

BRAIN INTERNATIONAL SCHOOL

REVISION SHEET

Subject : Mathematics

Class: XII

Q1. If $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, then the value of $|A A'|$ is

- (a) -9 (b) 3 (c) $\frac{1}{9}$ (d) 9

Q2. If $\begin{pmatrix} 2 & -8 & 3 \end{pmatrix} \begin{pmatrix} 2023 & 2025 & 1 \\ 2022 & 2024 & 0 \\ 2021 & 2022 & 1 \end{pmatrix} \begin{pmatrix} 2021 \\ 2022 \\ 2023 \end{pmatrix} = A$, then the order of matrix A is

- (a) 3 by 3 (b) 1 by 1 (c) 3 by 1 (d) 1 by 3

Q3. If $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{when } x \neq 1 \\ 2k, & \text{when } x = 1 \end{cases}$ is given to be continuous at $x = 1$, then the value of k is

- (a) -1 (b) -2 (c) 1 (d) 2

Q4. If $\left| \frac{A^{-1}}{2} \right| = \frac{1}{k|A|}$, where A is a 3×3 matrix, then the value of k is

- (a) $\frac{1}{8}$ (b) 8 (c) 2 (d) $\frac{1}{2}$

Q5. $\int \frac{x-1}{(x-2)(x-3)} dx$ equals

- (a) $2 \log|x-3| - \log|x-2| + C$ (b) $\log|x-3| - \log|x-2| + C$
(c) $\log|x-3| - 2 \log|x-2| + C$ (d) $\log|x-2| - \log|x-3| + C$

Q6. Let $\sin^{-1}(2x) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$. Then the value of x is

- (a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) 8 (d) $\frac{1}{4}$

Q7. $\int_e^{e^2} \frac{\log x}{x} dx$ equals

- (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) $-\frac{1}{2}$

Q8. Let $A = \{1, 2, 3\}$. Then total number of reflexive relations on set A is

- (a) 8 (b) 64 (c) 16 (d) 32

Q9. If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is

- (a) 6 (b) -6 (c) 0 (d) -7

Q10. Principal value of $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$ is

- (a) $\frac{7\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $-\frac{7\pi}{6}$

Q11. If $f(x) = \log x$, then $f'(x) + f'\left(\frac{1}{x}\right) =$

- (a) $\frac{x^2-1}{x}$ (b) $\frac{1-x^2}{x}$ (c) $\frac{x^2+1}{x}$ (d) $\frac{1+x}{x}$

Q12. If $y = 5e^{7x} + 6e^{-7x}$, such that $\frac{d^2y}{dx^2} - ky = 0$ then, the value of k will be

- (a) -49 (b) 1 (c) 49 (d) 0

Q13. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

- (a) 1 (b) 2 (c) 3 (d) 4

Q14. Value of $\int \frac{\cos \sqrt{x} dx}{\sqrt{x}}$ is

- (a) $-2 \sin \sqrt{x} + C$ (b) $\sin \sqrt{x} + C$ (c) $2 \cos \sqrt{x} + C$ (d) $2 \sin \sqrt{x} + C$

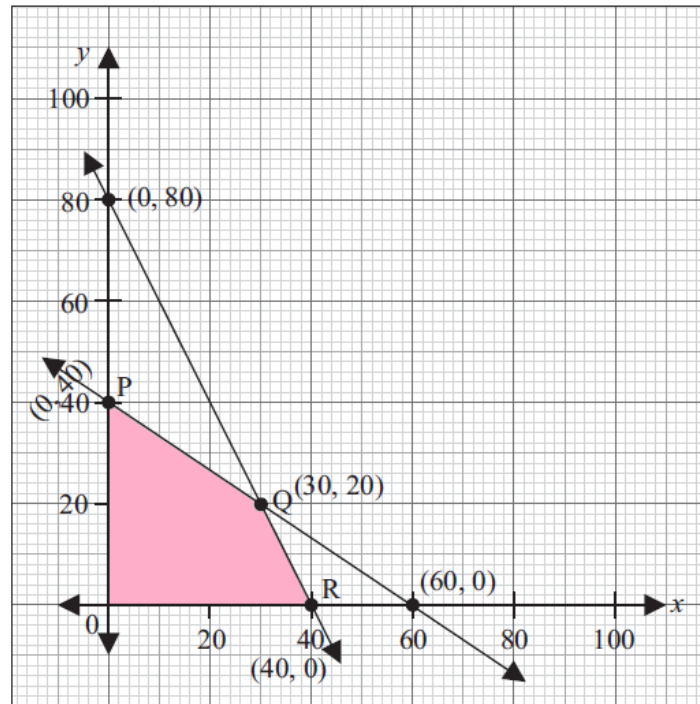
Q15. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x , is continuous at

- (a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$

Q16. For the function $y = \frac{x-7}{(x-2)(x-3)}$, the value of $\left(\frac{dy}{dx}\right)_{\text{at } x=7}$ is

- (a) -20 (b) $\frac{1}{20}$ (c) $-\frac{1}{20}$ (d) 20

Q17. For an L.P.P. the objective function is $Z = 4x + 3y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true?

- (a) Maximum value of Z is at R .
 (b) Maximum value of Z is at Q .
 (c) Value of Z at R is less than the value at P .
 (d) Value of Z at Q is less than the value at R .

Q18. If $A = \begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$ is written as $A = P + Q$, where P is a symmetric matrix and Q is skew symmetric matrix, then the matrix P will be given by

- (a) $\begin{pmatrix} 3 & 3 \\ 6 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 7 \\ 5 & 9 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

Q19. **Assertion:** The domain of the function $f(x) = \cos^{-1}\left(\frac{2x}{3}\right)$ is $x \in \left[-\frac{3}{2}, \frac{3}{2}\right]$.

Reason: $\cos^{-1} x$ is defined when $-1 \leq x \leq 1$.

Q20. **Assertion (A):** If $\begin{pmatrix} 2019 \\ 2023 \end{pmatrix} \begin{pmatrix} 2 & 0 & 2 & 4 \end{pmatrix} = X$, then order of X will be 2×4 .

Reason (R): Determinant value of a singular matrix is always non-zero.

Q21. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$, is increasing or decreasing in

$$x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right).$$

Q22. If a unit vector \vec{a} makes $\pi/4$ with \hat{i} , $\pi/3$ with \hat{j} and an acute angle θ with \hat{k} . Find the angle θ and hence components of \vec{a} .

Q23. Simplify : $\sin^{-1}\left[2x\sqrt{1-x^2}\right]$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$.

Q24. Let R be an equivalence relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ by $R = \{(x, y) : x, y \in A, x \text{ and } y \text{ are either both odd or both even}\}$. Write all the equivalence classes of set A.

Q25. Find the value of λ so that the lines are perpendicular $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$; $\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$

Q26. Three vectors \vec{a}, \vec{b} and \vec{c} satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$. If $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

Q27. Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$

Q28. Let a relation R on the set \square of real numbers be defined as $(a, b) \in R \Leftrightarrow 1 + ab > 0 \forall a, b \in \square$.

Show that R is reflexive and symmetric. Is it transitive? Justify your answer in each case.

Q29. Evaluate $I = \int \frac{x}{x^4+x^2+1} dx$

Q30. Solve the following Linear Programming Problem graphically:

$$\text{Minimise } Z = 13x - 15y$$

$$\text{subject to } x + y \leq 7, 2x - 3y + 6 \geq 0$$

$$x \geq 0, y \geq 0.$$

Q31. Evaluate $I = \int \frac{\sin x}{\sin(3x)} dx$

Q32. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB. Hence solve the system of equations $x - 2y = 10$, $2x + y + 3z = 8$ and $-2y + z = 7$.

Q33. Prove that the volume of the largest core that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Q34. Solve :

$$xy \log\left(\frac{x}{y}\right) dx + \left(y^2 - x^2 \log\left(\frac{x}{y}\right)\right) dy = 0.$$

Q35. Find the vector equation of the line through the point $(1, 2, -4)$ and perpendicular to the two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Q36. **Case-Study 1:** Let $f(x)$ be a real valued function. Then its

$$\text{Left Hand Derivative (L.H.D.) : } Lf'(a) = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{-h}$$

$$\text{Right Hand Derivative (R.H.D.) : } Rf'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Also, a function $f(x)$ is said to be differentiable at $x = a$ if its L.H.D. and R.H.D. at $x = a$ exist and both are equal.

$$\text{Let } f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}.$$

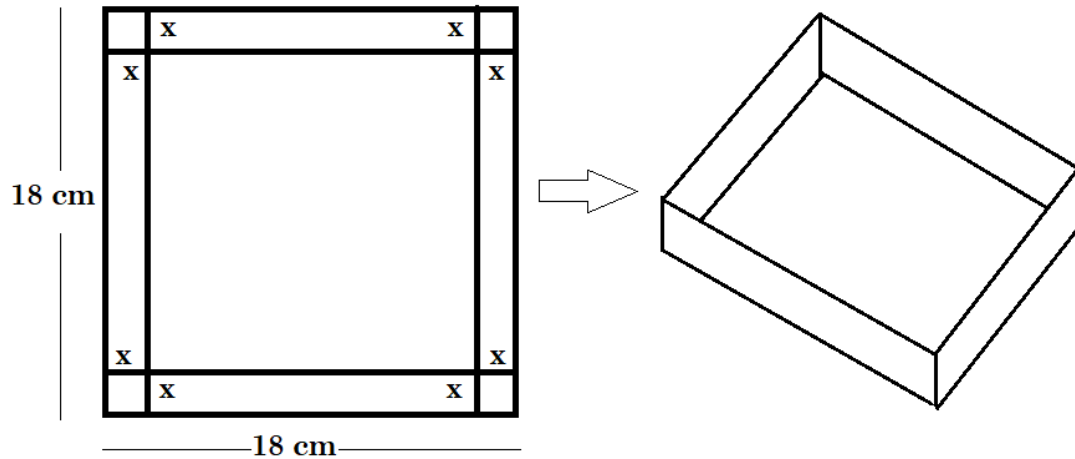
Based on the given information, answer the following questions.

(i) What is R.H.D. of $f(x)$ at $x = 1$?

(ii) What is L.H.D. of $f(x)$ at $x = 1$?

(iii) Check if the function $f(x)$ is differentiable at $x = 1$.

Q37. **Case-Study 2:** For an EMC project, a student of Class XII makes an open cardboard box for a jewellery shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flaps.



Assume that 'x' be the side of squares cut off from each corner (as shown in the diagram above).

Based on the given information, answer the following questions.

- (i) For the open box, find the length, breadth and height in terms of x .
- (ii) Write an expression for the volume of the open box.
- (iii) For what value of 'x', the open box will have maximum volume? Use derivatives.

Q38. Case-Study 3: An owner of a car rental company have determined that if they charge customers Rs x per day to rent a car, where $50 \leq x \leq 200$, then number of cars (n), they rent per day can be shown by linear function $n(x) = 1000 - 5x$. If they charge Rs. 50 per day or less they will rent all their cars. If they charge Rs. 200 or more per day they will not rent any car.



Based on the above information, answer the following question.

- (i) If $R(x)$ denote the revenue, then find the value of x at which $R(x)$ has maximum value.
- (ii) Find the Maximum revenue collected by company.