BRAIN INTERNATIONAL SCHOOL

SESSION 2024-25

CLASS: X **REVISION SHEET** **SUBJECT: MATHEMATICS**

REAL NUMBERS

1. If two positive integers a and b are written as $a = x^3 y^2$ and $b = x y^3$; x, y are prime numbers, then find HCF (a, b).
2. Find the HCF of $x^2 - 3x + 2$ and $x^2 - 4x + 3$.
3. Find the least positive integer divisible by first five natural numbers.
4. Find the HCF of the numbers: k, 2k, 3k, 4k and 5k, where k is any positive integer.
5. Three alarm clocks ring at intervals of 6, 10, and 14 minutes respectively. If they start ringing together, after how much time will they ring together?
6. What will be the least possible number of planks, if three pieces of timber 42 m, 49 m, and 63 m long have to be divided into planks of the same length.
7. Prove that $3 + \sqrt{7}$ is an irrational.
8. Check whether 4^n can end with the digit 0 for any natural number n.
9. In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.
10. Find the largest number which divides 320 and 457 leaving remainder 5 and 7 respectively.
11. If $(a \times 5)^n$ ends with the digit zero for every natural number n, then a is

    (a) only prime number (b) an even number (c) an odd number (d) none of these
12. The ratio of LCM and HCF of the least composite number and the least prime number is:

    (a) 3:2 (b) 2:7 (c) 2:1 (d) 1:2

POLYNOMIALS

1. Which are the zeroes of $p(x) = x^2 - 8x + 15$

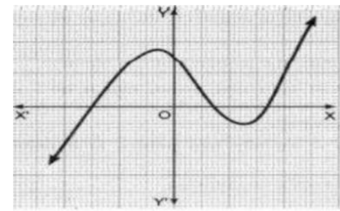
   (a) 5, –2 (b) –5, 2 (c) 5, 3 (d) 5, −3
2. Find the quadratic polynomial whose zeros are -3 and 4.

   (a) $x^2 - 7x - 12$ (b) $x^2 + x + 12$ (c) $x^2 - x - 12$ (d) $x^2 + 3x - 4$
3. If one zero of the polynomial $6x^2 + 37x - (k - 2)$ is reciprocal of the other, then, what is the value of k?

   (a) 4 (b) −6 (c) 6 (d) −4

4. If α and β are the zeroes of the polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$ then find the value of p.

5. The number of zeroes of the polynomial from the graph is



(a) 0 (b) 1 (c) 2 (d) 3

6. If α and β are zeroes of the quadratic polynomial $x^2 + 7x + 12$, then find $\frac{12}{\alpha} + \frac{12}{\beta} - 12\alpha\beta$.

7. If $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ are zeroes of $p(x) = x^2 + px + q$, then find a polynomial having $\frac{1}{\alpha}$ ***and*** $\frac{1}{\beta}$ as its zeroes; $q \neq 0$

8. If $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ are zeroes of the polynomial $f(x) = x^2 - p(x+2) - c$, then find the value of $(\boldsymbol{\alpha} + \mathbf{2})(\boldsymbol{\beta} + \mathbf{2})$.

9. On dividing a polynomial $p(x) = x^3 - 3x^2 + x + 2$ by another polynomial g(x), the quotient and remainder were $(x-2)$ and $(-2x + 4)$ respectively. Find g(x).

10. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

(i) c and b have opposite signs (iii) c and a have opposite signs
(ii) c and b have same signs (iv) c and a have same signs

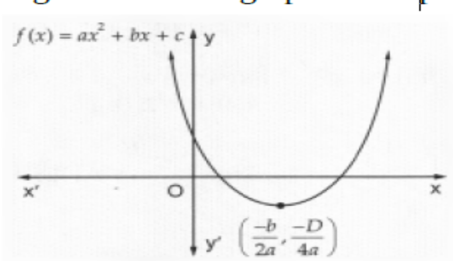
11. If the zeroes of the polynomial $x^2 + ax + b$ are double in value to the zeroes of the polynomial $2x^2 - 5x - 3$, then find the values of a and b.

12. If α and β are the zeros of the quadratic polynomial p(s) = 3s2 − 6s + 4, find the value of: $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

13. If α, β are zeroes of the quadratic polynomial $x^2 - 5x - 3$, then form a polynomial whose zeroes are $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$.

14.

Figure show the graph of the polynomial $f(x) = ax^2 + bx + c$ for which



a) $a > 0$, $b < 0$ and $c > 0$
b) $a < 0$, $b < 0$ and $c < 0$
c) $a < 0$, $b > 0$ and $c > 0$
d) $a > 0$, $b > 0$ and $c < 0$

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. The pair of equations $2x + 3y = 5$ and $4x + 6y = 10$ has:
(a) A unique solution (b) Infinitely many solutions
(c) No solution (d) Cannot be determined

2. The solution of the equations $x - 2y = 4$ and $3x + y = 5$ is:
(a) x = 3, y = -2 (b) x = 2, y = -1 (c) x = 4, y = 1 (d) x = 5, y = 0

3. Seven times a two-digit number is equal to four times the number obtained by reversing the order of its digit. If the difference between the digits is 3, then find the number.

4. If x = a, y = b is the solution of the equations x – y = 2 and x + y = 4, then find the values of a and b.

5. Find the solution of the linear pair px + qy = p – q and qx – py = p + q.

6. 4 men and 6 boys can finish a piece of work in 20 days while 3 men and 4 boys can finish it in 28 days. Find the time taken by one man alone to finish the work.

7. A motor boat takes 6 hours to cover 100 km downstream and 30 km upstream. If the motorboat goes 75 km downstream and returns back to the starting point in 8 hours, find the speed of the boat in still water and the speed of the stream.

8. The sum of a two-digit number and the number obtained by reversing its digits is 132. If the digits differ by 2, find the number.

9. A boat goes 30 km downstream in 2 hours and the same distance upstream in 3 hours. Find the speed of the boat in still water and the speed of the stream.

10. A shopkeeper sells a total of 400 pens of two types: A and B. If the cost of type A is ₹15 per pen and type B is ₹20 per pen, and the total sale amount is ₹7000, how many pens of each type were sold?

11. A person invested ₹15000 in two schemes: one offering 5% annual interest and the other 8%. If the total annual interest from both schemes is ₹960, find the amounts invested in each scheme.

12. The difference between the ages of two friends is 5 years. If the sum of their ages is 45 years, find their present ages.

13. A farmer has a rectangular plot of land. The length of the plot exceeds its breadth by 20 meters. If the perimeter of the plot is 220 meters, find the dimensions of the plot.

14. The ratio of incomes of two persons is 9:7, and the ratio of their expenditures is 4:3. If each saves ₹2000 per month, find their monthly incomes and expenditures.

15. For what value of k, the following pair of equations will have no solution?
$x - 2y = 3$ and $k y + 3x = 1$
16. Solve the following pair of linear elimination method.
$x - y + 1 = 0$; $3x + 2y - 12 = 0$

17. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or co incident.

18. (i) $3x + 45y - 4 = 0$
$10x + 150y = -8$
(ii) $12x - 8y + 5 = 0$
$3x - 24/5\, y + 3/5 = 0$

19. Two belts and three ties costs ₹400. Also, three belts and two ties costs ₹ 350. Find the cost of a belt and a tie.

20. Using substitution method, solve the following pair of equations
$x + 2y = -1$ and $2x - 3y = 12$

QUADRATIC EQUATIONS

1. In the Maths Olympiad of 2020 at Animal Planet, two representatives from the donkey's side, while solving a quadratic equation, committed the following mistakes.
i. One of them made a mistake in the constant term and got the roots as 5 and 9.
ii. Another one committed an error in the coefficient of x and he got the roots as 12 and 4.
But in the meantime, they realised that they are wrong and they managed to get it right jointly. Find the quadratic equation.

a) $2x^2 + 7x - 24 = 0$ b) $x^2 + 4x + 14 = 0$

c) $3x^2 - 17x + 52 = 0$ d) $x^2 - 14x + 48 = 0$

2. Solve: $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2, x \neq -\frac{1}{2}, 1$

3. ₹ 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got ₹ 160 less. Find the original number of persons.

4. For what value of k, is −2 a root of the equation $3x^2 + 4x + 2k=0$

5. Find the nature of the roots of the quadratic equation $4x^2-12x-9 = 0$.

6. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(a) $(x+4)^2 - 8x =0$ (b) $\sqrt{2}x^2-3/\sqrt{2}x + 1/\sqrt{2} = 0$

7. Find the roots of the following quadratic equations using the quadratic formula.

(a) $6a^2x^2 - 7abx - 3b^2 = 0, a \neq 0$ (b) $x^2 - 3\sqrt{5}x +10 = 0$

(c) $abx^2 + (b^2-ac)x - bc=0$ (d) $4/x - 3 = 2/2x + 3, x\neq 0,-3/2$

8. Solve for x: $36x^2 - 12ax + (a^2 - b^2) = 0$

9. Sum of the areas of two squares is $640m^2$. If the difference of their perimeters is 64m, find the sides of the two squares.

10. Find the roots of the equation $x^2 - 3x - m (m + 3) = 0$, where m is a constant.

11. Find the value of p so that the quadratic equation $px(x - 3) + 9 = 0$ has two equal roots.

12. Solve for x: $\sqrt{2x+9} + x = 13$.

13. Solve for x:

$$\frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6}, x \neq 3, -5.$$

14. Solve for x:

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0, \quad x \neq 3, -\frac{3}{2}$$

15. Solve for x:

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2}; x \neq 1, -2, 2$$

16. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers

17. If the roots of the quadratic equation $3x^2 - 5x + 2 = 0$ are α and β, what is the value of α + β?

(a) 5/3 (b) −5/3 (c) 5 (d) −5

18. The quadratic equation $x^2 + kx - 8 = 0$ has roots 4 and −2. The value of k is:

(a) 2 (b) −2 (c) −6 (d) 6

19. A train travels 60 km at a uniform speed. If the speed had been 10 km/h more, it would have taken 1 hour less for the journey. Find the speed of the train.

20. A rectangular plot of land has an area of 1200 m². If the length is 20 m more than the breadth, find its dimensions.

21. A person's age is three times that of their child's age. Four years ago, the square of their age was 169 more than the square of the child's age. Find their present ages.

22. The sum of the reciprocals of Reena's ages, three years ago and five years from now, is 1/3. Find her present age.

23. Two numbers differ by 4, and the sum of their reciprocals is 1/6. Find the numbers.

24. A motorboat, whose speed is 18 km/h in still water, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

25. Prove that if the roots of the quadratic equation $ax^2 + bx + c = 0$ are reciprocal to each other, then a = c.

ARITHMETIC PROGRESSIONS

1. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) −8 b) 4 c) −4 d) 8

2. The 15th term of the AP: 7, 10, 13, ... is:

(a) 45 (b) 50 (c) 52 (d) 49

3. The sum of the first n terms of an AP is $S_n = 3n^2 + 5n$. What is the first term (a) and the common difference (d)?

(a) 8, 6 (b) 5, 6 (c) 8, 3 (d) 5, 3

4. The sum of the first n terms of an AP is $S_n = 2n^2 + 3n$. Find the first term (a) and the common difference (d). Also determine the 10th term of the AP.

5. A student scored 40 marks in his first test and 50 marks in his second test. Assuming that his scores form an AP, find:
   (i) The total marks scored in 10 tests.
   (ii) In which test will he score 100 marks?

6. The sum of three numbers in an AP is 18, and their product is 80. Find the numbers.

7. The 5th term of an AP is 22, and the 15th term is 62. Find the common difference and the first term. Write the general term (a_n) of the AP.

8. A man saves ₹200 in January, ₹250 in February, ₹300 in March, and so on. Find the total savings in one year. In which month will his savings first exceed ₹2000?

9. The sum of the first n terms of an AP is given as $S_n = 2n^2 + 5n$. If the pth term is 99, find the value of p.

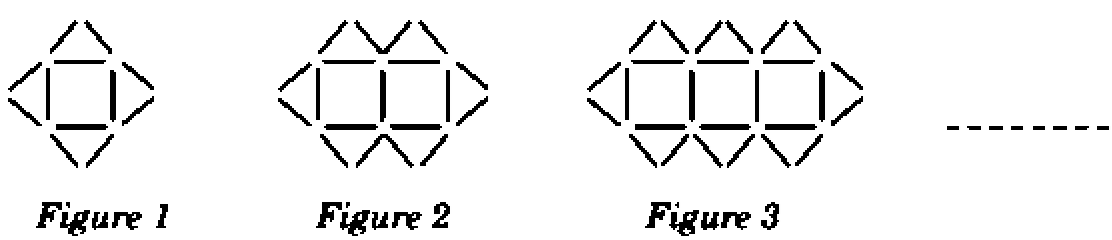
10. An AP consists of 21 terms. If the sum of the first 11 terms is equal to the sum of the last 11 terms, prove that the 11th term is the average of the first and the 21st term.

11. Find b in terms of a and c, such that a, b and c are in A.P.

12. In an A.P., the sum of first ten terms is −150 and the sum of its next ten terms is – 550. Find the A.P.

13. In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different figures.

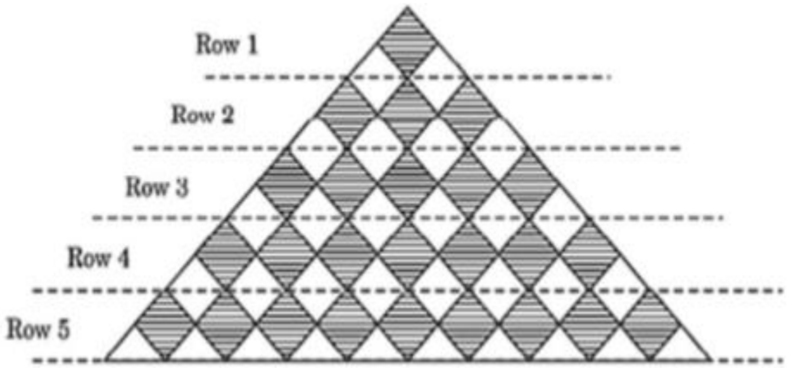
    One such patterns is shown below. Observe the pattern and answer the following questions using Arithmetic Progression:



(a) Write the AP for the number of triangles used in the figures. Also, write the n^{th} term of this AP.

(b) Which figure has 61 matchsticks?

14. A fashion designer is designing a fabric pattern. In each row, there are some shaded squares and unshaded triangles.



(a) Identify A.P. for the number of squares in each row.

(b) Identify A.P. for the number of triangles in each row.

(c) Write a formula for finding total number of triangles in n number of rows. Hence, find S_{10}.

(d) If each shaded square is of side 2 cm, then find the shaded area when 15 rows have been designed.

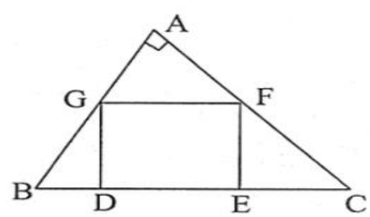
TRIANGLES

1. In fig., DEFG is a square in a triangle ABC right angled at A.

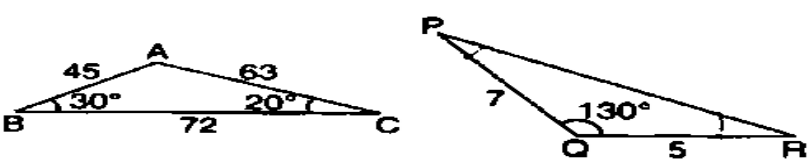
   Prove that

   i. Δ AGF ~ Δ DBG

   ii. Δ AGF ~ Δ EFC



2. In the figures find the measures of $\angle P$ and $\angle R$



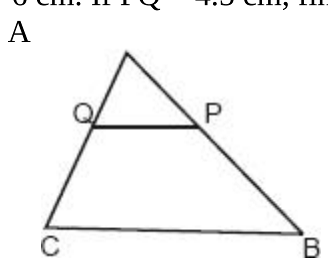
a) 20°, 30°.

b) 50°, 40°.

c) 30°, 20°.

d) 40°, 50°.

3. In the fig., P and Q are points on the sides AB and AC respectively of triangle ABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. If PQ = 4.5 cm, find BC.



4. The perimeter of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, then what is the length of AB?

5. ABC is a triangle in which AB = AC and D is any point in BC. Prove that : $(AB)^2 - (AD)^2 = BD \cdot CD$.

6. AD is the median of ΔABC, O is any point on AD. BO and CO produced meet AC and AB in E and F respectively. AD is produced to X such that OD = DX. Prove that AO : AX = AF : AB.

7. The perimeters of two similar triangles are 25cm and 15cm respectively. If one side of the first triangle is 9cm, determine the corresponding side of the other.

8. Triangle ABC is right angled at B and D is the mid-point of BC. Prove that:- $AC^2 = 4AD^2$. $3AB^2$

9. ABC is a triangle right angled at C. Let BC = a, CA= b , AB =c & p be the length of perpendicular from C on AB. Prove that

(i) CP = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

COORDINATE GEOMETRY

1. ABCD is a rectangle whose three vertices are B (4,0), C (4,3) and D (0,3). The length of one of its diagonals is

(a) 5 (b) 3 (c) 4 (d) 25

2. In what ratio, does x-axis divide the line segment joining the points A(3, 6) and B(−12, −3)?

(a) 2 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4

3. The area of a triangle formed by points (1, 1), (4, 5), (1, 6) is:

(a) 10 sq. units (b) 8 sq. units (c) 6 sq. units (d) 12 sq. units

4. If A (1, 2), B (4, y), C (6, −3) are collinear, then y is

(a) 3 (b) −3 (c) 0 (d) 1

5. Find the coordinates of the centroid of the triangle formed by points A(1, 2), B(4, 6), C(7, 8).

6. Determine whether the points (2, 3), (4, 7), (6, 11) are collinear.

7. If A (−3, −1), B (5, −1), and C (1, 7) are the vertices of a triangle, find the length of the median from C to AB.

8. Find the equation of the perpendicular bisector of the line segment joining (1, 3) and (5, 7).

9. Prove that the points (3, 0), (−2, −3), (2, 3) form a right-angled triangle.

10. Calculate the area of a quadrilateral formed by A(1, 2), B(4, 5), C(7, 8), D(6, 3).

11. If A(1, 1), B(4, −2), C(7, 1), and D(4, 4) form a parallelogram, find the coordinates of the point of intersection of the diagonals.

12. Three vertices of a parallelogram are A (2, 3), B (6, 7), C (8, 3). Find the coordinates of the fourth vertex D.

13. Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half of its length.

14. The vertices of a triangle are A (0, 0), B (3, 4), C (6, 0). Find the equation of the median drawn from vertex A and verify that it divides the triangle into two equal areas.

15. Show that the points A (1, 2), B (3, 6), C (−1, −2) and D(−3, −6) lie on the same straight line.

INTRODUCTION TO TRIGONOMETRY

1. **If sin 77° = x, then the value of tan 77° is**

(a) $\frac{1}{1+x^2}$ **(b)** $\frac{x}{\sqrt{1+x^2}}$ **(c)** $\frac{x}{\sqrt{1-x^2}}$ **(d)** $\frac{x}{1+x^2}$

2. Find the value of α and β if $\sin(\alpha + 2\beta) = \sqrt{3}/2$ and $\cos(\alpha + 4\beta) = 0$

3. Prove $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2 \sec A$, where the angles involved are acute angles for which the expressions are defined.

4. If 5 tan A = 4, then show that $\frac{5\sin A - 3\cos A}{5\sin A + 2\, Cos\, A} = \frac{1}{6}$

5. If $\tan(A) = \sqrt{2} - 1$ then show that $\sin(A) \cdot \cos(A) = \frac{\sqrt{2}}{4}$.

6. If $sec(\theta) = x + \frac{1}{4x}$ then prove that, $sec(\theta) + \tan(\theta) = 2x$ or $\frac{1}{2x}$.

7. If $\frac{1-\tan\theta}{1+\tan\theta} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, show that $\frac{\sin\theta}{\cos 2\theta} = 1$

8. Prove that $\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ} = \frac{1-2\sqrt{3}+\sqrt{2}}{2}$

9. If tan Θ + sec Θ = ℓ then prove that sec Θ = $\frac{\ell^2+1}{2\ell}$.

10. If $a\cot\theta + b\cos ec\theta = x^2, b\cot\theta + a\cos ec\theta = y^2$,
prove that $x^4 - y^4 = b^2 - a^2$.

11. If sec A = $\frac{5}{4}$, verify that $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

12. If $\sin\theta + \cos\theta = p$ and $\sec\theta + cosec\theta = q$, show that $q(p^2 - 1) = 2p$.

13. $\sqrt{(1 - \cos^2\theta)\sec^2\theta} = \tan\theta$. Is it true or false?

14. If x = a cos θ − b sin θ and y = a sin θ + b cos θ, then prove that $a^2 + b^2 = x^2 + y^2$.

APPLICATIONS OF TRIGONOMETRY

1. A man observes the top of a tower at an angle of elevation of 30°. If he is 50 m away from the tower, what is the height of the tower?

a) $25\sqrt{3}$ m b) $50\sqrt{3}$ m c) $100\sqrt{3}$ m d) 50 m

2. The angle of elevation of the sun is 45°. If the shadow of a tree is 10 m, what is the height of the tree?

a) 5 m b) 10 m c) 15 m d) 20 m

3. If the height of a building is 20 m and the angle of elevation of its top from a point on the ground is 60°, the distance of the point from the building is:

a) $20\sqrt{3}$ m b) $10\sqrt{3}$ m c) $20/\sqrt{3}$ m d) 10 m

4. From a point on the ground, the top of a flagpole is observed at an angle of 60°. If the height of the flagpole is 15 m, the horizontal distance to the point is:

a) $15\sqrt{3}$ m b) $5\sqrt{3}$ m c) 30 m d) 15 m

5. The angle of depression from the top of a 50 m tall building to a car parked on the ground is 45°. The distance of the car from the building is:

a) 25 m b) 50 m c) 75 m d) 100 m

6. If a ladder 13 m long is placed against a wall such that it reaches a height of 12 m, the angle it makes with the ground is approximately:

a) 30° b) 45° c) 60° d) 23°

7. A kite is flying at a height of 60 m above the ground. The string attached to the kite makes an angle of 30° with the ground. Find the length of the string, assuming it is taut.

8. From a point 40 m away from the foot of a tower, the angle of elevation of its top is 45°. Find the height of the tower.

9. The angle of depression of a car parked on the ground from the top of a 50 m high building is 30°. Find the distance of the car from the building.

10. A vertical pole is 6 m high. A man standing 4 m away from the pole observes the top of the pole at an angle of elevation of 60°. Verify if the observation is correct.

11. From the top of a 100 m tall building, the angle of depression of a person on the ground is 30°. Find the horizontal distance of the person from the building.

12. The shadow of a tower is 10 m when the angle of elevation of the sun is 45°. Find the height of the tower.

13. Two ships are sailing in the sea on either side of a lighthouse. The angles of elevation of the top of the lighthouse from the ships are 30° and 45° respectively. If the lighthouse is 100 m high, find the distance between the two ships.

14. A boy is standing on the top of a building 20 m high. He observes the angle of depression of the top and bottom of a pole as 30° and 60° respectively. Find the height of the pole if the pole and the building are on the same horizontal plane.

15. A person observes the top of a tower at an angle of elevation of 60° and moves 30 m closer to the tower. The angle of elevation becomes 75°. Find the height of the tower.

16. From the top of a building 80 m high, the angle of elevation of the top of another building is 30°, and the angle of depression of its base is 45°. Find the height of the other building.

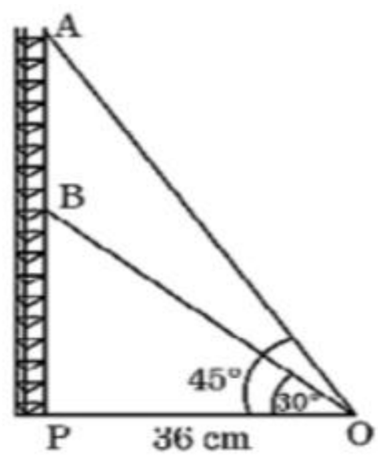
17. A drone flying at a height of 100 m observes two landmarks on opposite sides. The angles of depression of these landmarks are 45° and 60°. Find the distance between the two landmarks.

18. From the top of a 120 m tall tower, the angles of depression of the top and bottom of a vertical pole are observed to be 30° and 60° respectively. Find the height of the pole and the distance between the tower and the pole.

19. A hot air balloon is flying at a height of 150 m. The angles of elevation of the balloon from two points on opposite sides of it on the ground are 30° and 60°. Find the distance between the two points.

20. From the top of a 50 m high lighthouse, the angles of depression of two ships are 30° and 45°. Find the distance between the two ships if they are on opposite sides of the lighthouse.

CASE STUDY-BASED QUESTION



Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30˚ and the angle of elevation of the top of Section A is 45˚.

(a) Find the length of the wire from the point O to the top of Section B.

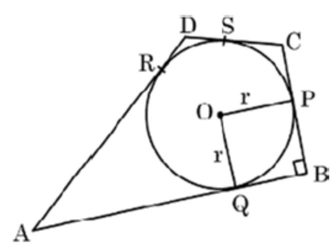
(b) Find the distance AB.

(c) Find the height of the Section A from the base of the tower.

(d) Find the area of △OPB.

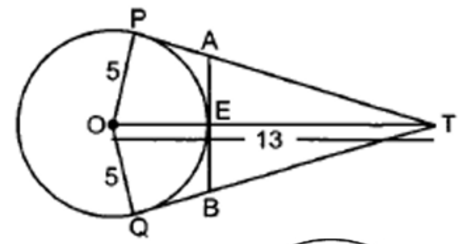
CIRCLES

1. From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.
2. In the given figure, a circle is inscribed in a quadrilateral ABCD in which B = 90˚. If AD = 17 cm, AB = 20 cm and DS = 3 cm, then find the radius of the circle.



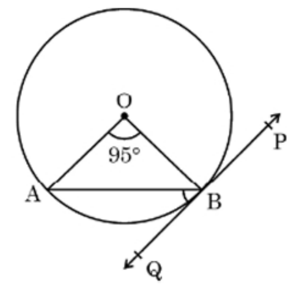
3. A tangent PT is drawn from an external point P to a circle of radius 3√2 cm such that distance of the point P from O is 6 cm. Find the value of ∠TPO.

4. In figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB. where TP and TQ are two tangents to the circle.



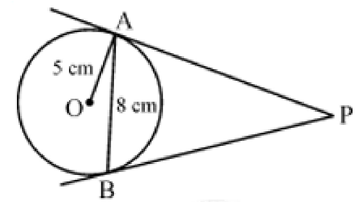
5. In the given figure, PQ is tangent to the circle centered at O. If AOB = 95˚, then the measure of ABQ will be

a) 85˚ b) 47.5˚ c) 95˚ d) 42.5˚



6. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

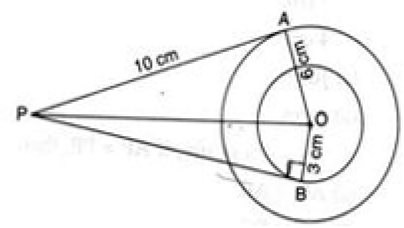
7. In a given figure, AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



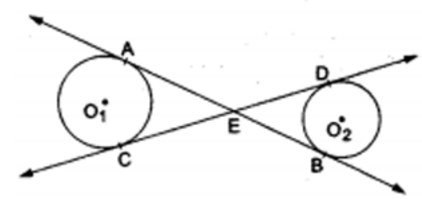
8. Two concentric circles with centre O are of radii 6 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles as shown in the figure. If AP = 10 cm, then BP is equal to

a) $\sqrt{91}$ b) $\sqrt{119}$ cm

c) $\sqrt{127}$ cm d) $\sqrt{109}$ cm

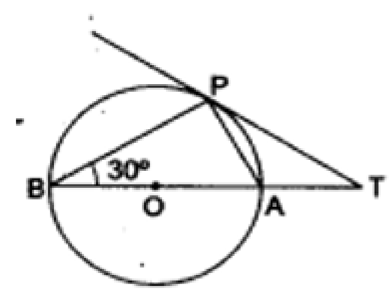


9. In the given figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.



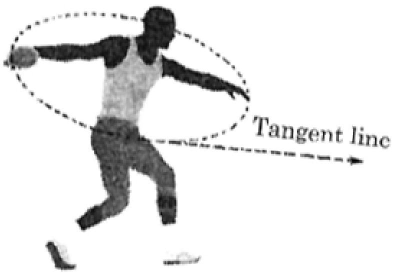
10.

In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT$ = 30°, prove that BA: AT = 2:1.

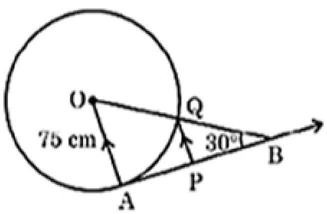


11.

The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and $\angle ABO = 30^o$. PQ is parallel to OA.



Find the length of AB. OB, AP and PQ.

AREA RELATED TO CIRCLES

1. Find the area of a circle that can be inscribed in a square of side 8 cm.

2. If sum of the areas of two circles with radii R1 and R2 is equal to area of a circle radius R, derive the relation among their radii.

3. If sum of the circumference of two circles with radii radii R1and R2 is equal to circumference of a circle radius R, derive the relation among their radii.

4. If the circumference of a circle and the perimeter of a square are equal, then write the relation between their radii.

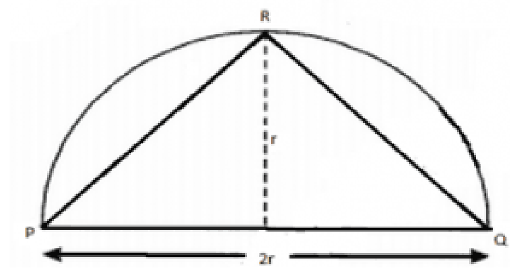
5. To build a single circular park equal in area to the sum of areas of two circular parks of diameter 8 m and 6 m in a locality. What would be the radius of new park?

6. Find the area of a square that can be inscribed in a circle of radius 6 cm.

7. Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of diameter 24 cm and 16 cm.

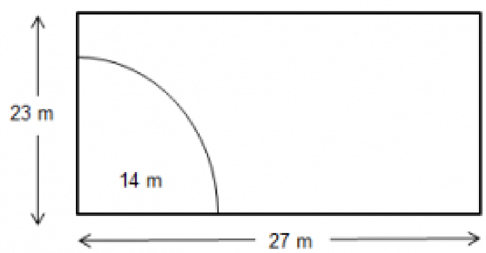
8. Find the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 20 cm and 15 cm.

9. Find the area of the largest triangle that can be inscribed in a semicircle of radius r.

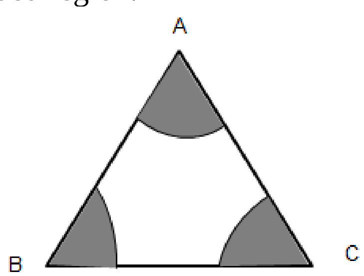


10. The wheel of a motor cycle is of radius 28cm. How many revolutions per minute must the wheel make so as to keep a speed of 60 Km/h?

11. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 27 m × 23 m. Find the area of the field in which the cow can graze**.**



12. In the given fig, arcs have been drawn with radii 7 cm each and with centers A, B and C. Find the area of the shaded region.



13. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 90° at its center. Find the radius of the circle**.**

14. The length of the minute hand of a clock is 7 cm. Find the area swept by the minute hand during the time period 6:05 A.M and 6:40 A.M.

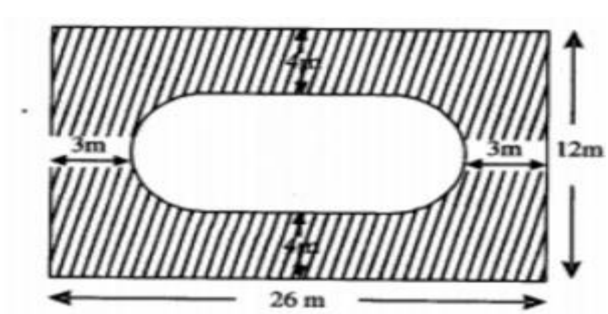
15. Find the area of the sector of a circle of radius 7 cm, if the corresponding arc length is 4 cm.

16. A circular pond is of diameter 20 m. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of Rs.30 per m^2.

17. Find the number of revolutions made by a circular tyre of area 1.32 m^2 in rolling a distance of 165 m.

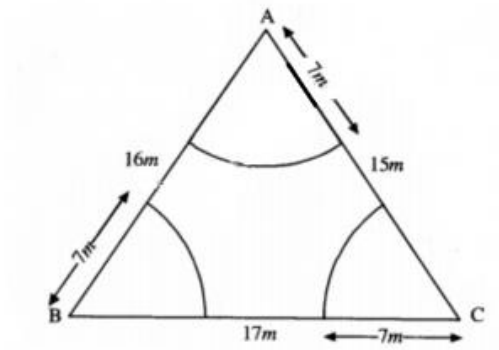
18. Find the difference of the areas of sectors of angle 90° and its corresponding major sector of a circle of radius 14 cm.

19. Find the area of the shaded region in the give figure.

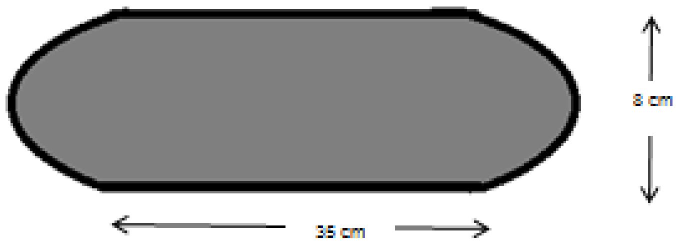


20. The diameter of front and rear wheels of a tractor 70 cm and 2 m respectively. Find the number of rotations that rear wheel will make in covering a distance in which the wheel makes 1200 rotations.

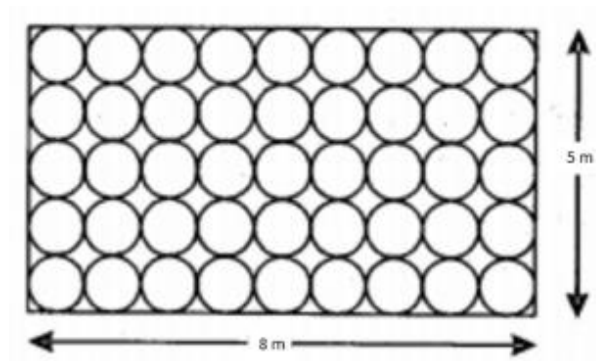
21. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.



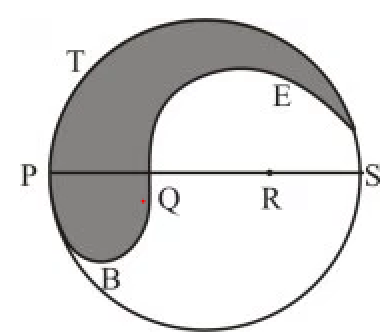
22. Find the area of the flower bed with semi- circular ends as shown in the figure.



23. Floor of a room is of dimension 8 m × 5 m and it is covered with circular tiles of diameters 40 cm each as shown in figure. Find the area of floor that remains uncovered with tiles.



23. PQRS is a diameter of a circle of radius 6cm. The lengths PQ, QR and RS are equal. Semi- circles are drawn on PQ and QS as diameters as shown in figure. Find the perimeter and area of shaded region.



SURFACE AREAS AND VOLUMES

1. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of ₹10 per dm^2.
2. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid (Use $\pi = 22/7$)
3. The curved surface area of a cylinder is 2640 sq.cm and its volume is 26400 cu.cm. Find the curved surface area of a right circular cone which has the same base and height as of the cylinder.

4. Water is flowing at 7m/s through a circular pipe of internal diameter 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Find the increase in water level in 30 minutes.

5. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the radius of the hemispherical ends is 18cm, find the cost of polishing the surface at 70p/ sq.cm.

6. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone is 6 cm and its height is 4 cm. Find the surface area of the toy. (Take $\pi = 3.14$)

7. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

8. From a solid cuboid 18 cm x 12 cm x 9 cm a hemisphere of radius 2.1cm is carved out. Find the whole surface area of the remaining solid.

9. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 10.5 cm, find the volume of water left in the tub.

10. A metal cube of edge 12 cm is melted and formed into 3 smaller cubes. If the edges of two smaller cubes are 6cm and 8cm, find the edge of the third smaller cube.

11. How many spherical lead shots each 4.2cm in diameter can be obtained from a rectangular solid of lead with dimensions 66 cm, 42 cm and 21 cm.

12. A well of diameter 7m is dug 15m deep and the earth dug out is used to form an embankment 7m wide around it. Find the height of the embankment.

13. A 12m deep well, with diameter 3.5 m is dug and the earth from it is spread evenly to form a platform 10.5m x 8.8m. Determine the height of the platform.
14. A cylindrical tub of radius 5cm and length 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed into the tub. If the radius of the hemisphere is 3.5 cm and the height of the cone is 5cm, find the volume of water left in the tub.

STATISTICS

1. The following are the ages of the students in a small community centre: 12, 14, 16, 18, 20, 22, 24, 26, 28, 30.
   (a) Calculate the mean age of the students.
   (b) If one new student joins with an age of 25, how will the mean age change? Calculate the new mean.

2. Find the mean from the following frequency distribution of marks at a test in statistics:

Marks (x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

3. The percentage of marks obtained by 100 students in an examination are given below:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	14	16	18	23	18	8	3

Determine the median percentage of marks.

4. Find the median of the given data

Marks	0-20	20-40	40-60	60-80	80-100
Frequency	5	15	30	8	2

5. Find the missing frequencies x and y if mean of 50 observations is 38.2

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	x	10	Y	8	5

6. The following table shows the number of students in different age groups:

Age Group	Number of Students (f)
10 - 15	8
16 - 20	15
21 - 25	10
26 - 30	5
31 - 35	2

Identify the mean, median and mode of the data. Verify the three values using the empirical formula.

7. Construct the frequency distribution table for the given data. Also find the median height.

Marks obtained	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than60
No. of students	14	22	37	58	67	75

PROBABILITY

1. In a bag, there are 4 red, 5 green, and 6 blue marbles. One marble is drawn at random. Find the probability that
   (a) the marble is red.
   (b) The marble is neither red nor green.

2. Two dice are rolled. What is the probability that:
   (a) The sum of the numbers on the dice is 7.
   (b) The sum of the numbers on the dice is even.
   (c) One die shows a 4.

3. A coin is tossed 100 times, and it lands on heads 56 times. What is the experimental probability of getting heads?

4. A jar contains 5 red balls, 7 green balls, and 8 blue balls. If a ball is picked at random, what is the theoretical probability of picking a green ball?

5. A card is drawn from a standard deck of 52 cards. What is the probability of drawing:

(a) A card that is a multiple of 3 (i.e., 3, 6, 9, or Queen).
(b) A card that is not a spade.

6. Two dice are rolled. What is the probability that:

(a) The sum of the numbers on the dice is 11.
(b) The product of the numbers on the dice is even.

7. Two cards are drawn one after the other from a well-shuffled deck of cards without replacement. What is the probability that:

(a) Both cards are aces.
(b) The first card is a heart and the second card is a diamond.

8. A fair die is rolled twice. What is the probability that the sum of the two rolls is less than or equal to 7?

DIRECTION: In the following questions, a statement of assertion (A)is followed by a statement of Reason (R). Choose the correct option

(a) Both assertion (A) and reason (R) are true and assertion reason R is the correct explanation of assertion A.

(b) Both assertion A and reason R are true but reason R is not the correct explanation of assertion A

(c) Assertion A is true but reason R is false.

(d) Assertion A is false but reason R is true.

(i) ASSERTION: 5 is an example of a rational number.

REASON: The square root of all positive integers is irrational numbers.

(ii) ASSERTION: For no value of n, where n is a natural number, the number 6^n ends with the digit zero.

REASON: For a number to end with digit zero, its prime factors should have 2 and 5.

(iii) ASSERTION: $3 \times 5 \times 7 + 7$ is a composite number.

REASON: A composite number has factors one, itself and any other natural number.

(iv) ASSERTION: A quadratic polynomial having $\frac{1}{2}$ and $\frac{1}{3}$ as its zeroes is $6x^2 - 5x + 1$

REASON: Quadratic polynomial having α and β as zeroes are given by $f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$ where k is a non-zero constant.

(v) ASSERTION: The system of equations $3x + 2y = 5$ and $6x + 4y = 10$ has infinitely many solutions.
REASON: If the two lines coincide, the system of equations has infinitely many solutions.

(vi) ASSERTION: The pair of equations $5x - y = 1$ and $10x - 2y = 2$ represents two parallel lines.
REASON: Two lines are parallel if their slopes are equal.

(vii) ASSERTION: The sum of the first n natural numbers is an arithmetic progression.
REASON: An arithmetic progression is a sequence of numbers in which the difference of any two successive members is constant.

(viii) ASSERTION: If the sum of the first n terms of an AP is given by $S_n = 4n^2 + 7n$, the nth term is $8n + 3$.
REASON: The nth term of an AP can be obtained by differentiating S_n.

(ix) ASSERTION: The roots of the quadratic equation $ax^2 + bx + c = 0$ are real and equal if $b^2 - 4ac = 0$.
REASON: The discriminant (Δ) determines the nature of the roots of a quadratic equation.

(x) ASSERTION: If the roots of $x^2 - 7x + 12 = 0$ are p and q, then $p + q = 7$ and $pq = 12$.
REASON: For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is $-b/a$, and the product of the roots is c/a.