

BRAIN INTERNATIONAL SCHOOL

SUBJECT : MATHEMATICS

CLASS : XI

NOV 2024

CHAPTER : SEQUENCES AND SERIES

Q1. The ratio of the A.M. and G.M. of two positive numbers a and b is $m : n$, show that:

$$a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2}).$$

Q2. If the p th, q th and r th term of a G.P. are a, b, c respectively, prove that

$$a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1.$$

Q3. Find the values of 'k' for which $-\frac{2}{7}, k, -\frac{7}{2}$ are in G.P. (geometric progression).

Q4. The sum of two numbers is '6' times their geometric mean, show that the numbers are in the ratio $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$.

Q5. Find the sum of 'n' terms of the series : $0.5 + 0.55 + 0.555 + \dots n$ terms.

Q6. In an increasing G.P., the sum of the first and last term is 66, and product of the second and last but one term is 128. If the sum of the series is 126, find the number of terms in the series.

Q7. If p, q, r are in G.P. and the equations $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

Q8. If a, b, c and d are in G.P. Prove that, $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

Q9. Let S be the sum, P the product and R the reciprocals of n terms in a G.P. Prove that $P^2 R^n = S^n$.

Q10. Find the natural number a for which $\sum_{k=1}^n f(a+k) = 16(2^n - 1)$, where the function f satisfies $f(x+y) = f(x) \cdot f(y)$ for all natural numbers x, y and further $f(1) = 2$.

Q11. If the p th and q th term of a G.P. are q and p respectively, show that its $(p+q)$ th term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.

Q12. If $a_1, a_2, a_3, \dots, a_n$ are in A.P., where $a_i > 0$ for all i , show that

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}.$$