ASSIGNMENT NO. 3

SUBJECT: MATHEMATICS **CLASS-XII** **MAY,2025**

CHAPTER : CONTINUITY & DIFFERENTIATIONS

Ques1 Find which of the following function is continuous & discontinuous:

a) $\begin{cases} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, if\ x \neq 0 \\ 0 \quad , if\ x = 0 \end{cases}$ **b)** Let f(x) = $\begin{cases} \frac{1-cos4x}{x^2}, if\ x < 0 \\ a \quad , if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, if\ x > 0 \end{cases}$ For what values of a,f is continuous at x=0

ques 2 Direct Derivative

a) $\frac{8^x}{x^8}$ **b)** $\mathrm{Sin}\sqrt{x} + \cos^2\sqrt{x}$ **c)** $\sin x^2 + \sin^2 x + \sin^2(x^2)$ **d)** $\sin^{-1}\left(\frac{1}{\sqrt{x+!}}\right)$

e) $\tan^{-1}(x^2 + y^2) = a$

ques 3 Derivative of Trigonometry

I $tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right]$ **ii.** $tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$ iii $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ **iv** $tan^{-1}\sqrt{\frac{1-x}{1+x}}$

v $tan^{-1}\left[\frac{x}{1+\sqrt{1-x^2}}\right]$ vi .$sec^{-1}\left(\frac{x+1}{x-1}\right) + sin^{-1}\left(\frac{x-1}{x+1}\right)$ vii . $sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$

viii.$cos^{-1}\left\{\frac{\sin x+\cos x}{\sqrt{2}}\right\}$ ix. $tan^{-1}\left(\frac{x}{1+6x^2}\right)$

Ques 4 Differentiate $sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Ques 5 If$x^p . y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

Ques 6 Given that: $\mathrm{Cos}\frac{x}{2} . cos\frac{x}{4} . cos\frac{x}{8} \ldots .. = \frac{sinx}{x}$, prove that: $\frac{1}{2^2}sec^2\frac{x}{2} + \frac{1}{2^4}sec^2\frac{x}{4} + \cdots = cosec^2 x - \frac{1}{x^2}$

Ques 7 Differential coefficient of $sec(tan^{-1}x)$ w. r. t is

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\frac{x}{1+x^2}$ (c) $x\sqrt{1+x^2}$ (d) $\frac{1}{\sqrt{1+x^2}}$

Ques 8. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is:

(a) $\frac{3}{2}$ (b) $\frac{3}{2t}$ (c) $\frac{3}{4t}$ (d) $\frac{4}{3t}$

Ques 9 If $y = log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{4x^3}{1-x^4}$ (b) $-\frac{4x}{1-x^4}$ (c) $\frac{4x^3}{1-x^4}$ (d) $\frac{x^3}{1-x^4}$

CHAPTER : APP OF DERIVATIVES

Ques1. The function $f(x) = x^3 - 3x^2 - 9x - 7$ has a stationary point at:

(a) -1 only (b) 3 only (c) -1 and 3 (d) None

Ques2. The function $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x - 9$ has a stationery point at

(a) 2 only (b) 3 only (c) 2 and 3 (d) None

Ques3. The sides of an equilateral triangle are increasing at the rate of 2cm/s. The rate at which the area increases, when side i 10cm is:

(a) 10 cm^2/s (b) $\sqrt{3}\ cm^2/s$ (c) $10\sqrt{3}\ cm^2/s$ (d) $\frac{10}{3}\ cm^2/s$

Ques 4. The function $y = logx$ does not has local maxima or local minima.

Ques5 An airforce plane is ascending vertically at the rate of 100km/hr. If the radius of earth is r km. How fast is the area of the earth, visible from the plane, increasing at 3minutes after it started ascending? Given that the visible area A at height h is given by A=$\frac{2\pi r^2 h}{r+h}$

Ques 6 Show that f(x) = Cos (2x + $\pi/4$) is an increasing function on $(3\pi/8, 7\pi/8)$.

Ques7 An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units?

Ques8 A rectangle is inscribed in a semi-circle of radius r with one of sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area

Ques 9 Prove that area of right angled triangle of given hypotenuse is max when the triangle is isosceles

Ques 10 Show that the surface area of a closed cuboid with square base and given volume is minimum when it is cube

Ques 11 A given quantity of metal is to be cast into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of cylinder to the diameter of its semicircular ends is $\pi : (\pi + 2)$

Ques 12 A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300/— per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Re 1/— one subscriber will discontinue the service. Find what increase will bring maximum profit?