

## BRAIN INTERNATIONAL SCHOOL

**SUBJECT : MATHEMATICS**

**CLASS : XII**

**JULY 2024**

### **CHAPTER : APPLICATIONS OF DERIVATIVES**

**Q1.** Find the rate of change of volume of a cone of constant height with respect to radius of the base.

**Q2.** The side of an equilateral triangle is increasing at the rate of 0.5 cm/s. Find the rate of increase of its area when side is  $4\sqrt{3}$  cm.

**Q3.** Show that the function  $f(x) = (3x + 5)^3$  is increasing in R.

**Q4.** Show that the function  $f(x) = \log(\cos x)$  is decreasing in  $(0, \frac{\pi}{2})$ .

**Q5.** Show that the function  $f(x) = \log_e x$  is an increasing function for  $x > 0$ .

**Q6.** The surface area of a spherical bubble is increasing at the rate of  $2 \text{ cm}^2/\text{s}$ . Find the rate at which the volume is increasing at the instant if its radius is 6 cm.

**Q7.** A particle moves along a straight line in such a way that its distance from fixed origin is the square root of the quadratic function of time. Prove that the acceleration varies inversely as the cube of the distance.

**Q8.** What will be the height of a variable cone when its volume and radius are changing at the rate of  $100 \text{ cm}^2/\text{s}$  and  $20 \text{ cm}/\text{s}$  respectively and its radius is always 5 times of its height?

**Q9.** A particle moves in a straight line according to the formula  $s = t^3 - 6t^2 - 15t$ , where s represents the distance in meters and t represents the time in seconds. Find the time interval during which the speed of the particle decreases. ( $\text{speed} = \frac{ds}{dt}$ ).

**Q10.** Find the interval in which the function  $f(x) = 2x^3 - 15x^2 + 36x + 17$  is strictly increasing or strictly decreasing.

**Q11.** Prove that  $\frac{x}{1+x} < \log(1+x) < x$  for all  $x > 0$ .

**Q12.** Find the interval in which the function  $f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$ , is strictly increasing or strictly decreasing in  $(0, 2\pi)$ .

**Q13.** Show that function  $f(x) = x^2$  is strictly increasing function in  $(0, \infty)$  without using derivatives.

**Q14.** Find whether the function  $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ ,  $\frac{3\pi}{8} < x < \frac{7\pi}{8}$  is increasing or decreasing.

**Q15.** A rectangle of the greatest area is inscribed in ellipse  $x^2 + 4y^2 = 16$ . Find the dimensions of the rectangle.

**Q16.** The combined resistance  $R$  of two resistors  $R_1$  and  $R_2$  ( $R_1, R_2 > 0$ ) is given by  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ . If  $R_1 + R_2 = C$ ,  $C$  is a constant, find  $R_1$  and  $R_2$  so that  $R$  is maximum.

**Q17.** Show that the height  $h$  of a right circular cylinder of maximum total surface area including the two ends that can be inscribed in a sphere of radius  $r$  is given by  $h^2 = 2r^2 \left(1 - \frac{1}{\sqrt{5}}\right)$ .