BRAIN INTERNATIONAL SCHOOL

SESSION 2024-25

CLASS : XII **TERM 1 REVISION SHEET** **SUBJECT : MATHEMATICS**

INVERSE TRIGONOMETRIC FUNCTIONS

Q1. Show that $sin^{-1}\left(\sqrt{\frac{a-x}{2a}}\right) = \frac{1}{2}\ cos^{-1}\frac{x}{a}$.

Q2. Write the principle value :

(i) $cosec^{-1}(2)$

(ii) $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(iii) $tan^{-1}(-\sqrt{3})$

(iv) $tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Q3. Write the value in

(i) $cosec^{-1}(\sqrt{2}) + sec^{-1}(\sqrt{2})$

(ii) $cos^{-1}\left(cos\ \frac{2\pi}{3}\right) + sin^{-1}\left(cos\ \frac{2\pi}{3}\right)$

(iii) $tan^{-1}(\sqrt{3}) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Q4. What is the domain of the function $cosec^{-1}x$?

Q5. Write one branch of $tan^{-1}x$ other than the principle branch.

Q6. Evaluate in

(i) $sin^{-1}\left\{\cos\left(sin^{-1}\frac{3}{2}\right)\right\}$

(ii) $cosec^{-1}\left\{\text{cosec}\left(-\frac{\pi}{4}\right)\right\}$

(iii) $\cos\left\{\frac{\pi}{3} - cos^{-1}\left(\frac{1}{2}\right)\right\}$

(iv) $sec^2\ (tan^{-1}\ 2)$

(v) $cos^{-1}\left(\cos\frac{5\pi}{3}\right)$

(vi) $sec^{-1}\left(\frac{x-3}{x+3}\right)+sin^{-1}\left(\frac{x+3}{x-3}\right)$

(vii) $tan^{-1}\{\cos\pi\}$

Q7. Prove that in

(i) $2\,tan^{-1}x = sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(ii) $2\,cos^{-1}x = sec^{-1}\left(\frac{1}{2x^2-1}\right)$

(iii) $sin^{-1}x = cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

(iv) $cos^{-1}x = 2\,cos^{-1}\sqrt{\frac{1+x}{2}}$

Q8. Find the value of $cosec\left(cot^{-1}\frac{y}{2}\right)$ in terms of y alone.

Q9. Prove that

(i) $2\,tan^{-1}\frac{1}{5}+tan^{-1}\frac{1}{8}=tan^{-1}\frac{4}{7}$

(ii) $tan^{-1}\frac{1}{5}+tan^{-1}\frac{1}{3}+tan^{-1}\frac{1}{7}+tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

(iii) $cos^{-1}\frac{4}{5}+cos^{-1}\frac{12}{13}=cos^{-1}\frac{33}{65}$

(iv) $cos^{-1}\frac{4}{5}+tan^{-1}\frac{3}{5}=tan^{-1}\frac{27}{11}$

(v) $sin^{-1}\frac{4}{5}+2\,tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

(vi) $tan^{-1}\left[\frac{3a^2x-x^3}{a\,(a^2-3x^2)}\right]=3\,tan^{-1}\left(\frac{x}{q}\right)$

(vii) $sec^2\,(tan^{-1}3)+cosec^2(cot^{-1}4)=27$

(viii) $sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)=\frac{\pi}{4}+sin^{-1}x, -1\le x\le 1.$

Q10. Write in the simplest form

(i) $cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$

(ii) $tan^{-1}\left(\frac{8x}{1+20x^2}\right)$

(iii) $cot^{-1}\sqrt{\frac{1+\cos 5x}{1-\cos 5x}}$

(iv) $sin^{-1}\left(x^2\sqrt{1-x^2}+x\sqrt{1-x^4}\right)$

MATRICES

Q1. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y.

Q2. Construct a 3 × 2 matrix A, if $A = [a_{ij}], where\ a_{ij} = \begin{Bmatrix} i+j, & if\ i \geq j \\ i+j, & if\ i < j \end{Bmatrix}$.

Q3. If matrix $A = [1\ 2\ 3]$, write matrix AA' where A' is transpose of matrix A.

Q4. Find the number of all possible matrices of order 2 × 2 with each entry 1 or 2.

Q5. If $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ $and\ B = [b_{ij}]$ $\begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$, then find $3a_{12} - 5b_{21}$.

Q6. Write a square matrix of order 2 which is both symmetric and skew symmetric.

Q7. If $\begin{bmatrix} 2x-1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ x+y \end{bmatrix}$, find x and y.

Q8. Simplify, $sin\ \theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} + cos\ \theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} - diag\ [1, -1]$.

Q9. If $X_{m\times 3}Y_{p\times 4} = Z_{2\times b}$, find the values of m, p and b.

Q10. For what value of k, the matrix $\begin{bmatrix} 0 & -1 & k \\ 1 & 0 & 5 \\ 4 & -5 & 0 \end{bmatrix}$ is skew symmetric?

DETERMINANTS

Q1. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$, find the value of x.

Q2. Evaluate $\begin{vmatrix} 1 & 0 & 0 \\ 2 & cos\ x & \sin x \\ 3 & -\sin x & cos\ x \end{vmatrix}$.

Q3. Find the minor of a_{12} in the following determinant : $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

Q4. Evaluate $\begin{vmatrix} \sec 35^0 & \tan 35^0 \\ \cot 55^0 & \text{cosec}\, 55^0 \end{vmatrix}$.

Q5. For what value of k, the matric $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ is invertible?

Q6. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 23 & 33 & 44 \end{vmatrix}$.

Q7. If $A = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$, then find the value of k if $|2A| = k\ |A|$.

Q8. Given a non-zero real number k and a determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}, then\ k\Delta = \begin{vmatrix} ka & b \\ c & kd \end{vmatrix}$. State true or false. If false give one determinant which can be true.

Q9. Write $|A^{-1}|$ for the matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

Q10. Area of a triangle with vertices $(k, 0), (1, 1)\ and\ (0, 3)$ is 5 sq units. Find the value (s) of k.

CONTINUITY AND DIFFERENTIABILITY

Q1. Differentiation each of the following with respect to x

(i) $\sin \log\ x$

(ii) $cos^{-1} \sqrt{x}$

(iii) $\log_a (\sin x)$

(iv) $e^{sin^{-1} x}$

(v) $\sin\ [\log\ (x^2 - 1)]$

Q2. Differentiate $\cos x$ with respect to e^x.

Q3. If $= sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right), find\ \frac{dy}{dx}$.

Q4. Given $f(0) = -2, f(0) = 3$. Find $h\ '(0)$, where $h(x) = xf(x)$.

Q5. Find $\frac{dy}{dx}$ at (4, 9), when $\sqrt{x} + \sqrt{y} = 5$.

Q6. Find the second derivative of $\log\ x$ with respect to x

Q7. Show that the function $f(x) = \frac{\sin x}{|x|}$ is discontinuous at $x = 0$.

Q8. Show that the function $f(x) = x - |x|$ is continuous at $x = 0$.

Q9. Let f be the function defined as $(x) = \begin{cases} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}, & if\ x \neq 0 \\ 3k, & if\ x = 0 \end{cases}, a > 0$. For what value of k, function is continuous at $x = 0$?

Q10. A function f is defined as $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , if\ x < 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , if\ x > 0 \end{cases}$. Is the function continuous at $x = 0$? If not how should the function be defined at $x = 0$, so that the function is continuous at $x = 0$?

APPLICATION OF DERIVATIVES

Q1. Find the rate of change of volume of a cone of constant height with respect to radius of the base.

Q2. The side of an equilateral triangle is increasing at the rate of 0.5 cm/s. Find the rate of increase of its area when side is $4\sqrt{3}\ cm$.

Q3. Show that the function $f(x) = (3x+5)^3$ is increasing in R.

Q4. Show that the function $f(x) = \log(\cos x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$.

Q5. Show that the function $f(x) = \log_e x$ is an increasing function for $x > 0$.

Q6. At what point on the curve $y = x^2$ does the normal make an angle of 30^0 clockwise with the x-axis?

Q7. Find the point (s) on the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where tangent is parallel to the y-axis.

Q8. The surface area of a spherical bubble is increasing at the rate of $2\ cm^2/s$. Find the rate at which the volume is increasing at the instant if its radius is 6 cm.

Q9. A particle moves along a straight line in such a way that its distance from fixed origin is the square root of the quadratic function of time. Prove that the acceleration varies inversely as the cube of the distance.

Q10. What will be the height of a variable cone when its volume and radius are changing at the rate of $100\ cm^2/s$ and $20\ cm/s$ respectively and its radius is always 5 times of its height?

INTEGRALS

Q1. Evaluate each of the following integrals in

(i) $\int \frac{x^2}{1+x^3}\, dx$

(ii) $\int \frac{2\cos x}{3\, sin^2 x}\, dx$

(iii) $\int \frac{x+\cos 6x}{3x^2+\sin 6x}\, dx$

(iv) $\int \frac{sec^2 \sqrt{x}}{\sqrt{x}}\, dx$

(v) $\int \frac{1}{\sqrt{ax+b} - \sqrt{ax-d}} dx, b \neq -d$

(vi) $\int \sec x \log(\sec x + \tan x)\, dx$

(vii) $\int \frac{2+3x}{3-2x}\, dx$

(viii) $\int \sqrt{\frac{a+x}{x}}\, dx$

(ix) $\int \frac{x-\sin x}{1-\cos x}\, dx$

(x) $\int \frac{1}{\sin x\,(2+\cos x)} dx$

(xi) $\int \frac{1}{(2x+3)\,\sqrt{x+1}} dx$

(xii) $\int \frac{1}{(x+1)\,\sqrt{x^2-1}} dx$

(xiii) $\int \frac{\sin 4x}{(2+\sin 2x)^2} dx$

(xiv) $\int cot^3 x\, dx$

(xv) $\int \frac{\sin x}{sin^2 x - 3\cos x - 1} dx$

(xvi) $\int \frac{1}{(\cos x + 2\sin x)^{\,2}} dx$

(xvii) $\int \frac{1}{(x+1)\,\sqrt{2x-3}} dx$

(xviii) $\int x\,\sqrt{\frac{1+x}{1-x}} dx$

(xix) $\int \frac{1}{1-cos^4 x} dx$

(xx) $\int_{-2}^{2} f(x)\, dx,\ \ where\ f(x) = \begin{cases} 2x-1, & -2 \le x \le 1 \\ 3x-2, & 1 \le x \le 2 \end{cases}$

(xxi) $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\cos x}\, dx$

(xxii) $\int_0^{\pi} |\cos x|\, dx$

(xxiii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \cos^2 x\ \, dx$

(xxiv) $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} [\sin|x| - \cos|x|]\, dx$

(xxv) $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{sin^4 x + cos^4 x}\, dx$

(xxvi) $\int_{0}^{\frac{\pi}{2}} \frac{1}{a \sin x + b \cos x}\, dx, \quad a,\ b > 0$

(xxvii) $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \log \left|\frac{2-\sin x}{2+\sin x}\right| dx$

(xxviii) $\int_{0}^{1} \frac{\log x}{\sqrt{1-x^2}}\, dx$

(xxix) $\int_{0}^{\frac{\pi}{2}} (\sin x - \cos x) \log(\sin x + \cos x)\, dx$

(xxx) $\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{3\, cos\ x + sin\ x}\, dx$