temperature to reach$40°C$ ? Given that $k = 0 \cdot 03, \log_e 4 = 1 \cdot 3863$ . **(2)**

## Ch-10 VECTORS ALGEBRA

1 The scalar product of the vector $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ is equal to 1. Find the value of $\lambda$ .

2 if $|\vec{a} + \vec{b}| = 60, |\vec{a} - \vec{b}| = 40 and |\vec{b}| = 46, find |\vec{a}|$

3 If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{c} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

4 If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, prove that $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$

5 If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$ .

6 Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$.

7 A vector $\vec{r}$ has magnitude 14 units and direction ratios 2, 3, - 6. Find the direction cosines and components of $\vec{r}$ , given that $\vec{r}$ makes an acute angle with x - axis.

8 If $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k},\ \vec{b} = 2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ find (i) Magnitude of $\vec{a} \times \vec{b}$ (ii) A unit vector which is $\perp$ *to both* $\vec{a}$ and $\vec{b}$ (iii) The cosine and sine of the angle b/wthe vectors $\vec{a}$ and $\vec{b}$ .

9 If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

10 **Read the following text carefully and answer the questions that follow:**

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, - 3, - 2) and ( - 2, 2, 6) respectively.


14. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then find $\vec{a} + \vec{b} + \vec{c}$ . (1)

15. What is the Area of $\triangle$ ABC. (1)

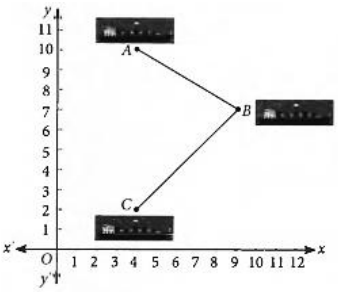
16. Suppose, if the given slogans are to be placed on a straight line, then find the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ . (2)

**OR**

If $\vec{a} = 2\,\hat{\imath} + 3\,\hat{\jmath} + 6\,\hat{k}$ , then find the unit vector in the direction of vector $\vec{a}$ . (2)

11 **Read the following text carefully and answer the questions that follow:**

A barge is pulled into harbour by two tug boats as shown in the figure.

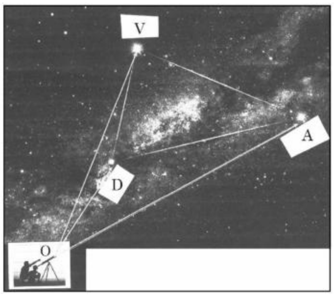



17. Find position vector of A.(1)
18. Find position vector of B. (1)
19. Find the vector$\overrightarrow{AC}$ in terms of $\hat{\imath}, \hat{\jmath}$ . (2)

**OR**

If $\vec{A} = 4\hat{\imath} + 3\hat{\jmath}$ and $\vec{B} = 3\hat{\imath} + 4\hat{\jmath}$ , thenfind $|\vec{A}| + |\vec{B}|$ (2)

11 An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O(0,0,0)$ and the three stars have their locations at the points $D, A$ and V having position vectors $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}, 7\hat{\imath} + 5\hat{\jmath} + 8\hat{k}$ and $-3\hat{\imath} + 7\hat{\jmath} + 11\hat{k}$ respectively.




Based on the above information, answer the following questions:

20. How far is the star V from star A? **(1)**

21. Find a unit vector in the direction of $\overrightarrow{DA}$ . **(1)**

22.
 a. Find the measure of $\angle VDA$ . **(2) OR**

 b. What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$ ? **(2)**

## Ch-11 THREE DIMENSIONAL GEOMETRY

1 If the equation of a line is x = ay + b, z = cy + d, then find the direction ratios of the line and a point on the line.

2 Find the equation of the line passing through the point (1, 2, - 4) and parallel to the line $\frac{x-3}{4} = \frac{y-5}{2} = \frac{z+1}{3}$.

3 Find the image of the point $(-1,5,2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$ . Find the length of the line segment joining the points (given point and the image point).

4 Two vertices of the parallelogram ABCD are given as $A(-1,2,1)$ and $B(1,-2,5)$ . If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$ , then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

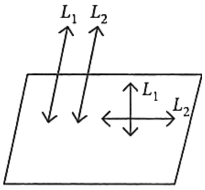
5 Find the equation of a line in vector and cartesian form which passes through the point $(1,2,-4)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ , and $\vec{r} = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k} + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$.

6 Find the co - ordinates of the foot of the perpendicular drawn from the point $(2,3,-8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ .

Also, find the perpendicular distance of the given point from the line.

7 **Read the following text carefully and answer the questions that follow:**

If $a_1$ , $b_1$ , $c_1$ and $a_2$ , $b_2$ , $c_2$ are direction ratios of two lines say $L_1$ and $L_2$ respectively. Then $L_1 \parallel L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$.




23. Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining B(1, 4, 6) and C(5, 4, 4). (1)

24. Find the direction ratios of the line which is perpendicular to the lines with direction ratios proportional to (1, - 2, - 2) and (0, 2, 1). (1)

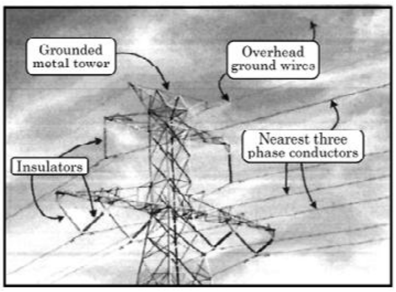
25. What is the relation between lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ and $\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2}$.

    (2)

    **OR**

    If $l_1$ , $m_1$ , $n_1$ and $l_2$ , $m_2$ , $n_2$ are direction cosines of $L_1$ and $L_2$ respectively, then what is the condition for $L_1$ parallel to $L_2$. (2)

8 Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.




Two such wires lie along the following lines:

$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$

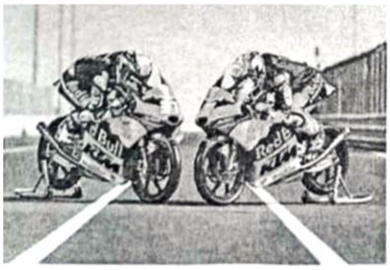
$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$

Based on the given information, answer the following questions:

26. Are the lines $l_1$ and $l_2$ coplanar? Justify your answer. (2)

27. Find the point of intersection of the lines $l_1$ and $l_2$ . (2)

9 Two motorcycles Aand Bare running at the speed more than the allowed speed on the roads represented by the lines$\vec{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = (3\hat{\imath} + 3\hat{\jmath})$ $+\mu(2\hat{\imath} + \hat{\jmath} + \hat{k})$ respectively.



28. Find the shortest distance between the given lines. (2)

29. Find the point at which the motorcycles may collide.(2)

**Ch-12 LINEAR PROGRAMMING**

1 Solve the following linear programming problem graphically:

Maximise $Z = 20x + 30y$

Subject to the constraints:

$$x + y \leq 80$$

$$2x + 3y \geq 100$$

$$x \geq 14$$

$$y \geq 14$$

2 Solve the following LPP graphically:

Maximize and Minimize Z = 3x + 5y

Subject to 3x - 4y + 12 $\geq$ 0

2x - y + 2 $\geq$ 0

2x + 3y - 12 $\geq$ 0

0 $\leq$ x $\leq$ 4

y $\geq$ 2

3 Maximise Z = 3x + 4y, subject to the constraints:$x + y \leq 1, x \geq 0, y \geq 0$ .

4 Solve the following linear programming problem graphically.Minimise$z = 3x + 5y$ subject to the constraints

$$x + 2y \geq 10$$

$$x + y \geq 6$$

$$3x + y \geq 8$$

$\mathbf{x, y \geq 0}$ .

5 Minimize and Maximize Z = x + 2y subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0$ .

6 Show that the minimum of Z occurs at more than two points.

Minimize and Maximize Z = 5x + 10y subject to $x + 2y \leq 120,\ x + y \geq 60$ , $x - 2y \geq 0,\ x,\ y \geq 0$ .

7 **Read the following text carefully and answer the questions that follow:**

Sheetal rides her car at 25 km/hr. She has to spend ₹ 2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹ 5 per km. She has ₹ 100 to spend on diesel.

30. Formulate above information mathematically. (1)

31. Represent the given information graphically. (1)

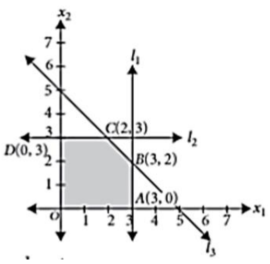
32. Find the maximum distance covered by her in hour? (2)

**OR**

If Z = 6x - 9y be the objective function, then findmaximum value of Z. (2)

8 **Read the following text carefully and answer the questions that follow:**

Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let Z = 4x - 6ybe the objective function.




33. At which corner point the minimum value of Z occurs? (1)

34. At which corner point the maximum value of Z occurs?(1)

35. What is the value of (maximum of Z - minimum of Z)?(2)

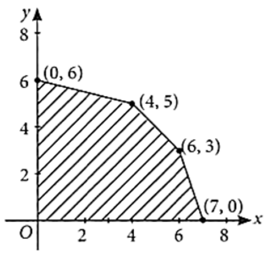
**OR**

The corner points of the feasible region determined by the system of linear inequalities are(2)

9 **Read the following text carefully and answer the questions that follow:**

Linear programming is a method for finding the optimal values (maximum or minimum) of quantities subject to the constraints when a relationship is expressed as linear equations or inequations.

36. At which points is the optimal value of the objective function attained? (1)
37. What does the graph of the inequality 3x + 4y < 12 look like?(1)
38. Where does the maximum of the objective function Z = 2x + 5y occur in relation to the feasible region shown in the figure for the given LPP?(2)




**OR**

What are the conditions on the positive values of p and q that ensure the maximum of the objective function Z = px + qy occurs at both the corner points (15, 15) and (0, 20) of the feasible region determined by the given system of linear constraints?(2)

**Ch-13 PROBABILITY**

1 Some students are having a misconception while comparing decimals. For example, a student may mention that$78.56 > 78.9$ as $7856 > 789$ . In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question: In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table:

| Name of student | Distance of javelin (in meters) |
|---|---|
| Ajay | 47.7 |
| Bijoy | 47.07 |
| Kartik | 43.09 |
| Dinesh | 43.9 |
| Devesh | 45.2 |

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconceptions answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

39. What is the probability of a student not having misconception but still answers Bijoy in the test? **(1)**

40. What is the probability that a randomly selected student answers Bijoy as his answer in the test? **(1)**

41.
   a. What is the probability that a student who answered as Bijoy is having misconception? **(2) OR**

   b. What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception? **(2)**

2 **Read the following text carefully and answer the questions that follow:**

A shopkeeper sells three types of flower seeds $A_1$, $A_2$, $A_3$. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

42. Calculate the probability that a randomly chosen seed will germinate. (1)

43. Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. (1)

44. A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card. (2)

    **OR**

    If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B). (2)

3 Based upon the results of regular medical check - ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let $A_1$ : People with good health,

$A_2$ : People with average health,

and $A_3$ : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category $A_1$, $A_2$ and $A_3$ are $25\%$, $35\%$ and $50\%$ , respectively.

Based upon the above information, answer the following questions:

45. A person was tested randomly. What is the probability that he/she has contracted the disease? **(2)**

46. Given that the person has not contracted the disease, what is the probability that the person is from category$A_2$ ? **(2)**

3 **Read the following text carefully and answer the questions that follow:**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New - born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

47. Find the probability that daughter is at one end, given that father and mother are in the middle. (1)

48. Find the probability that mother is at right end, given that son and daughter are together. (1)

49. Find the probability that father and mother are in the middle, given that son is at right end. (2)

    **OR**

    Find the probability that father and son are standing together, given that mother and daughter are standing together. (2)

4 A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively.

The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

50. Calculate the probability that a randomly chosen seed will germinate.(2)

51. Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.(2)

5 Events A and B are such that P(A) = $\frac{1}{2}$ , P(B) = $\frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$ .State whether A and B are independent?..

6 The probability of simultaneous occurrence of at least one of the two events A and B is p. If the probability that exactly one of A, B occurs is q, then prove that P(A') + P(B') = 2 – 2p + q

7 13. If P(A) = $\frac{3}{5}$ , P(B) = $\frac{1}{5}$ , find (A ∩ B), if (i) A and B are mutually exclusive (ii) A and B are independent

8 A black and a red die are rolled together. Find the conditional probability of obtaining the sum8, given that the red die resulted in a number less than 4.

9 Two cards are drawn from a well shuffled pack of 52 cards one after the other without replacement. Find the probability that one of them is a queen and the other is a king of opposite colour

10 A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

11 Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution

12 The random variable X has a probability distribution P(X) of the following form,

where k is some number :

$$P(X) = \begin{cases} k \ \ if \ \ x = 0 \\ 2k \ \ if \ \ x = 1 \\ 3k, \ \ if \ \ x = 2 \\ 0, otherwise \end{cases}$$

(a) Determine the value of k.

(b) Find P (X < 2), P (X ≤ 2), P(X ≥ 2).

13 A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.