Brain International School

Vikas Puri, New Delhi

REVISION SHEET

SUBJECT: MATHEMATICS **CLASS-XII** **TERM 1**

CHAPTER 1 RELATIONS

Ques1 Let Z be the set of all integers and R be the relation on Z defined on R = {(a, b) : a, b ∈ Z, and (a – b) is divisible by 5}. Prove that R is an equivalence relation.

Ques 2 Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

Ques 3 Let A = | R – {3} and B = | R – {1}. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f isone-one and onto

Ques 4 Show that the function f in A = |R – $\left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

Ques 5 Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible.

Ques 6 Let A = R – {2} and B = R – {1}. If $f: A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto.

Ques7 Show that the function $f: R \to \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function.

Q**ues 8** Consider $f: R - R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective.

Ques 9 Let $f: IR - \left\{-\frac{4}{3}\right\} \to IR$ be a function as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also check whether f is an onto function or not.

CHAPTER 2 INVERSE TRIGO

Ques1 Evaluate the following:

a) $\cos^{-1} x$ for x = $\frac{\sqrt{3}}{2}$ **b)** $\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ **c)** $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

d) $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$ **e)** Sec $\left(\tan^{-1}\frac{y}{2}\right)$ **f)** $\sin\left(2\tan^{-1}\frac{2}{3}\right) + cos\left(\tan^{-1}\sqrt{3}\right)$

g) Sin $(2\tan^{-1}(.75)$ **h)** Cos$\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right]$ **i)** $\sin\left[\cot^{-1}\left(cos(\tan^{-1}1)\right)\right]$

j) $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

CHAPTER MATRICES AND DETERMINANTS

Ques1 If A = $\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, find $A^2 - 5A + 4I$ & hence find matrix X such that $A^2 - 5A + 4I + X = O$.

Ques2 If $\begin{bmatrix}2x & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix} = 0$, find x.

Ques3 Let A = $\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, B = $\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$, C = $\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$, find a matrix D such that CD – AB = 0.

Ques4 Find the values of x and y from the following matrix equation:

$2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix} + \begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix} = \begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$.

Ques5 Find matrix A, if $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A = \begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$.

Ques6 If A = $\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

Q**ues7** If A =$\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

Ques 8 If A = $\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = 0$

Q**ues9** Express the matrix A = $\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

Ques10 Show that all the diagonal elements of a skew symmetric matrix are zero.

Ques 11 If A is a skew symmetric matrix of order 3, then prove that det A = 0.

Ques12 To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their locality, where they sold paper bags, scrap-books and pastel-sheets made by them using recycled paper at the rate of RS 20, Rs 15 and Rs 10 per unit respectively. School A sold 25 paper bags, 10 scrap-books and 30 pastel-sheets. School B sold 20 paper bags, 15 scrap-books and 30 pastel-books. While School C sold 25 paper bags, 18 scrap-books and 35 pastel-sheets. Using matrices, find the total amount raised by each school. By such exhibition, which values are inculcated in the student?

Ques 13 Using matrix method, solve the following system of equations:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4;\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1;\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2;\ x, y, z \neq 0$

Ques 14 Using matrices, solve the following system of equations:

$4x + 3y + 2z = 60, \qquad x + 2y + 3z = 45, \qquad 6x + 2y + 3z = 70$

Q15 A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs 6,000. Three times the award money for Hard work added to that given for honesty amounts to Rs 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using for each value, using matrix method.

Q16 The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping other and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of the all awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category.

Q17 Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of Rs x, Rs y and Rs z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of RS 29,000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of Rs 30,500. If the three prizes per person together cost Rs 9500 then

(1) represent the above situation by a matrix equation and form linear equations using matrix multiplication.

(2) solve these equations using matrices

(3) Which values are reflected in this question?

Q18 Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find the value of k if D(k, 0) is a point such that area of triangle ABD is 3 square units.

Q19 Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$

Q 20 Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations:

$x - y + z = 4$; $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

CHAPTER CONTINUITY AND DIFFERENTIABILITY

Ques1 Show that the function f defined as follows, is continuous at $x = 2$ but not differentiable there

at $f(x) = \begin{cases} 3x - 2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x - 4, & x > 2 \end{cases}$

Ques2 Find the relationship between a and b that the function f defined by

$f(x) = \begin{cases} ax + b, & x \le 3 \\ bx + 3, & x > 3 \end{cases}$ is continuous at $x = 3$

Ques4 Show that function $f(x) = |x - 3|$ is a continuous but not differentiable at $x = 3$.

Ques5 Show that the function $f(x) = |x - 1| + |x + 1|$ is not differentiable at $x = 1$ & $x = -1$.

Ques6 Find the value of 'p' for which the function f defined as f(x) = $\begin{cases} \frac{1 - Sin^3 x}{3Cos^2 x}, & x < \frac{\pi}{2} \\ p, & x = \frac{\pi}{2} \\ \frac{q(1-Sin\, x)}{(\pi - 2x)^2}, & x > \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$

Ques1 Find $\frac{dy}{dx} y = sin^{-1}\{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\}$

Ques2 Find $\frac{dy}{dx}$ if $y = (cosx)^x + (sinx)^{\frac{1}{x}}$

Ques3 Differentiate $x^{xcosx} + \frac{x^2-1}{x^2-1}$

Ques4 If $x = a(\theta - Sin\theta), y = a(1 + Cos\theta)$, find $\frac{d^2y}{dx^2}$

Ques5 If $(Cosx)^y = (Cosy)^x$ find $\frac{dy}{dx}$

Ques6 If $x = a(cost + t\, sint)$ and $y = a(sint - t\, cost)$, find $\frac{d^2y}{dx^2}, \frac{d^2x}{dt^2}\frac{d^2y}{dt^2}$

Ques7 Differentiate with respect to x: $(logx)^x + x^{logx}$

Ques8 Differentiate with respect to x: $sin^{-1}\left(\frac{2^{x+1}3^x}{1+(36)^x}\right)$

Ques9 If $x = a\, cos^3\theta$ and $y = aSin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$

Ques10 Differentiate $tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $cos^{-1}(2x\sqrt{1-x^2})$

Ques11 $tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ find $\frac{dy}{dx}$

Ques12 If x = a Sin2t(1 + Cos2t) and y = b Cos2t(1 – Cos2t) find the value of $\frac{dy}{dx}$ at t = $\frac{\pi}{4}$ and t = $\frac{\pi}{3}$

Ques13 Find $\frac{dy}{dx}$ at x = 1 and y = $\frac{\pi}{4}$ if $Sin^2y + Cosxy = k$

Ques14 Differentiate the function $(Sin\, x)^x + \sin^{-1}\sqrt{x}$ w.r.t x

Ques15 If $x^y + y^x = a^b$ find $\frac{dy}{dx}$

Ques16 Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ w.r.t. x

Ques17 If $(x^2 + y^2)^2 = xy$ find $\frac{dy}{dx}$

Ch AOD

Q 1 The volume of a sphere is increasing at the rate of 3 cubic centimetres per second. Find the rate of increase of its surface area, when the radius is 2cm.

Q 2 Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) strictly increasing (b) strictly decreasing

Q3Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is

(a) strictly increasing (b) strictly decreasing

Q4 If the length of three sides of a trapezium other than the base is 10cm each, find the area of the trapezium, when it is maximum.

Q 5 An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units

Q6 A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure is 10m, find its dimensions in order that the area may be maximum.

CHAPTER INTEGRAL

Q1 Evaluate: $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$ **Q2** Evaluate: $\int \frac{1-x^2}{x(1-2x)}dx$

Q3 Evaluate the following: $\int \frac{x+2}{\sqrt{(x-2)(x-3)}}dx$ **Q4** Evaluate: $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ **Q5** Evaluate: $\int \frac{2x}{(x^2+1)(x^2+3)}dx$

Q6 Evaluate: $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$ **Q7** Evaluate: $\int \sin x \sin 2x \sin 3x\, dx$ **Q8** Evaluate: $\int \frac{2}{(1-x)(1+x^2)}dx$

Q9 Evaluate: $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ **Q10** Evaluate: $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$ **Q11** Evaluate: $\int \frac{\sin(x-a)}{\sin(x+a)}dx$ **Q12** Evaluate: $\int \frac{5x-2}{1+2x+3x^2}dx$ **Q13** Evaluate: $\int \frac{\sin^6 x+\cos^6 x}{\sin^2 x+\cos^2 x}dx$ **Q14** Evaluate: $\int (x-3)\sqrt{x^2+3x-18}\,dx$

Q15 Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$ **Q16** Evaluate: $\int \frac{1}{\cos^4 x+\sin^4 x}dx$ **Q17** Find: $\int (x+3)\sqrt{3-4x-x^2}\,dx$

Q18 Find: $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$ **Q19** Find: $\int \frac{x^3}{x^4+3x^2+2}dx$

Q20 Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$ **Q21** Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$ **Q22** $\int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Q23 Evaluate: $\int_0^{\pi/2} 2\sin x \cos x \tan^{-1}(\sin x)\,dx$. **Q24** Evaluate: $\int_0^{\pi/2} \frac{x\sin x \cos x}{\sin^4 x+\cos^4 x}dx$**Q 25** Evaluate: $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}dx$

Q26 Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **Q27** Prove that: $\int_0^{\pi/4}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx = \sqrt{2}.\frac{\pi}{2}$

Q28 Evaluate: $\int_0^{\frac{\pi}{4}} \frac{dx}{cos^3 x\sqrt{2sin2x}}$ **Q29** Evaluate: $\int_0^{\pi/2} \frac{dx}{1+\sqrt{tanx}}$ **Q 30** Evaluate: $\int_0^{\pi/4} \log(1+tanx)\,dx$

Q 31 Find: $\int_0^{1/\sqrt{2}} \frac{sin^{-1}x}{(1-x^2)^{3/2}} dx$ **Q 32** Evaluate: $\int_0^{\pi} e^{2x}.sin\left(\frac{\pi}{4}+x\right)dx$ **Q 33** Evaluate: $\int_1^2 |x^3-x|\,dx$ **Q 34** Evaluate: $\int_0^{\pi/2} \frac{sin^2 x}{sinx+cosx} dx$ **Q 35** Evaluate: $\int_0^{3/2} |xcos\,\pi x|\,dx$

CH AOI

Q1 , find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Q 2 Using integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4).

Q3 Using integration, find the area of the triangular region whose sides have equations $y = 2x + 1,\ y = 3x + 1\ and\ x = 4$

Q 4 Sketch the graph of $y = |x + 3|$ and evaluate the area under $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$.

Q 5 Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0,\ 2x + 3y - 21 = 0\ and\ x - 5y + 9 = 0$.